



# **Units 3 and 4 Maths Methods (CAS): Exam 1**

## **Practice Exam Solutions**

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email [practiceexams@ee.org.au](mailto:practiceexams@ee.org.au).

Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

### Question 1a

$$\int 4(2x - 5)^3 dx = \frac{4(2x-5)^4}{4 \times 2} + c' [1]$$

$$= \frac{1}{2} (2x - 5)^4 + c [1] \text{ (the } c \text{ is not required as asked for an anti-derivative, } c \text{ can be any value)}$$

### Question 1b

$$\frac{d}{dx} \left( \frac{2\sin(x)}{x} \right) = \frac{x \times \frac{d}{dx}(2\sin(x)) - 2\sin(x) \times \frac{d}{dx}(x)}{(x)^2} [1]$$

$$= \frac{2x \cos(x) - 2\sin(x)}{(x)^2} [1]$$

$$f'(\pi) = \frac{2(\pi) \cos(\pi) - 2\sin(\pi)}{(\pi)^2} = \frac{2(\pi)(-1)}{(\pi)^2} = -\frac{2}{\pi} [1]$$

### Question 2a

$$\mu = 15, \sigma = 4$$

$$\Pr(X > 19) = \Pr(X > \mu + \sigma)$$

$$\Pr(\mu - \sigma > X > \mu + \sigma) = 0.68$$

$$\Pr(X > 19) = \frac{1 - \Pr(\mu - \sigma > X > \mu + \sigma)}{2} = \frac{1 - 0.68}{2} = 0.16 [1]$$

### Question 2b

$$Z = \frac{x - \mu}{\sigma} = \frac{10 - 15}{4} = -\frac{5}{4}$$

$$\Pr(X < 10) = \Pr(Z < -\frac{5}{4}) [1]$$

$$\Pr(Z < -\frac{5}{4}) = \Pr(Z > \frac{5}{4}) \text{ (due to symmetry around } Z=0)$$

$$\Pr(X < 10) = \Pr(Z > \frac{5}{4}), c = \frac{5}{4} \text{ or } c = 1.25 [1]$$

### Question 3

$$\begin{bmatrix} x^1 \\ y^1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \end{bmatrix} [1]$$

$$x^1 = \frac{1}{2}x - 2 \Rightarrow x = 2(x^1 + 2)$$

$$y^1 = -3y + 1 \Rightarrow y = -\frac{1}{3}(y^1 - 1) [1]$$

$$y = \frac{1}{x} \text{ sub in for } -\frac{1}{3}(y^1 - 1) = \frac{1}{2(x^1 + 2)} [1]$$

$$f(x) = 1 - \frac{3}{2(x^1 + 2)} [1]$$

### Question 4

$$\log_e(x^2) - \log_e(x + 4) = \log_e(2)$$

$$\log_e\left(\frac{x^2}{x+4}\right) = \log_e(2) [1]$$

$$\frac{x^2}{x+4} = 2 [1]$$

$$x^2 - 2x - 8 = 0$$

$$(x - 4)(x + 2) = 0$$

$$\Rightarrow x = 4, x = -2 [1]$$

Since  $x$  must be positive, the answer is:

$$x = 4 [1]$$

**Question 5**

$$\Pr(\text{different colours} \mid \text{first blue}) = \frac{\Pr(\text{different colours} \cap \text{first blue})}{\Pr(\text{first blue})} [1]$$

$$\Pr(\text{different colours} \cap \text{first blue}) = \Pr(BR) + \Pr(BG) = \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{6} = \frac{2}{9} [1]$$

$$\Pr(\text{first blue}) = \frac{1}{3}$$

$$\Pr(\text{different colours} \mid \text{first blue}) = \frac{2/9}{1/3} = \frac{2}{3} [1]$$

**Question 6a**

$$\frac{\sin(2x)}{\cos(2x)} = \sqrt{3} \frac{\cos(2x)}{\cos(2x)} [1]$$

$$\frac{\sin(2x)}{\cos(2x)} = \sqrt{3}$$

$$\tan(2x) = \sqrt{3}$$

$$2x = -\frac{2\pi}{3}, \frac{\pi}{3}$$

$$x = -\frac{\pi}{3}, \frac{\pi}{6} [1]$$

**Question 6b**

$$\text{Range of } f(x) = [1 - 2, 1 + 2] = [-1, 3] [1]$$

$$\text{End-points: } y = 2 \cos\left((0) - \frac{\pi}{3}\right) + 1$$

$$y = 2 \cos\left(\frac{1}{2}\right) + 1 = 2$$

$$y = 2 \cos\left((2\pi) - \frac{\pi}{3}\right) + 1$$

$$y = 2 \cos\left(\frac{1}{2}\right) + 1 = 2$$

$$\text{Endpoints: } (0, 2), (2\pi, 2) [1]$$

**Question 7**

$$\int_0^a ax - x^2 dx = \frac{9}{2} [1]$$

$$\left[\frac{1}{2}ax^2 - \frac{1}{3}x^3\right]_0^a = \left(\frac{1}{2}a^3 - \frac{1}{3}a^3\right) - \left(\frac{1}{2}(0)^3 - \frac{1}{3}(0)^3\right) = \frac{9}{2} [1]$$

$$\Rightarrow \frac{1}{6}a^3 = \frac{9}{2} \Rightarrow a^3 = 27$$

$$\Rightarrow a = 3 [1]$$

**Question 8**

$$\frac{dU}{dt} = 2 \text{ m/s}$$

$$L = \sqrt{25 - U^2} [1]$$

$$\Rightarrow \frac{dL}{dU} = \frac{-U}{\sqrt{25 - U^2}} = \frac{3}{4} \text{ m/m when } U = 3 \text{ cm} [1]$$

$$\frac{dL}{dt} = \frac{dL}{dU} \frac{dU}{dt} [1]$$

$$= \frac{3}{4} \frac{2}{1} = \frac{3}{2} \text{ m/s} [1]$$

**Question 9**

Shape of  $f$  and  $f^{-1}$  correct ( $f^{-1}$  is given by  $f$  reflected in both axes) [1]

For inverse let  $f(x) = y$  and swap  $x$  and  $y$ ,  $x = 4e^{y-2} + 1$  [1]

$$f^{-1}(x) = y = \log_e\left(\frac{x-1}{4}\right) + 2 [1]$$

Intercept for  $f$  at  $\left(0, \frac{4}{e^2} + 1\right)$  and intercept for  $f^{-1}$  at  $\left(\frac{4}{e^2} + 1, 0\right)$  [1]

**Question 10a**

$$\frac{dy}{dx} = m = 3x^2 - 2x - 1$$

$$\text{At pt } (0,1) \ m = 3(0)^2 - 2(0) - 1 = -1[1]$$

$$\text{Gradient of normal} = -\frac{1}{m} = -\frac{1}{(-1)} = 1[1]$$

$$y = x + c \text{ pt } (0,1) \Rightarrow 1 = c$$

$$y = x + 1 [1]$$

**Question 10b**

$$x + 1 = x^3 - x^2 - x + 1 [1]$$

$$x^3 - x^2 - 2x = x(x - 2)(x + 1) = 0$$

$$x = -1, 0, 2 \text{ sub into } y = x + 1 [1]$$

$$(-1,0), (0,1), (2,3) [1]$$