

UNIT 4 MATHEMATICAL METHODS 2005 TRIAL EXAMINATION SOLUTIONS

WRITTEN EXAMINATION 1

FACTS, SKILLS AND APPLICATION TASKS

PART I – MULTIPLE CHOICE QUESTIONS

	Answer		Answer
Question 1	C	Question 16	C
Question 2	D	Question 17	B
Question 3	B	Question 18	D
Question 4	B	Question 19	A
Question 5	C	Question 20	A
Question 6	D	Question 21	C
Question 7	D	Question 22	E
Question 8	A	Question 23	D
Question 9	B	Question 24	B
Question 10	A	Question 25	E
Question 11	E	Question 26	D
Question 12	C	Question 27	C
Question 13	D		
Question 14	D		
Question 15	B		

QUESTION 1 Answer is C

For tangent functions: $T = \frac{\pi}{\text{magnitude of coefficient of } x} = \frac{\pi}{2}$

QUESTION 2 Answer is D

$$y = a(\sin x - 1) = a \sin x - a$$

$$\text{Max value} = a + -a = 0$$

$$\text{Min value} = -a + -a = -2a$$

Range is $[-2a, 0] = D$

QUESTION 3 Answer is B

$$a \cos(x - b) = c$$

$$\cos(x - b) = \frac{c}{a}$$

A solution exists if $-1 \leq \frac{c}{a} \leq 1$.

If $\frac{c}{a} < -1$, no solution exists i.e. $c < -a$ OR

If $\frac{c}{a} > 1$, no solution exists i.e. $c > a$

QUESTION 4 Answer is B

$$\log_a 3 + 2 \log_a (2x + 1) = \log_a 48$$

$$\log_a 3 + \log_a (2x + 1)^2 = \log_a 48$$

$$\log_a 3(2x + 1)^2 = \log_a 48$$

$$3(2x + 1)^2 = 48$$

$$(2x + 1)^2 = 16$$

$$(2x + 1) = \pm 4$$

$$x = -\frac{5}{2}, \frac{3}{2} \quad \text{Check validity of solutions: } x \neq -\frac{5}{2}$$

QUESTION 5 Answer is C

$$(x + a)(x^2 + b)(x - c)^2 = 0$$

$$\begin{array}{lll} (x + a) = 0 & \text{or} & (x^2 + b) = 0 & \text{or} & (x - c)^2 = 0 \\ x = -a & & \text{No solution} & & x = c \end{array}$$

Two real number solutions will be obtained.

QUESTION 6 Answer is D

QUESTION 7 Answer is D

$$\begin{aligned} 4 - at &> 0 \\ -at &> -4 \\ at &< 4 \\ \therefore t &= \frac{4}{a} \end{aligned}$$

QUESTION 8 Answer is A

Sign of graph = $(+) \times (+) \times (-)^2 = +$. Therefore, options A, B or C.

X intercept occurs at $(x + b) = 0 \therefore x = -b$.

Curve cuts at this point as power on brackets is 1.

X intercept occurs at $(a - x)^2 = 0 \therefore x = a$.

Curve touches at this point as power on brackets is 2.

QUESTION 9 Answer is B

QUESTION 10 Answer is A

QUESTION 11 Answer is E

Negative hyperbola, therefore, coefficient of x must be negative. Answer is B, D or E.

Y asymptote is negative $(-b)$, therefore, vertical translation is b units in the negative direction. There must be a -1 separate from the fraction. Answer is D or E.

X asymptote is positive (a) . The denominator of the fraction, when made equal to 0 must result in an answer of $x = a$. Answer is E.

QUESTION 12 Answer is C

Turning point is $(-a, -b)$ therefore, maximal domain is based on the value $x = -a$, and can be either greater than or less than this value.

QUESTION 13 Answer is D

$$f(x) = 2x^3 - x + \sqrt{x} = 2x^3 - x + x^{1/2}$$

$$f'(x) = 6x^2 - 1 + \frac{1}{2\sqrt{x}}$$

$$f'(a) = 6a^2 - 1 + \frac{1}{2\sqrt{a}}$$

QUESTION 14 Answer is D

$$y = \cos^3(2x) = (\cos(2x))^3$$

$$\frac{dy}{dx} = 3 \times -2 \sin(2x) \times \cos^2(2x) = -6 \sin(2x) \cos^2(2x)$$

QUESTION 15 Answer is B

$$y = \frac{\cos x}{e^{2x}} \quad \text{Apply Quotient Rule}$$

$$\frac{dy}{dx} = \frac{(e^{2x} \times -\sin x) - (\cos x \times 2e^{2x})}{(e^{2x})^2} = \frac{-e^{2x} \sin x - 2 \cos x e^{2x}}{(e^{2x})^2}$$

$$= \frac{-e^{2x}(\sin x + 2 \cos x)}{(e^{2x})^2} = \frac{-(\sin x + 2 \cos x)}{e^{2x}}$$

QUESTION 16 Answer is C

Container is filled from the bottom up.

