

VCE
2005 Mathematical Methods
Trial Examination 2

Suggested Solutions

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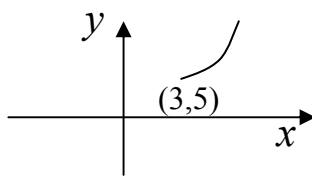
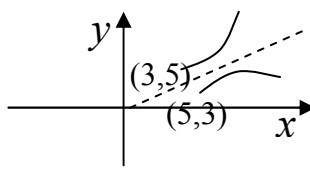


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Question 1

a. $f(x) = 2 \left[x^2 - 6x + \frac{23}{2} \right]$ $f(x) = 2 \left[x^2 - 6x + 9 + \frac{23}{2} - 9 \right] \quad (1 \text{ mark})$ $f(x) = 2 \left[(x-3)^2 + \frac{5}{2} \right]$ $f(x) = 2(x-3)^2 + 5$ $A = 2, B = 3, C = 5 \quad (1 \text{ mark})$	b.  $f(x)$ must have one - one correspondence for $f^{-1}(x)$ to exist. $a = 3 \quad (1 \text{ mark})$
c. $x = 2(y-3)^2 + 5$ $x - 5 = 2(y-3)^2$ $\frac{x-5}{2} = (y-3)^2$ $y-3 = \pm \sqrt{\frac{x-5}{2}}$ $y = 3 \pm \sqrt{\frac{x-5}{2}} \quad (1 \text{ mark})$ But $y \geq 3$ $\therefore y = 3 + \sqrt{\frac{x-5}{2}}$ $\therefore f^{-1}(x) = 3 + \sqrt{\frac{x-5}{2}} \quad x \geq 5 \quad (1 \text{ mark})$ Domain $[5, \infty)$ (1 mark) Range $[3, \infty)$ (1 mark)	d. 1 mark for each shape with its end point.  e.(i) $f'(x) = 4x - 12$ When $x = 4$, $f'(x) = 4$ (1 mark)

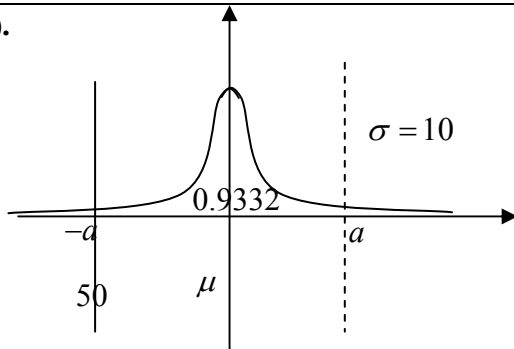
e.(ii) $f^{-1}(x) = 4 = 3 + \sqrt{\frac{x-5}{2}}$ $\sqrt{\frac{x-5}{2}} = 1$ $\frac{x-5}{2} = 1$ $x-5 = 2$ $x = 7 \quad (1 \text{ mark})$ $f'^{-1}(x) = \frac{1}{2} \left(\frac{x-5}{2} \right)^{-\frac{1}{2}} \times \frac{1}{2} = \frac{1}{4} \left(\frac{x-5}{2} \right)^{-\frac{1}{2}} \quad (1 \text{ mark})$ When $x = 7$, $f'^{-1}(x) = \frac{1}{4} \quad (1 \text{ mark})$	f. Let point of intersection be (a, b) For $f^{-1}(x)$ when $x = 7, y = 4$ $\therefore$ for $f(x)$ when $x = 4, y = 7$ Gradient of $f(x) = 4$ $\therefore \frac{7-b}{4-a} = 4$ $\therefore 7-b = 16-4a$ $4a-b = 9 \quad (1) \quad (1 \text{ mark})$ But gradient of $f^{-1}(x) = \frac{1}{4}$ $\therefore \frac{4-b}{7-a} = \frac{1}{4}$ $\therefore 7-a = 16-4b$ $-a+4b = 9$ $-4a+16b = 36 \quad (2)$ $(1) + (2)$ $15b = 45$ $b = 3$ Sub $b = 3$ in (1) $a = 3$ $(3, 3) \quad (1 \text{ mark})$
g. All the points will be of the form (a, a), so the equation of the line will be $y = x$ (1 mark)	

