

MATHS METHODS EXAM 2: SOLUTIONS

Question 1

a. $2\pi r^2 + 2\pi r h = 98\pi + 140$ **1M**

$$2\pi r h = 98\pi + 140 - 2\pi r^2$$

$$h = \frac{98\pi + 140 - 2\pi r^2}{2\pi r}$$
 1M

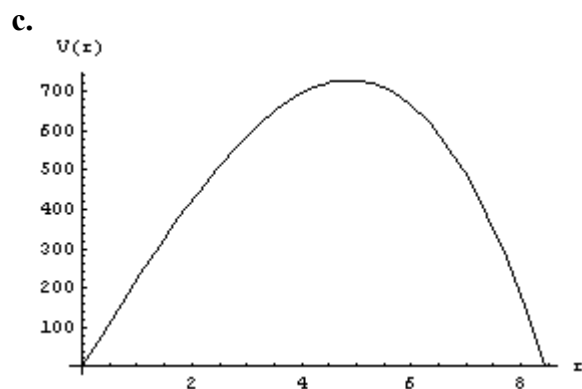
$$= \frac{70 + 49\pi - \pi r^2}{\pi r} \text{ as required}$$

b. $V = \pi r^2 h$

$$= \pi r^2 \frac{70 + 49\pi - \pi r^2}{\pi r}$$
 1M

$$= r(70 + 49\pi - \pi r^2)$$
 1M

$$= (70 + 49\pi)r - \pi r^3 \text{ as required.}$$



shape **1M**

open circles at (0, 0) and (8.4, 0) **1M**

maximum marked (4.9, 727.7) **1M**

d. $V'(r) = 0$

$$70 + 49\pi - 3\pi r^2 = 0$$
 1M

$$r^2 = \frac{70 + 49\pi}{3\pi}$$

$$r = \sqrt{\frac{70 + 49\pi}{3\pi}} \text{ cm}$$
 1A

$$h = \frac{98\pi + 140 - 2\pi r^2}{2\pi r}$$

$$= \frac{98\pi + 140 - 2\pi \frac{70 + 49\pi}{3\pi}}{2\pi \sqrt{\frac{70 + 49\pi}{3\pi}}} \text{ cm}$$
 1M

$$= 2(10 + 7\pi) \sqrt{\frac{7}{30\pi + 21\pi^2}} \text{ cm as required.}$$

e. i $C = 0.005(98\pi + 140) + 0.002V$
 $= 0.49\pi + 0.7 + 0.002V$ **1A**

ii Dilation of a factor of 0.002 from the r axis, **1A**
followed by a translation of $0.49\pi + 0.7$ cm parallel to the V axis. **1A**

iii $C = 0.49\pi + 0.7 + 0.002V_{\max}$
 $= 0.49\pi + 0.7 + 0.002 \times 727.721$
 $= \$3.69$ **1A**

