

2006**MATHEMATICAL METHODS****Written examination 1****STUDENT NAME:****QUESTION AND ANSWER BOOK****Reading time: 15 minutes****Writing time: 1 hour****Structure of book**

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring notes of any kind, sheets of paper, white out liquid/tape or a calculator into the examination.

Materials provided

- The question and answer book of 9 pages, with a separate sheet of miscellaneous formulas.
- Working space is provided throughout the question book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

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Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

- a. Use the factor theorem to show that $(x + 2)$ is a factor of $9x^3 + 12x^2 - 11x + 2$.

- b. The equation $y = 9x^3 + 12x^2 - 11x + 2$ can be written in the form $y = (x + 2)(ax - b)^2$ where $\{a, b\} > 0$. State the values of a and b .

2 + 2 = 4 marks

Question 2

- a. The graph of a function g is obtained from the graph of the function f which has the rule $f(x) = 2(x - 2)^5$ by a performing a translation of -4 units parallel to the x -axis. Write down the rule for g .

- b. The graph of a function h is obtained from the graph of g by a reflection in the y -axis. Write down the rule for h .

- c. The graph of a function k is obtained from the graph of h by a dilation by a scale factor of $\frac{1}{2}$ along the y -axis. Write down the rule for k .

1 + 1 + 1 = 3 marks

Question 3

Solve the equation $\sqrt{3} \sin(2x) + \cos(2x) = 0$ for $x \in [0, 2\pi]$, giving exact values in terms of π .

3 marks

Question 4

Let $f(x) = x^2$ and $g(x) = 3x - 5$.

- a. Write down the rule of $f(g(x))$.

- b. Find the derivative of $f(g(x))$.

QUESTION 4 – continued
TURN OVER

- c. Hence, find the coordinates of the point P on the curve with the equation $y = f(g(x))$ at which the tangent is parallel to the line $2y - 12x = 7$.