Question 2

a. $\frac{35}{100} \times 18,000 = 6,300$ (1 mark)	b. $4k^2 + k + 6k^2 + 4k - 14k^2 = 1$ $-4k^2 + 5k - 1 = 0$ $4k^2 - 5k + 1 = 0$ $(4k - 1)(k - 1) = 0 \quad (1 \text{ mark})$ $k = \frac{1}{4} \text{ or } 1$ But $0 < k < 1$ $\therefore k = \frac{1}{4} \quad (1 \text{ mark})$
c. $\Pr = 4k^2 + k + 6k^2 = 10k^2 + k = \frac{7}{8} \quad (1 \text{ mark})$	d.(i) $\Pr(X = 1) = \binom{10}{1} \left(\frac{3}{10}\right)^1 \left(\frac{7}{10}\right)^9 = 0.121$ (1 mark) d.(ii). $\Pr(X \geq 2) = 1 - [\Pr(X = 0) + \Pr(X = 1)] \quad (1 \text{ mark})$ $= 1 - \left[\binom{10}{0} \left(\frac{3}{10}\right)^0 \left(\frac{7}{10}\right)^{10} + \binom{10}{1} \left(\frac{3}{10}\right)^1 \left(\frac{7}{10}\right)^9 \right]$ $= 0.851 \text{ to 3 dec. places. } (1 \text{ mark})$

Question 2(continued)

e(i).



$$\Pr(X > 50) = 0.9332$$

$$\Pr(Z > -a) = 0.9332$$

$$\Pr(Z < a) = 0.9332$$

$$a = 1.5$$

$$-a = -1.5 \quad (1 \text{ mark})$$

$$Z = \frac{x - \mu}{\sigma}$$

$$-1.5 = \frac{50 - \mu}{10}$$

$$\mu = 65 \quad (1 \text{ mark})$$

e(ii).

$$\Pr(X > 82 / X > 50)$$

$$= \frac{\Pr(X > 82 \cap X > 50)}{\Pr(X > 50)}$$

$$= \frac{\Pr(X > 82)}{0.9332} \quad (1 \text{ mark})$$

$$Z = \frac{x - \mu}{\sigma} = \frac{82 - 65}{10} = 1.7$$

$$\Pr(X > 82 / X > 50) = \frac{\Pr(Z > 1.7)}{0.9332}$$

$$= \frac{1 - \Pr(Z < 1.7)}{0.9332} = \frac{1 - 0.9554}{0.9332} = 0.05 \quad (1 \text{ mark})$$

f.

$$\Pr(X = 4) = \frac{\binom{20}{7} \binom{10}{4}}{\binom{30}{11}} \quad (1 \text{ mark})$$

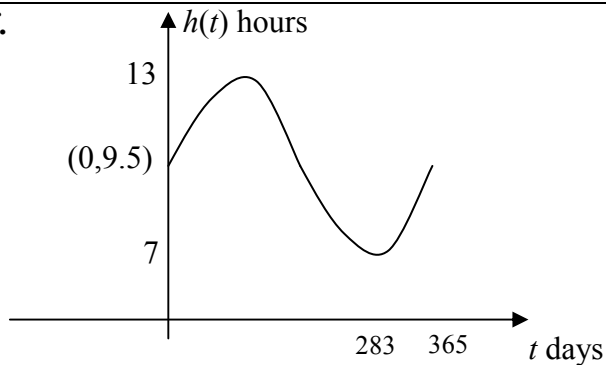
$$= 0.298 \quad (1 \text{ mark})$$

Question 3

a. Maximum $h = 10 + 3 = 13$ hours (1 mark)	b. Minimum $h = 10 - 3 = 7$ hours (1 mark)
c. Period = $\frac{2\pi}{n}$ where $n = \frac{2\pi}{365}$ Period = $2\pi \div \frac{2\pi}{365} = 365$ days (1 mark)	d. Minimum occurs when $\cos \frac{2\pi(t-100.5)}{365} = -1$ (1 mark) $\frac{2\pi(t-100.5)}{365} = -\pi, \pi, 3\pi, 5\pi, \dots$ (1 mark) $\frac{2(t-100.5)}{365} = -1, 1, 3, 5, \dots$ $2(t-100.5) = -365, 365, \dots$ $t - 100.5 = -182.5, 182.5, \dots$ $t = 283 \quad t > 0$ Minimum daylight hours occur on the 283rd day of the year. (1 mark)
e. $10 + 3 \cos \frac{2\pi(t-100.5)}{365} = 12$ $\cos \frac{2\pi(t-100.5)}{365} = \frac{2}{3}$ $\frac{2\pi(t-100.5)}{365} = -0.8411, 0.8411, 5.442$ (1 mark) $2\pi(t-100.5) = -306.99, 307.0015, 1986.37$ $t = 51.64, 149.36$ $t = 52, 149$ (1 mark) 52nd day of the year is February 21 149th day of the year is May 29 (1 mark)	

Question 3 (continued)

f.



When $t = 0, h = 10 + 3\cos\left[\frac{2\pi(-100.5)}{365}\right] = 9.5$
(1 mark)

g.

