



Units 3 and 4 Maths Methods (CAS): Exam 1

Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

Question 1a

$$\frac{dy}{dx} = 3x^2 e^{-3x} - 3x^3 e^{-3x}$$

1 mark for using product rule

$$= x^2 e^{-3x} (3 - 3x)$$

1 mark-answer

Question 1b

$$\frac{dy}{dx} = \frac{3x^2 + 8x}{(3x+4)^2}$$

1 mark for correct dy/dx

$$\text{at } x=-1, \frac{dy}{dx} = -5$$

1 mark-answer

Question 2a

$$\frac{3x+2}{x+1} = \frac{3x+3-1}{x+1} = 3 - \frac{1}{x+1}$$

or use long division 1 mark

$$a = 3, b = -1$$

1 mark-answer

Question 2a

$$\int \frac{3x+2}{x+1} dx = \int 3 - \frac{1}{x+1} dx = \int 3 dx - \int \frac{1}{x+1} dx = 3x - \ln(|x+1|)$$

1 mark for $3x$ and 1 mark for $-\ln(|x+1|)$ **Question 3**

$$2 \cos\left(3x + \frac{\pi}{6}\right) = -\sqrt{3}$$

$$\Rightarrow \cos\left(3x + \frac{\pi}{6}\right) = \frac{-\sqrt{3}}{2}$$

$$\Rightarrow 3x + \frac{\pi}{6} = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}$$

1 mark

$$\Rightarrow 3x = \frac{2\pi}{3}, \pi, \frac{8\pi}{3}, 3\pi$$

$$x = \frac{2\pi}{9}, \frac{\pi}{3}, \frac{8\pi}{9}, \pi$$

1 mark-(3 correct answers) 2 marks-(4 correct answers)

Question 4a

Full proof required.

$$\text{Need, } p^2 + 1.6p + 0.2 = 1$$

1 mark

$$\Rightarrow (p - 0.4)(p + 2) = 0$$

1 mark

$$\Rightarrow p = 0.4 \text{ or } p = -2$$

$$\text{as } 0 \leq p \leq 1, \Rightarrow p = 0.4$$

1 mark

Question 4b

$$E(X) = 0.2 \times 0 + 0.4 \times 1 + 0.24 \times 2 + 0.16 \times 3$$

1 mark

$$= 1.36$$

1 mark-answer

Question 4c

$$\Pr = (0.2 + 0.4)^2 = 0.36$$

1 mark-answer

Question 5a

$$x = 3 - e^{\frac{y-2}{3}}$$

1 mark

$$3 - x = e^{\frac{y-2}{3}}$$

$$y - 2 = \ln(3 - x)$$

1 mark

$$y = 3 \ln(3 - x) + 2$$

$$\Rightarrow f^{-1}(x) = 3 \ln(3 - x) + 2$$

1 mark

Question 5b

$$\text{Domain } f^{-1}(x): (-\infty, 3)$$

1 mark

Question 6

$$x^3 - 13x + 12 = 0$$

$$(1)^3 - 13(1) + 12 = 1 - 12 + 13 = 0$$

$$\text{use long division or another technique to get } \frac{x^3 - 13x + 12}{(x - 1)} = x^2 + x - 12$$

$$\Rightarrow (x - 1)(x^2 + x - 12) = 0$$

$$\Rightarrow (x - 1)(x + 4)(x - 3) = 0$$

$$\Rightarrow x = -4, 1, 3$$

1 mark for each correct solution

Question 7

$$\ln(x + 4) - 2 \ln(x + 1) + \ln(x - 1) = \ln\left(\frac{(x+4)(x-1)}{(x+1)^2}\right)$$

1 mark

$$\Rightarrow \ln\left(\frac{(x+4)(x-1)}{(x+1)^2}\right) = 0$$

$$\Rightarrow \frac{(x+4)(x-1)}{(x+1)^2} = 1$$

1 mark

$$\Rightarrow (x + 4)(x - 1) = (x + 1)^2$$

$$\Rightarrow x^2 + 3x - 4 = x^2 + 2x + 1$$

$$\Rightarrow 3x - 4 = 2x + 1$$

$$\Rightarrow x = 5$$

1 mark

Question 8

$$\sin(x) = \sqrt{3} \cos(x)$$

$$\Rightarrow \tan(x) = \sqrt{3}$$

$$\Rightarrow x = \frac{\pi}{3}, \frac{4\pi}{3} \quad 1 \text{ mark}$$

$$Area = \int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} \sin(x) - \sqrt{3} \cos(x) dx \quad 1 \text{ mark}$$

$$Area = [-\cos(x) - \sqrt{3}\sin(x)]_{x=\frac{\pi}{3}}^{x=\frac{4\pi}{3}} \quad 1 \text{ mark}$$

$$Area = \left(\frac{1}{2} + \frac{3}{2}\right) - \left(-\frac{1}{2} - \frac{3}{2}\right)$$

$$Area = 4 \quad 2 \text{ marks}$$

Question 9

Full proof required.

$$gradient = -\frac{1}{(-\frac{1}{3})} = 3$$

$$\frac{dy}{dx} = -2(1-x) \quad 1 \text{ mark}$$

$$gradient = \frac{dy}{dx} \Rightarrow 3 = -2(1-x) \quad 1 \text{ mark}$$

$$\Rightarrow x = \frac{5}{2} \Rightarrow y = \left(1 - \frac{5}{2}\right)^2 = \frac{9}{4} \quad 1 \text{ mark}$$

$$\frac{9}{4} = -\frac{1}{3}\left(\frac{5}{2}\right) + c \Rightarrow c = \frac{37}{12} \quad 1 \text{ mark}$$

Question 10a

Full Proof required

$$\int_{-6}^6 k \left(1 - \left|\frac{x}{6}\right|\right) dx = 1 \quad 1 \text{ mark}$$

$$\int_{-6}^6 k \left(1 - \left|\frac{x}{6}\right|\right) dx = k \int_0^{-6} \left(1 + \frac{x}{6}\right) dx + k \int_0^6 \left(1 - \frac{x}{6}\right) dx = -k(-6 + 3) + k(6 - 3)$$

$$\Rightarrow 3k + 3k = 1$$

$$\Rightarrow k = \frac{1}{6} \quad 1 \text{ mark}$$

Question 10b

$$\frac{3}{4} = \frac{1}{6} \int_{-q}^q \left(1 - \left|\frac{x}{6}\right|\right) dx$$

$$\frac{1}{6} \int_{-q}^q \left(1 - \left|\frac{x}{6}\right|\right) dx = \frac{1}{3} \int_0^q \left(1 - \frac{x}{6}\right) dx = \frac{1}{3} \left[x - \frac{x^2}{12} \right]_{x=0}^{x=q} \quad 1 \text{ mark}$$

$$\Rightarrow \frac{1}{3} \left(q - \frac{q^2}{12} \right) = \frac{3}{4}$$

$$\Rightarrow 12q - q^2 = 27$$

$$\Rightarrow (q - 9)(q - 3) = 0$$

$$\text{but } 0 \leq q \leq 6, \text{ so } q = 3 \quad 1 \text{ mark}$$