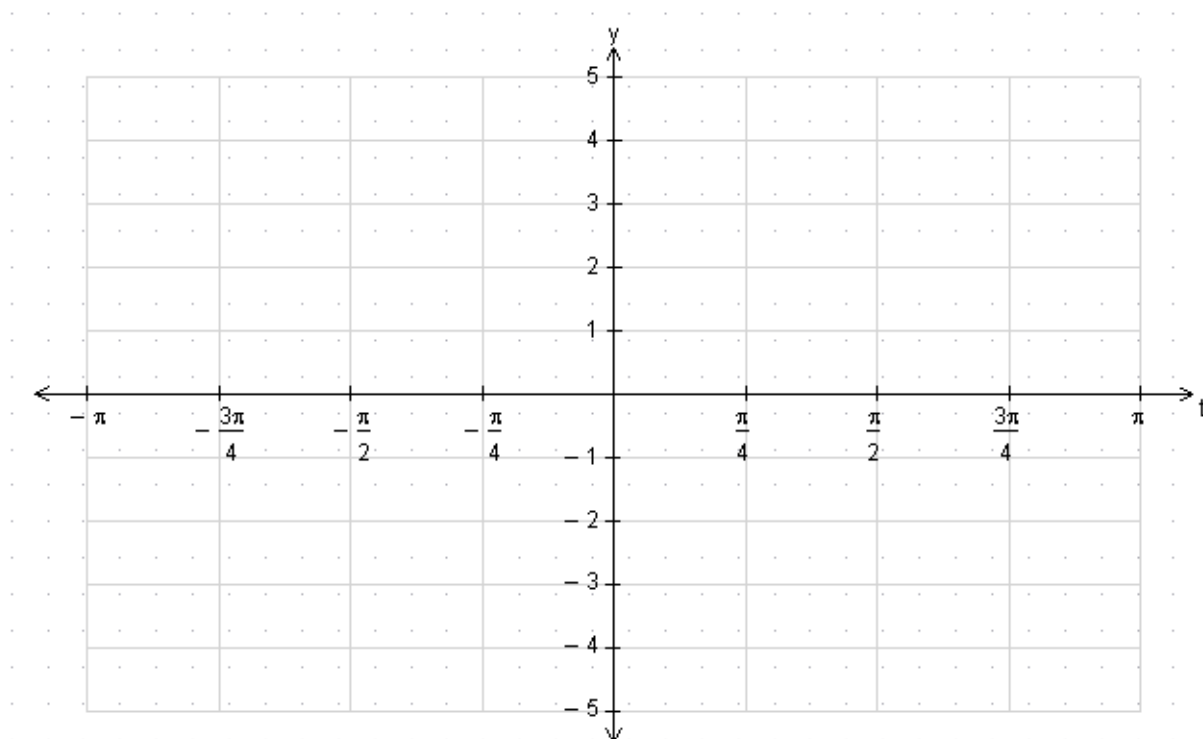
1 + 1 + 2 = 4 marks

Question 5

For the function $f : [-\pi, \pi] \rightarrow \mathbb{R}$, $f(x) = -2\cos(2x + \frac{\pi}{2})$

- a. Write down the period of the function.

- b. On the set of axes below, sketch the graph of the function f .

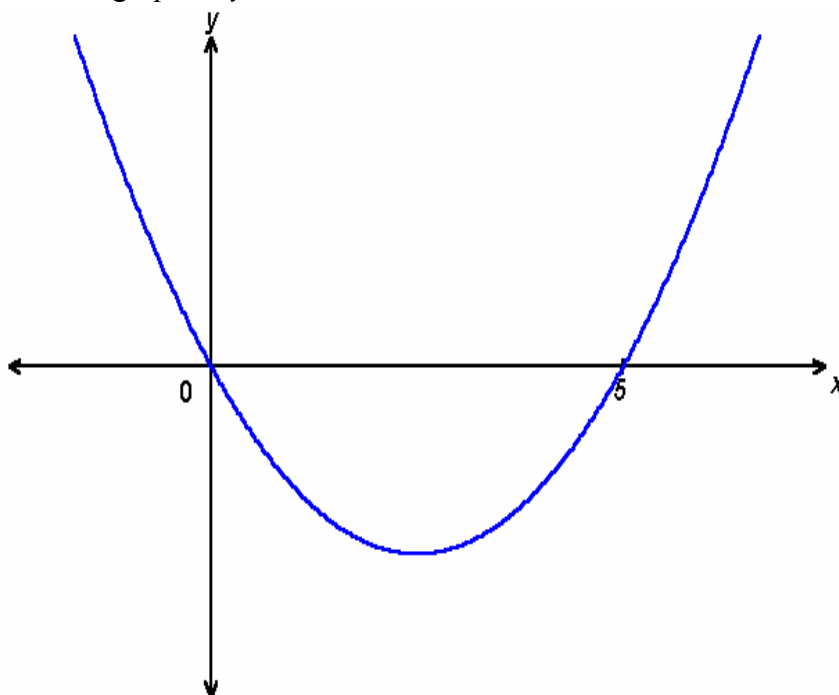


- c. State the number of solutions to the equation $\cos(2t + \frac{\pi}{2}) = \frac{1}{2}$, where $-\pi \leq x \leq \pi$.

1 + 2 + 1 = 4 marks

Question 6

Part of the graph of $y = x^2 - 5x$ is shown below.



- a. On the same set of axes sketch the graph of $y = |x^2 - 5x|$.
- b. Find the set of values of x for which $|x^2 - 5x| \geq 6$.

1 + 2 = 3 marks

Question 7

For the function $f(x) = 2e^{1-x}$,

- a. find the rule of the inverse function f^{-1} .

- b. find the domain of the inverse function f^{-1} .

2 + 1 = 3 marks

Question 8

The random variable X has the following probability distribution.

x	-1	0	1	2
$Pr(X = x)$	a	2a	3a	0.4

- a. Find the value of a .

