



Units 3 and 4 Maths Methods (CAS): Exam 1

Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

Question 1a

$$\frac{dy}{dx} = \ln(x) + 1 \quad [1]$$

Question 1b

$$\frac{dy}{dx} = 2(3x^2 - 2)(x^3 - 2x) \quad [1]$$

$$\text{At } x = 2, \frac{dy}{dx} = 80 \quad [1]$$

Question 2

$$\int \cos(-2x + 3)dx = -\frac{1}{2}\sin(-2x + 3) + c \quad [1]$$

$$\text{One such antiderivative is } -\frac{1}{2}\sin(-2x + 3) + 1 \quad [1]$$

Question 3a

The inverse of $g(x)$ can be found by swapping x and y :

$$x = 2e^{3y} + 1 \quad [1]$$

$$e^{3y} = \frac{x - 1}{2}$$

$$y = \frac{1}{3}\ln\frac{x-1}{2} \quad [1]$$

Question 3b

$$g(g^{-1}(x)) = 2e^{3(\frac{1}{3}\ln\frac{x-1}{2})} + 1 \quad [1]$$

$$= 2e^{\ln\frac{x-1}{2}} + 1$$

$$= x \quad [1]$$

Question 4a

$$E(X) = 0.1 + 2(0.25) + 3(0.4) = 1.8 \quad [1]$$

Question 4b

$$\text{Pr} = 0.25 \times 0.25 \times 0.25 = \frac{1}{64} \quad [1]$$

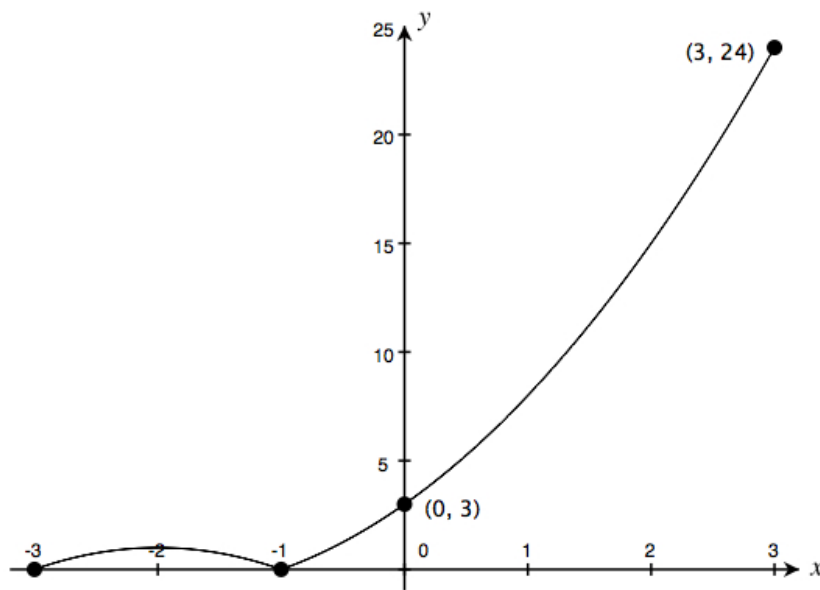
Question 4c

$\text{Pr}(3 \text{ calls over } 2 \text{ day span})$

$$= \text{Pr}(1 \text{ call then } 2 \text{ calls}) + \text{Pr}(2 \text{ calls then } 1 \text{ call}) + \text{Pr}(0 \text{ calls then } 3 \text{ calls}) + \text{Pr}(3 \text{ calls then } 0 \text{ calls}) \quad [1]$$

$$= 0.1 \times 0.25 + 0.25 \times 0.1 + 0.25 \times 0.4 + 0.4 \times 0.25 \quad [1]$$

$$= 0.25 \quad [1]$$

Question 5a

Graph should contain the following:

- End points are at $(-3, 0)$ and $(3, 24)$ [1]
- Axis-intercepts are at $(-3, 0)$, $(-1, 0)$ and $(0, 3)$ [1]
- Correct shape [1]

Question 5b

$(0, -1)$ [1]

Question 5c

The mapping is $(x, y) \rightarrow (-x, y)$ [reflection in y axis] $\rightarrow (-x, y + 2)$ [translation 2 units up]. This maps to (x', y') . So $x = -x'$ and $y = y' - 2$. So $y' - 2 = | -(-x')^2 - 4(-x') - 3 |$.