Initial part of container – height of water increases by the same amount per unit time.
Therefore h vs t graph is an oblique line. Answer is B or C.

Middle part of container is becoming wider. The height occupied by water will decrease per unit time. Gradient of curve is decreasing, therefore, shape of graph resembles:



Top part of container is becoming narrower. The height occupied by water will increase per unit time. Gradient of curve is increasing, therefore, shape of graph resembles:



QUESTION 17 Answer is B

QUESTION 18 Answer is D

$$f'(x) = 2 \sin\left(2x - \frac{\pi}{2}\right)$$

$$\text{As } f(x) = \int f'(x) dx$$

$$\therefore f(x) = \int 2 \sin\left(2x - \frac{\pi}{2}\right) dx = 2 \int \sin\left(2x - \frac{\pi}{2}\right) dx = \frac{2 \times -\cos\left(2x - \frac{\pi}{2}\right)}{2} + c$$

$$\therefore f(x) = -\cos\left(2x - \frac{\pi}{2}\right) + c$$

$$\text{When } x = \frac{\pi}{4}, y = 1$$

$$\therefore 1 = -\cos\left(\frac{2\pi}{4} - \frac{\pi}{2}\right) + c$$

$$1 = -\cos(0) + c$$

$$\therefore 1 = -1 + c \quad \therefore c = 2$$

$$\therefore f(x) = -\cos\left(2x - \frac{\pi}{2}\right) + 2$$

QUESTION 19 Answer is A

$$\int (5 - 3x)^3 dx = \frac{(5 - 3x)^4}{4 \times -3} + c = -\frac{(5 - 3x)^4}{12} + c$$

QUESTION 20 Answer is A

$$\int_0^a \left(\frac{2}{4x+1}\right) dx = \frac{1}{2} \int_0^a \left(\frac{4}{4x+1}\right) dx = \frac{1}{2} [\log_e(4x+1)]_0^a = \frac{1}{2} [\log_e(4a+1) - \log_e 1]$$

$$= \frac{1}{2} \log_e(4a+1) = \log_e(4a+1)^{1/2}$$

$$\log_e(4a+1)^{1/2} = \log_e k$$

$$k = \sqrt{4a+1}$$

QUESTION 21 Answer is C

The antiderivative of $-ax$ results in $-\frac{ax^2}{2} + c$ i.e. a negative quadratic curve is required.

QUESTION 22 Answer is E

Note that: $\int_b^c [g(x) - f(x)]dx = -\int_b^c [f(x) - g(x)]dx$

QUESTION 23 Answer is D

General term $= {}^5C_r (4)^{5-r} (-ax)^r$

For x^3 , $r = 3$

$${}^5C_3 (4)^{5-3} (-ax)^3 = 10 \times 16 \times -a^3 x^3 = -160a^3 x^3$$

The coefficient of $x^3 = -160a^3 = -4320 \therefore a = 3$

QUESTION 24 Answer is B

QUESTION 25 Answer is E

n is the same for both distributions.

A is positively skewed so $p < 0.5$. B is negatively skewed so $p > 0.5$.

As $\mu = np$, $\mu_B > \mu_A$. Answer is A or E.

As curves are mirror images about $X = 10$, sum of p values will equal to 1. Therefore, σ 's are equal. Answer is E.

QUESTION 26 Answer is D

As order is important the standard probability formula cannot be applied.

$$\Pr(\text{bull's eye last 2 rounds}) = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} = \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^2$$

QUESTION 27 Answer is C

z values to the left of the mean are negative. $\therefore$ A is correct.

Normal curves are symmetrical about the mean, irrespective of whether z or X values are being used. $\therefore$ B is correct.

C cannot be the correct answer as a and b represent z values – not X .