- b. If $Pr(X \leq k) > 0.5$, find the minimum value of k .

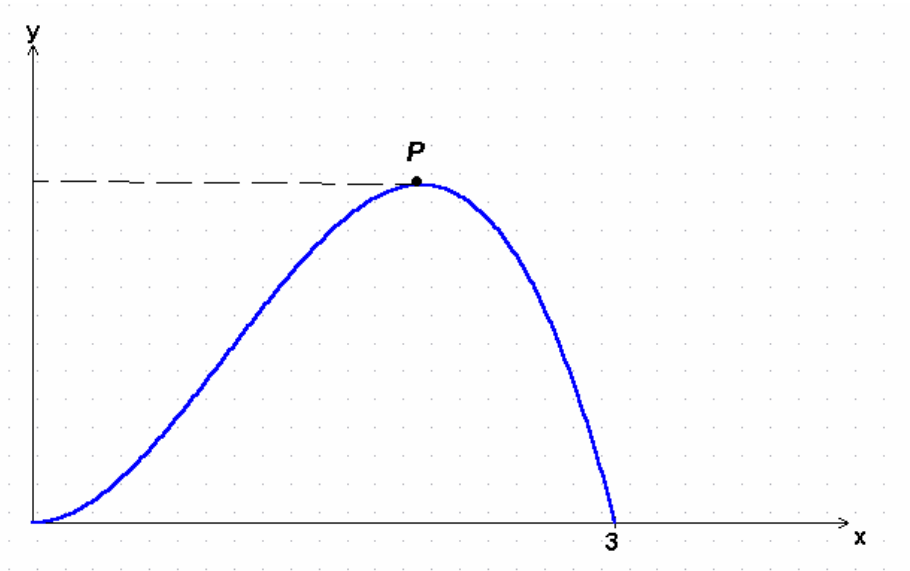
1 + 1 = 2 marks

Question 9

A continuous random variable X has the probability density function given by

$$f(x) = \frac{4}{27}(3x^2 - x^3), \quad 0 \leq x \leq 3$$

The graph of f , as shown below, has a maximum point at P .



- a. Find the value of the x -coordinate of P .

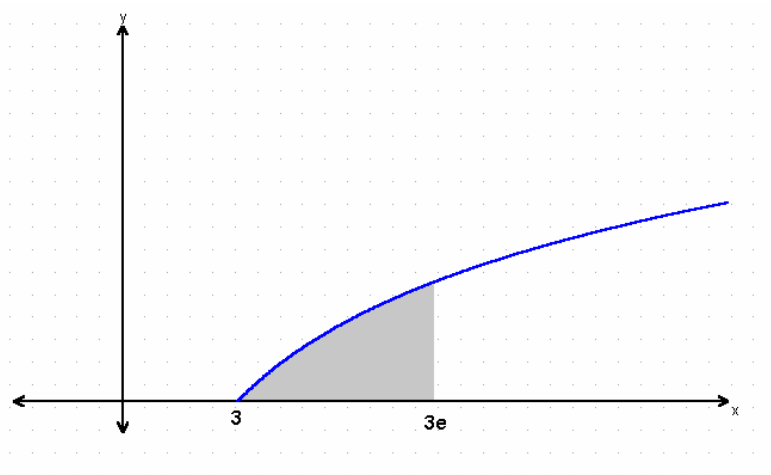
- b. Find the $\Pr(0 < X < 2)$.

- c. Find the mean value of X .

3 + 3 + 3 = 9 marks

Question 10

The graph of the function $f : [3, \infty) \rightarrow \mathbb{R}$, $f(x) = \log_e \frac{x}{3}$ is shown below.



- a. If $y = x \log_e \frac{x}{3} - x$, find $\frac{dy}{dx}$.

- b. Hence, find the exact area of the shaded region.

2 + 3 = 5 marks

END OF QUESTION AND ANSWER BOOK