Question 2

a. $10\% > 25$ gives $Z_1 = 1.28155$
 $15\% < 10$ gives $Z_2 = -1.03643$ **1M**

$$\frac{25 - \mu}{\sigma} = 1.28155; \frac{10 - \mu}{\sigma} = -1.03643$$
 2A

solve simultaneously **1M**

$$\mu = 16.71, \sigma = 6.47 \text{ as required}$$

b. $\text{normalcdf}(-E99, 15, 16.71, 6.47)$ **1M**
 $= 0.3958$ **1A**

c. Binomial **1M**

$$N = 10, p = 1 - 0.3958, x \geq 3$$
 1A

$$\Pr(X \geq 3) = 0.9886$$
 1A

d. Conditional probability

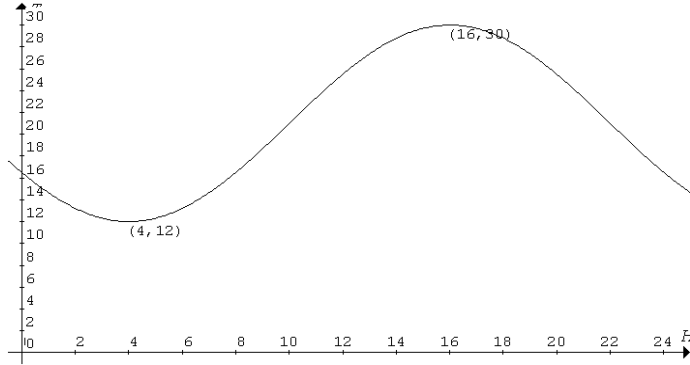
$$\Pr(X > 20 \mid X > 15) = \frac{\Pr(X > 20)}{\Pr(X > 15)}$$
 1M

$$= \frac{0.3056}{0.6042}$$
 1A

$$= 0.5058$$
 1A

Question 3

a.



Minimum **1A**
 Maximum **1A**
 Domain $[0, 24]$ **1A**
 Axes scaled and shape **1A**

b. amplitude $= \frac{30-12}{2}$; $A = 9$ **1A**

period $= \frac{2\pi}{n} = 24$; $n = \frac{\pi}{12}$ **1A**

median value $= 12 + 9$; $c = 21$ **1A**

horizontal translation $= 16$ to the right or 8 to the left; $b = -16$; or $+8$ **1A**

c. When $H = 0$, $T = 16.5$;
 when $H = 12$, $T = 25.5$ **2A**

d. $\frac{dT}{dH} = -\frac{3\pi}{4} \sin\left(\frac{\pi}{12}(H+8)\right)$ **1H**

e. $\frac{dT}{dH}$ is maximum

when $\sin\left(\frac{\pi}{12}(H+8)\right) = -1$ **1H**

when $\frac{\pi}{12}(H+8) = \frac{3\pi}{2}$, $H = 10$ **1A**

Answer: at 10 am **1A**

f. At 10 am $\frac{dT}{dH} = \frac{3\pi}{4}$ **1A**

Question 4

a. $\sin(2x)e^x = 0$

$e^x \neq 0$,

$\sin(2x) = 0$

$x = 0, \frac{\pi}{2}, \pi$

1A

b. i $\frac{d}{dx} e^x (\sin(2x) - 2 \cos(2x)) =$

$e^x (\sin(2x) - 2 \cos(2x)) + e^x (2 \cos(2x) + 4 \sin(2x))$ **1M**

$= 5 \sin(2x) e^x$ as required

ii $\int_0^{\frac{\pi}{2}} (\sin(2x) e^x) dx - \int_{\frac{\pi}{2}}^{\pi} (\sin(2x) e^x) dx$ **1M**

$= \frac{1}{5} [(e^x (\sin(2x) - 2 \cos(2x)))]_0^{\frac{\pi}{2}} - [(e^x (\sin(2x) - 2 \cos(2x)))]_{\frac{\pi}{2}}^{\pi}$ **1M**

$= \frac{2}{5} (e^{\pi} + 2e^{\frac{\pi}{2}} + 1)$ **1A**

$= \frac{2}{5} (1 + e^{\frac{\pi}{2}})^2$ units²

c. i $\int_0^{\frac{\pi}{2}} (\sin(2x) e^x) dx$ **1M**
 $= 2.32$ units² **1A**

ii Use trial and error with the calculator.

$\int_{2.09225}^{2.9550} (-7.5 - f(x)) dx \approx 2.32687$ units² **2M**

7.5 units **1A**

d. i Area of Ann's and John's land

$= 2 \int_0^{\frac{\pi}{2}} (\sin(2x) e^x) dx \approx 4.648$ **1M**

Use trial and error with the calculator.

$\int_{1.96101}^{3.01811} (-5 - f(x)) dx \approx 4.66728$ **1M**

$\Rightarrow 5$ units **1A**

ii $a = -7.5$ units from c. ii **1A**