PART II - SHORT ANSWER QUESTIONS

QUESTION 1

a. As $P(2) = 0$: $2(2)^3 + 3(2)^2 + 2a + b = 0$
 $2a + b = -28$

As $P(-3) = 0$: $2(-3)^3 + 3(-3)^2 - 3a + b = 0$
 $-3a + b = 27$

Solve equations simultaneously: $2a + b = -28$ —
 $-3a + b = 27$ —

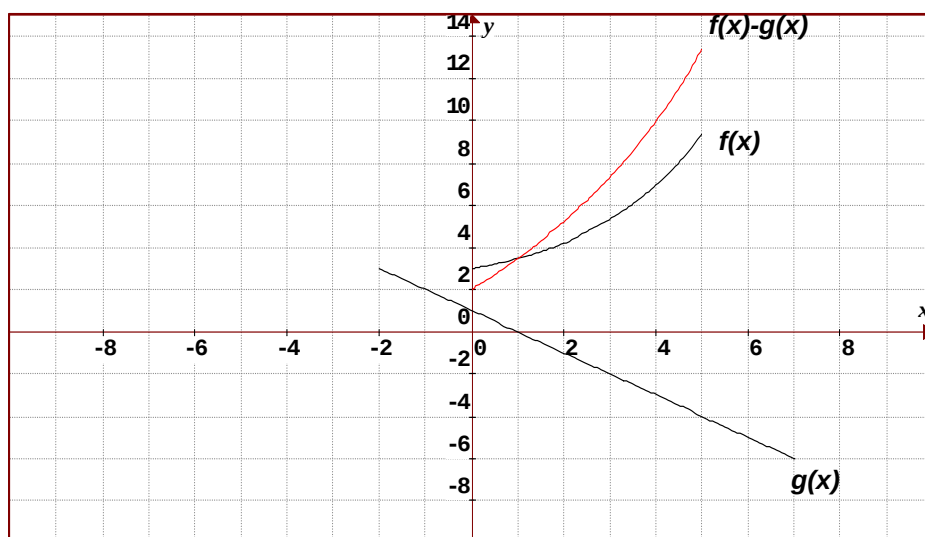
$\therefore a = -11$ and $b = -6$

b. To write $f(x)$ in the form $a + \frac{b}{x+2}$: Long Division is required.

$$\begin{array}{r} 1 \\ x+2 \overline{) x-2} \\ \underline{x+2} \\ -4 \end{array}$$

Therefore, $f(x) = \frac{x-2}{x+2} = 1 - \frac{4}{x+2}$.

QUESTION 2



Note: Functions can only be added, subtracted (multiplied and divided) across the common domain.

QUESTION 3

$$\sin\left(2x + \frac{\pi}{3}\right) = -\sqrt{3} \cos\left(2x + \frac{\pi}{3}\right)$$

Divide both sides by $\cos\left(2x + \frac{\pi}{3}\right)$:

$$\frac{\sin\left(2x + \frac{\pi}{3}\right)}{\cos\left(2x + \frac{\pi}{3}\right)} = -\frac{\sqrt{3} \cos\left(2x + \frac{\pi}{3}\right)}{\cos\left(2x + \frac{\pi}{3}\right)}$$

$$\tan\left(2x + \frac{\pi}{3}\right) = -\sqrt{3}$$

1st Quadrant angle: $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

Solutions to lie in Quadrants 2 and 4.

$$\left(2x + \frac{\pi}{3}\right) = \pi - \frac{\pi}{3} \quad \text{or} \quad \left(2x + \frac{\pi}{3}\right) = 2\pi - \frac{\pi}{3}$$

$$x = \frac{\pi}{6}, \frac{2\pi}{3}$$

QUESTION 4

a. $y = -3e^{2x} + 1$

When $x = 0$: $y = -3e^0 + 1 = -2 \therefore (0, -2)$

$$\frac{dy}{dx} = -6e^{2x} \quad \text{When } x = 0: \frac{dy}{dx} = -6e^0 = -6$$

$$m_{\text{normal}} = -\frac{1}{m_{\text{tangent}}} = \frac{1}{6}$$

Equation Normal: $y - y_1 = m(x - x_1)$

$$y + 2 = \frac{x}{6}$$

$$y = \frac{x}{6} - 2$$

b. $y = -3e^{2x} + 1$

X intercept, let $y = 0$:

$$-3e^{2x} + 1 = 0$$

$$e^{2x} = \frac{1}{3}$$

$$\log_e e^{2x} = \log_e \frac{1}{3}$$

$$2x = \log_e \frac{1}{3}$$

$$x = \frac{1}{2} \log_e \frac{1}{3}$$

Coordinates: $\left(\frac{1}{2} \log_e \frac{1}{3}, 0 \right)$

QUESTION 5

a. $\int_1^5 \frac{f(x)}{2} dx = \frac{1}{2} \int_1^5 f(x) dx = \frac{1}{2} \times 20 = 10$

