

STUDENT:	TEACHER:
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YEAR 12 – OCTOBER 2006

MATHEMATICAL METHODS

Written test 1

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
11	11	40

- Students are permitted to bring into the test room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are **NOT** permitted to bring into the test room: blank sheets of paper and/or white out liquid/tape.
- No calculator is allowed in this test.

Materials supplied

- Question and answer book of 8 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **name** in the space provided above.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or other electronic communication devices into the test room.

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

Given the functions

$$f: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}, f(x) = \frac{1}{x} \quad \text{and}$$

$$g: [2, +\infty) \rightarrow \mathbb{R}, g(x) = \sqrt{3x-4}$$

- a. Find the largest domain for which $g(f(x))$ is defined

1 mark

- b. Write down the rule of the composite function $g(f(x))$

1 mark

- c. Find the derivative of $g(f(x))$

1 mark

Question 2 (4 marks)

Solve the equations

- a. $\ln \sqrt{x+2} - \ln x = 0$

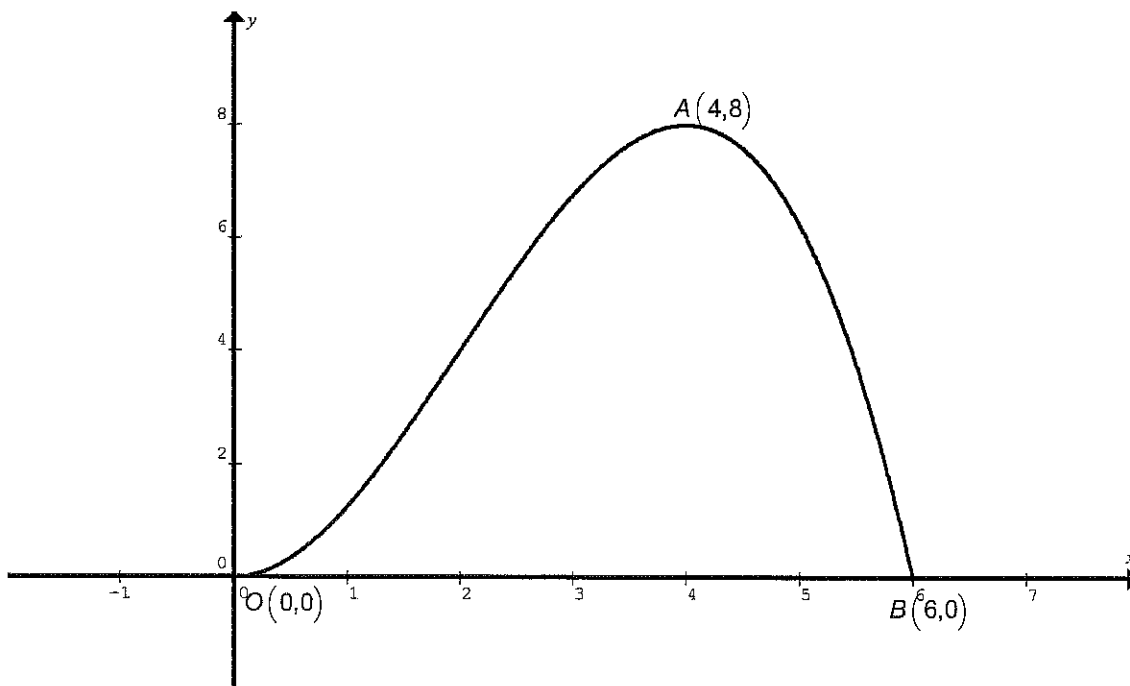
2 marks

b. $\left| \sin\left(x - \frac{\pi}{6}\right) \right| = \frac{\sqrt{3}}{2}$ for $x \in [0, 2\pi]$, giving exact answers.

2 marks

Question 3 (3 marks)

The graph of the function $y = f(x)$, which passes through the points O (0, 0), A (4, 8) and B (6, 0) is shown



Sketch, on the same set of axes, the graph of $y = f(2(x+1))$ with the images of O, A, B clearly shown and their coordinates labelled.

3 marks

Question 4 (4 marks)

If $f: \mathbf{R} \setminus \{2\} \rightarrow \mathbf{R}$ is such that $f(x) = \frac{1}{x-2}$, find the equations of the tangents to $y = f(x)$ which have a gradient of -4 .

[illegible]

4 marks

Question 5 (4 marks)

Considering that the difference between $\left(\frac{\pi}{3}\right)^\circ$ and 1° is small, use linear approximation to find $\sin 1^\circ$, giving your answer in exact form.

