

YEAR 12 – OCTOBER 2006

MATHEMATICAL METHODS

Written test 2

ANSWERS & SOLUTIONS BOOK

SECTION 1 – Multiple choice questions

1 B	6 A	11 A	16 E	21 A
2 B	7 C	12 E	17 B	22 A
3 D	8 C	13 B	18 B	
4 D	9 D	14 C	19 C	
5 A	10 E	15 D	20 B	

SECTION 2

Question 1

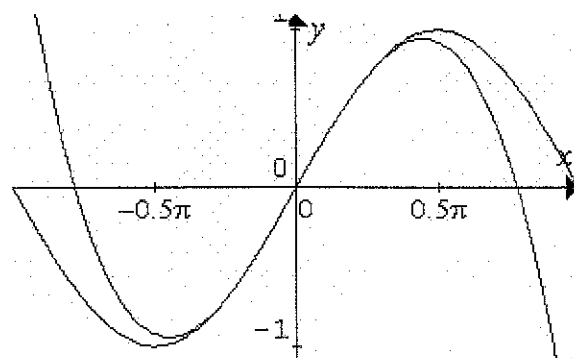
- a. $x = 4 \cos t, \frac{dx}{dt} = -4 \sin t$
 $y = 4 \sin t, \frac{dy}{dt} = 4 \cos t$
- b. $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = -\frac{4 \cos t}{4 \sin t} = -\cot t$
- c. $\frac{dy}{dx} = -\cot \frac{5\pi}{6} = \sqrt{3}$
- d. $-1 = -\cot t \Rightarrow \tan t = 1 \Rightarrow t = \frac{\pi}{4}, \frac{5\pi}{4}$
- e. $y - 4 \sin t = -\cot t(x - 4 \cos t)$
 $y = -\cot t \times x + \frac{4 \cos^2 t}{\sin t} + 4 \sin t$
 $y = -\cot t \times x + \frac{4 \cos^2 t + 4 \sin^2 t}{\sin t}$
 $y = -\cot t(x) + \frac{4}{\sin t}$
- f. $B(0, \frac{4}{\sin t}), C(\frac{4}{\cos t}, 0)$
- g. $A = \frac{1}{2} b \times h = \frac{1}{2} \times \frac{4}{\sin t} \times \frac{4}{\cos t}$
- h. $A = 4(\sin 2t)^{-1} \Rightarrow \frac{dA}{dt} = -\frac{8 \cos 2t}{\sin^2 2t}$
- i. St. points when
 $\frac{dA}{dt} = 0 \Rightarrow \cos 2t = 0 \Rightarrow t = \frac{\pi}{4},$

Also the gradient changes from negative to zero to positive so that the S.P. is a local minimum.
 (students should provide the sign diagram)

Question 2

- a. $P(x) = \frac{x}{6}(6 - x^2) = 0 \Rightarrow x \text{ int} = 0, \pm\sqrt{6}$
- b. $P'(x) = 1 - \frac{x^2}{2}$
- c. TPs are when $P'(x) = 0 \Rightarrow x = \pm\sqrt{2}$
 TPs are at $(\pm\sqrt{2}, \pm\frac{2\sqrt{2}}{3})$

d.



Note that the cubic is above the wave on left of O, and then the cubic is below the wave on the right of O.

Endpoints are $(\mp\pi, \pm 2.026)$

- e. Shade the area between curves.
 between $-\pi$ and π

Question 2 (continued)

$$f. \text{ Area} = 2 \int_0^{\pi} \sin x - (x - \frac{x^3}{6}) dx$$

$$\text{Area} = 2 \left[-\cos x - \frac{x^2}{2} + \frac{x^4}{24} \right]_0^{\pi}$$

$$= 4 - \pi^2 + \frac{\pi^4}{12}$$

$$g. S(\frac{\pi}{4}) - P(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} - \frac{\pi}{4} + \frac{\pi^3}{384}$$

h. Total error = Area between curves.

Question 3

a. Normalcdf(105,1e99,100,10)=0.3085

b. Normalcdf(87,115,100,10)=0.8364

c. invnormalcdf(0.9,100,10)=112.82=113

d. 95% means 2 sd from the mean so the interval is [80,120].

e. $\Pr(\text{life} < 90) = 0.15866$

If Y = the number of globes lasting less 90 hours, then Y is Binomially distributed

$$\Pr(Y = 2) = \text{binomialpdf}(12, 0.15866, 2)$$

$$= 0.2953$$

f. $\Pr(Y > 2) = 1 - \Pr(Y \leq 2)$

$$= 1 - \text{binomialcdf}(12, 0.15866, 2)$$

$$= 1 - 0.7057$$

$$= 0.2943$$

g. $E(\text{profit}) = 4 \times 0.7057 - 2 \times 0.2943 = 2.2342$

Expected profit is \$2.23 per box

$$\text{Var}(\text{profit}) = 16 \times 0.7057 + 4 \times 0.2943 - 2.2342^2$$

$$= 7.7468$$

Variance is \$7.75 per box.

h. Expected profit = \$2234

Question 4

a. Endpoints $(\pm 1, \frac{1}{2}(e + e^{-1}))$

$$b. y = -\frac{1}{2}(e^x + e^{-x})$$

c. Centre (0,0) height is when $x=0$
 $y = 502.3$, so the height is 502 feet.

d. Dilation from the y-axis scale factor w.
 Reflection in x-axis

e.

$$\text{height} = \frac{1}{2}(e + \frac{1}{e}) - 1 = \frac{1}{2}(\frac{e^2 + 1 - 2e}{e})$$

$$\text{height} = \frac{(e-1)^2}{2e}$$

$$f. 2 = 4 - 0.5(e^x + e^{-x})$$

Leads to $e^x + e^{-x} = 4$, let $w = e^x$

$$\text{Gives } w^2 - 4w + 1 = 0 \Rightarrow w = 2 \pm \sqrt{3} = e^x$$

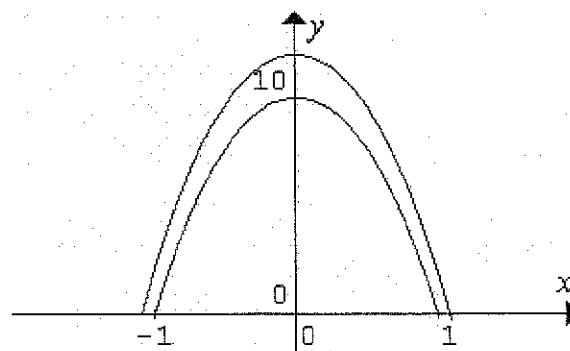
$$\text{So that } x = \ln(2 \pm \sqrt{3})$$

$$g. \text{ Area} = \int_{-c}^c b - \frac{a}{2}(e^{\frac{x}{a}} + e^{-\frac{x}{a}}) dx$$

$$= \left[bx - \frac{a^2}{2}(e^{\frac{x}{a}} - e^{-\frac{x}{a}}) \right]_{-c}^c$$

$$= 2bc - a^2(e^{\frac{c}{a}} - e^{-\frac{c}{a}})$$

h.



The y intercepts are 12 and 10.

The x intercepts are $\pm 1, \pm 1.087$