b. $\int_1^5 [f(x) + 1] dx = \int_1^5 f(x) dx + \int_1^5 1 dx = 20 + [x]_1^5$
 $= 20 + (5 - 1) = 24$

c. $\int_5^1 f(x) dx + 1 = -\int_1^5 f(x) dx + 1 = -20 + 1 = -19$

QUESTION 6

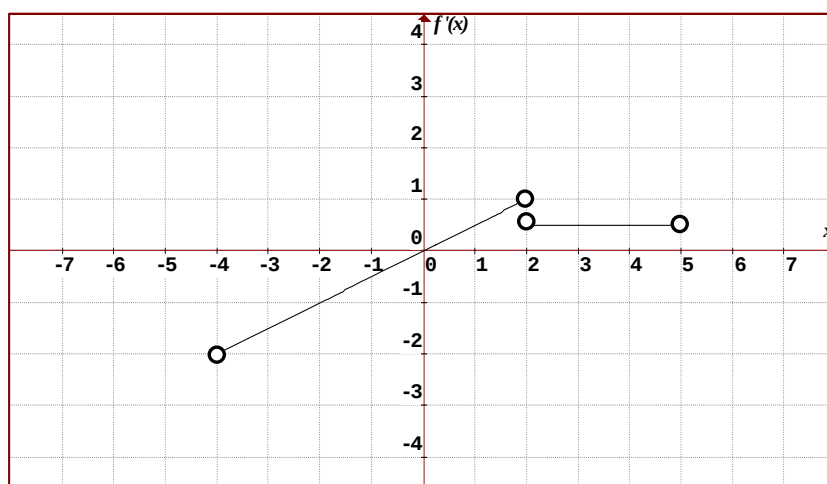
$$\text{Gradient of line} = \frac{\text{rise}}{\text{run}} = \frac{1.5}{3} = 0.5.$$

Draw tangents at $x = -4$ and $x = 2$, select 2 points that lie on each tangent and calculate the corresponding gradients.

At $x = -4$, $m \approx -2$.

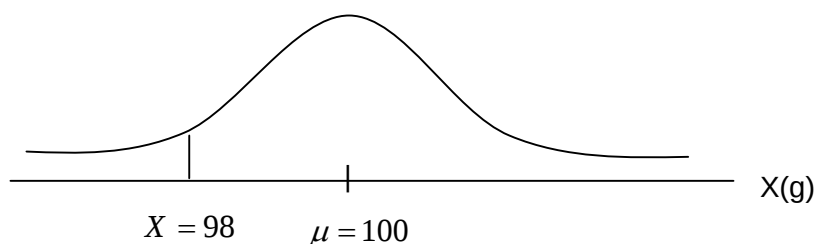
At $x = 2$, $m \approx 1$.

As a stationary point exists at $x = 0$ for $f(x)$, the graph of $f'(x)$ will cut the X axis at $x = 0$.



Note: Gradients are undefined at the end points of a domain.
Therefore, use open circles at the end points.

QUESTION 7



a. $\Pr(X < 98) = \text{normalcdf}(-1 \times 10^{99}, 98, 100, 3) = 0.252$

Note: Variance is 9, therefore, standard deviation is 3.

b. Hypergeometric as we are sampling without replacement.

Let X = number of underweight packets.

$$\Pr(X = 2) = \frac{\binom{6}{2} \binom{24}{0}}{\binom{30}{2}} = \frac{15 \times 1}{435} = \frac{1}{29}$$

c. Box is rejected if:

(The 2 packets removed from the first sampling are underweight) AND
(1, 2 or 3 packets are underweight from the second sampling).

Second Sampling

$$= \frac{1}{29} \times [\Pr(X = 1) + \Pr(X = 2) + \Pr(X = 3)]$$

$$= \frac{1}{29} \times [1 - \Pr(X = 0)] = \frac{1}{29} \times \left[1 - \frac{\binom{4}{0} \binom{24}{3}}{\binom{28}{3}} \right]$$

$$= \frac{1}{29} \times \left(1 - \frac{2024}{3276} \right) = 0.013$$

Note: This part of the question requires hypergeometric manipulations as we are continuing to sample without replacement. However, a number of variables have changed in value. The sample size has decreased from 30 to 28 as 2 packets were removed in the first sampling. The number of underweight packets has decreased from 6 to 4 as 2 underweight packets were removed in the first sampling.