[illegible]

4 marks

MATHEMATICAL METHODS AND MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

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Version 2 – March 2006

Mathematical Methods and Mathematical Methods CAS Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc \sin A$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

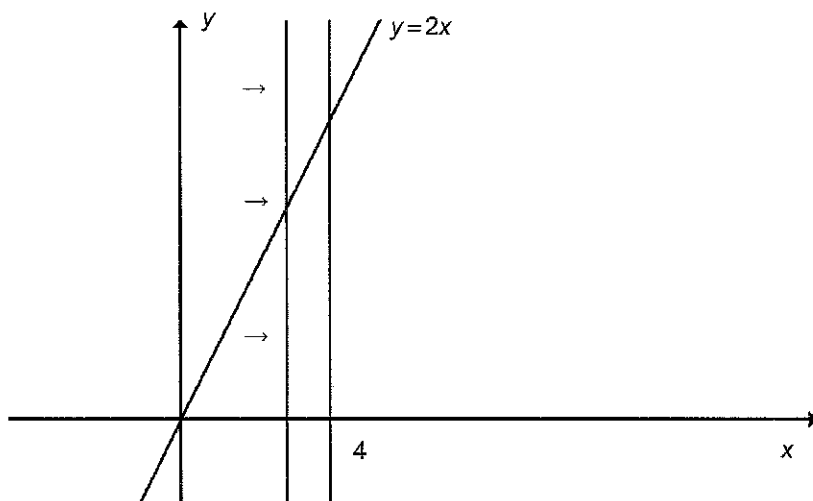
probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

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END OF FORMULA SHEET

Question 6 (4 marks)

A right angled triangle is defined by the line $y=2x$, the positive side of the x – axis and a vertical line from $y=2x$ to the x – axis.



If the vertical line is moving to the right from the origin at a constant rate of 2 units/min, find the rate of increase of the area of the right angled triangle when $x=4$ units.

4 marks

Question 7 (4 marks)

a. Find $\frac{dy}{dx}$ if $y=2\log_e \frac{1}{|2x-5|}$, $x \neq \frac{5}{2}$.

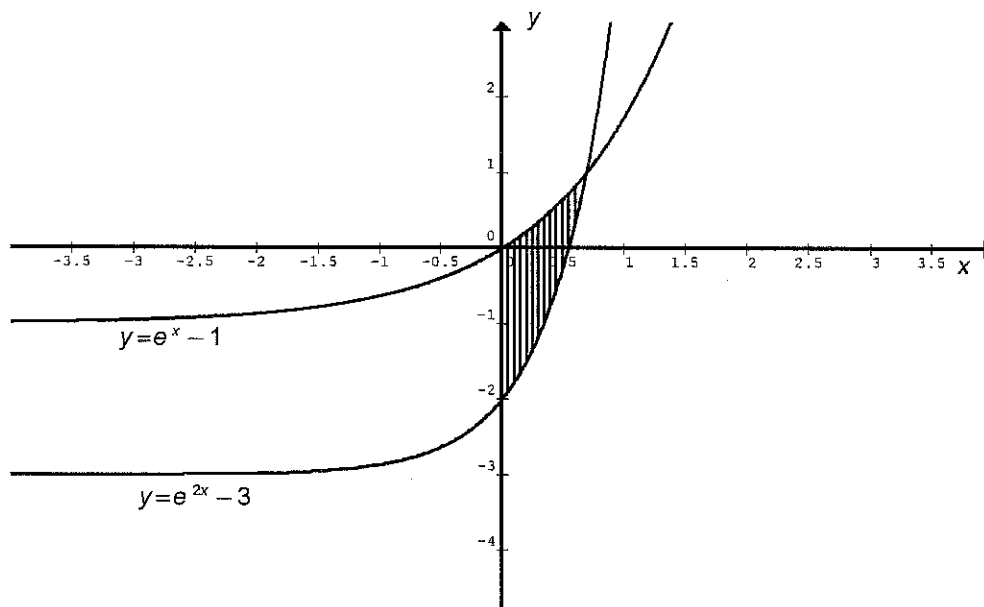
2 marks

- b. If $f: \mathbb{R} \rightarrow \mathbb{R}$ is such that $f'(x) = \sin\left(2x - \frac{\pi}{3}\right)$ and $f(0) = \frac{1}{2}$, find the rule of $f(x)$.

2 marks

Question 8 (4 marks)

Find the exact area bounded by the curves $y = e^{2x} - 3$, $y = e^x - 1$ and the line $x = 0$.



4 marks

Question 9 (3 marks)

A random variable $X = \{1, 2, 3, 4\}$ has a probability distribution function $f(x) = kx$.

- a. Find the value of k .

1 mark

- b. Find the value of $E(X)$.

1 mark

- c. Find the value of $E(2X + 3)$.

1 mark
Question 10 (3 marks)

If X is a binomial random variable with parameters n and p .

- a. Find $Pr(X=1)$

1 mark

- b. Show that $Pr(X=1 | X \leq 1) = \frac{np}{q + np}$

2 marks

END OF QUESTION AND ANSWER BOOK