Image of $y = | -x^2 + 4x - 3 | + 2$ [1]

Question 6

$$\sin\left(2x - \frac{\pi}{2}\right) = -\frac{\sqrt{2}}{2} [1]$$

$$2x - \frac{\pi}{2} = -\frac{\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{13\pi}{4} [1]$$

$$x = \frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8} [1]$$

Question 7

$$\ln\left(\frac{(x-3)(2x-1)}{(x+1)^2}\right) = 0 \quad [1]$$

$$(x-3)(2x-1) = (x+1)^2$$

$$x^2 - 9x + 2 = 0 \quad [1]$$

Reject $x = \frac{9}{2} - \frac{\sqrt{73}}{2}$ since $x > 1$, so:

$$x = \frac{9}{2} + \frac{\sqrt{73}}{2} \quad [1]$$

Question 8a

$$\Pr(X > 161 | X > 150) = \frac{\Pr(X > 161)}{\Pr(X > 150)} \quad [1]$$

$$\text{Now } P(X > 161) = P(Z > \frac{(161-150)}{11}) = P(Z > 1) = P(Z < -1)$$

$$\text{and } P(X < 139) = P(Z < \frac{((139-150))}{11}) = P(Z < -1) = q.$$

So $P(X > 161) = q$. Since $P(X > 150) = P(Z > 0) = 0.5$,

$$\frac{\Pr(X > 161)}{\Pr(X > 150)} = \frac{q}{0.5} = 2q \quad [1]$$

Question 8b

$$\Pr(X \leq k) = \int_2^k \frac{x-2}{8} dx \quad [1]$$

$$= \frac{1}{8} \left(\frac{k^2}{2} - 2k + 2 \right) = \frac{1}{2}$$

$$k^2 - 4k - 4 = 0 \quad [1]$$

Reject $k = 2 - 2\sqrt{2}$ since $2 < k < 6$

$$k = 2 + 2\sqrt{2} \quad [1]$$

Question 9a

$$f'(x) = 2x \ln(x) + x \quad [1]$$

Question 9b

$$x \ln(x) = \frac{f'(x)-x}{2} \quad [1]$$

$$\int_1^e (x \ln x) dx = \int_1^e \frac{f'(x)-x}{2} dx = \int_1^e \left(\frac{f'(x)}{2} - \frac{x}{2} \right) dx = \left[\frac{f(x)}{2} - \frac{x^2}{4} \right]_1^e = \left[\frac{1}{2} x^2 \ln(x) - \frac{x^2}{4} \right]_1^e \quad [1]$$

$$= \frac{e^2+1}{4} \quad [1]$$

Question 10

Area below curve: first we must find x-intercepts.

$$x^3 - kx = 0$$

$$x = 0, \pm\sqrt{k} [1]$$

So the area underneath the x-axis for $x > 0$ is:

$$-\int_0^{\sqrt{k}} (x^3 - kx) dx = -\left[\frac{x^4}{4} - \frac{kx^2}{2}\right]_0^{\sqrt{k}} = \frac{k^2}{4} [1]$$

Area above the curve: first we must find m , the point of intersection

$$f(x) = g(x)$$

$$x^3 - 2kx = 0$$

$$x = 0, \pm\sqrt{2k} [1]. \text{ As } m > 0, m = \sqrt{2k}$$

To find the area above the x-axis, we find the total area between $g(x)$ and $f(x)$, then subtract the area under the x-axis as found earlier:

$$\begin{aligned} \text{Area between } g(x) \text{ and } f(x) &= \int_0^m (kx - (x^3 - kx)) dx \\ &= \int_0^m (2kx - x^3) dx \\ &= \int_0^{\sqrt{2k}} (2kx - x^3) dx \\ &= \left[kx^2 - \frac{x^4}{4} \right]_0^{\sqrt{2k}} = k(2k) - \frac{4k^2}{4} = k^2 \end{aligned}$$

Therefore the area bounded by f and g is four times the size of the area bounded by the g and the x-axis.