There are 12 hours of sunlight on day 52 and day 149.
Number of days from day 52 to day 149 = 97
More than 12 hours of sunlight = 96 days
because day 52 and day 149 should not be included.

(1 mark)

h.

$$\frac{dh}{dt} = -3 \sin\left[\frac{2\pi(t-100.5)}{365}\right] \times \frac{2\pi}{365} \quad (1 \text{ mark})$$

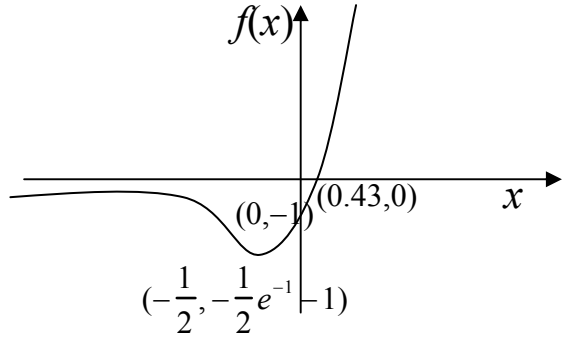
When $t = 30$,

$$\frac{dh}{dt} = -3 \sin\left[\frac{2\pi(-70.5)}{365}\right] \times \frac{2\pi}{365}$$

$$\frac{dh}{dt} = 0.048 \text{ hours / day} = 2.89 \text{ min / day}$$

$$\frac{dh}{dt} = 3 \text{ min / day} \quad (1 \text{ mark})$$

Question 4

a. $f(0) = 0 \times 1 - 1 = -1 \quad (1 \text{ mark})$	b. $f'(x) = xke^{kx} + e^{kx} \quad (1 \text{ mark})$
c. Turning Point occurs when $f'(x) = 0$ $e^{kx}(kx + 1) = 0 \quad (1 \text{ mark})$ $e^{kx} \neq 0$ $\therefore kx + 1 = 0$ $kx = -1$ $x = -\frac{1}{k} = -\frac{1}{2}$ $\therefore k = 2 \quad (1 \text{ mark})$	d.  Asymptote: $y = 0$ When $x = -\frac{1}{2}, y = -\frac{1}{2}e^{-1} - 1$ When $x = 0, y = -1$ 1 mark for shape and x, y intercepts 1 mark for equation of asymptote 1 mark for exact value of minimum turning point.

Question 4 (continued)

e. $\int x e^{5x}$ $f(x) = x e^{kx} - 1$ $f'(x) = x k e^{kx} + e^{kx}$ $\int (x k e^{kx} + e^{kx}) dx = x e^{kx} - 1 \quad (1 \text{ mark})$ If $k = 5$ $\int 5 x e^{5x} dx + \int e^{5x} dx = x e^{5x} - 1$ $5 \int x e^{5x} dx + \frac{1}{5} e^{5x} = x e^{5x} - 1$ $5 \int x e^{5x} dx = x e^{5x} - 1 - \frac{1}{5} e^{5x}$ $\int x e^{5x} dx = \frac{1}{5} (x e^{5x} - 1 - \frac{1}{5} e^{5x}) + c \quad (1 \text{ mark})$ where c is a constant. So $\int x e^{5x} dx = \frac{1}{5} \left(x e^{5x} - \frac{1}{5} e^{5x} \right) + c_1$ where c_1 is a constant. $\int x e^{5x} dx = \frac{1}{25} (5x - 1) e^{5x} + c_1$	f. $A = \left \int_0^{0.43} f(x) dx \right + \int_{0.43}^1 f(x) dx$ $\int f(x) dx = \int (x e^{2x} - 1) dx$ $A = \left \frac{1}{4} (2x - 1) e^{2x} - x \right _0^{0.43} + \frac{1}{4} (2x - 1) e^{2x} - x \Big _{0.43}^0 \quad (1 \text{ mark})$ $A = -0.2627 + 1.36 \quad (1 \text{ mark})$ $A = 0.2627 + 1.36$ $A = 1.62 \text{ to 2 decimal places.} \quad (1 \text{ mark})$
g. $g(x)$ and $g^{-1}(x)$ intersect on the line $y = x$ $\therefore x = x e^{5x}$ $x - x e^{5x} = 0 \quad (1 \text{ mark})$ $x(1 - e^{5x}) = 0$ $\Rightarrow x = 0 \text{ or } e^{5x} = 1$ $\Rightarrow x = 0 \text{ or } 5x = 0$ $\Rightarrow x = 0$ When $x = 0, y = 0$ $\therefore \text{ point is } (0, 0) \quad (1 \text{ mark})$	

END OF SUGGESTED SOLUTIONS
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