

VCE
2005 Mathematical Methods
Trial Examination 1

Suggested Solutions

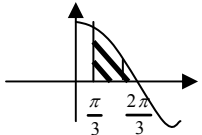
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Question 1 C $\Pr(X=1) + \Pr(X=4) = 0.1 + 0.3 = 0.4$	Question 2 D $2\Pr(X > a) = 2[1 - \Pr(X < a)] = 2 - 2\Pr(X < a)$
Question 3 B $E(X) = n \frac{D}{N} = 6 \times \frac{4}{12} = 2$	Question 4 A $\Pr = \text{BBBG} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32}$
Question 5 A $\Pr(YY) + \Pr(RY) = \frac{4}{10} \times \frac{3}{9} + \frac{6}{10} \times \frac{4}{9} = \frac{2}{5}$	Question 6 C $2 \cos\left(\frac{x}{2}\right) = 1$ $\cos\left(\frac{x}{2}\right) = \frac{1}{2}$ $\frac{x}{2} = \frac{\pi}{3}$ $x = \frac{2\pi}{3}$
Question 7 E Amplitude = 2 Translated 1 unit up $\therefore$ not A or B Period = $\frac{2\pi}{n} = \frac{\pi}{3} \Rightarrow n = 6 \therefore$ not C When $\theta = 0$, graph is a maximum $\therefore$ a cos graph $\therefore$ E	$2 \cos\left(\frac{x}{2}\right) = \sqrt{3}$ $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$ $\frac{x}{2} = \frac{\pi}{6}$ $x = \frac{\pi}{3}$ $\{x : \frac{\pi}{3} \leq x \leq \frac{2\pi}{3}\}$ 
Question 8 D $2 - 2 \cos^2 \theta = 3 \cos \theta$ $2 \cos^2 \theta + 3 \cos \theta - 2 = 0$ $(2 \cos \theta - 1)(\cos \theta + 2) = 0$ $\cos \theta = \frac{1}{2}$ or $\cos \theta = -2$ $-1 \leq \cos \theta \leq 1$ $\therefore \cos \theta = \frac{1}{2}$ $\Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$ Sum of solutions = $\frac{\pi}{3} + \frac{5\pi}{3} = 2\pi$	Question 9 E $g(x) = f(x-1) = (x-1)^3 - 2(x-1)^2 - 8(x-1) - 5$ $= x^3 - 3x^2 + 3x - 1 - 2(x^2 - 2x + 1) - 8x + 8 - 5$ $= x^3 - 5x^2 - x$ Question 10 A $y = a(x+1)^2 + 4$ $y = a(x-b)^2 + c$ $\therefore b = -1, c = 4$ When $x = 0, y = 2$ $\Rightarrow 2 = a(0+1)^2 + 4$ $\Rightarrow a + 4 = 2 \Rightarrow a = -2$

Question 11 C $g(x) = -(e^x - 2) = -e^x + 2 = -f(x) + 2$	Question 12 B $x = (y - 1)^2 - 5$ $x + 5 = (y - 1)^2$ $y - 1 = \pm\sqrt{x + 5}$ $y = 1 \pm \sqrt{x + 5} \quad y \geq 1, x \geq -5$ $\therefore y = 1 + \sqrt{x + 5}$
Question 13 E To have an inverse, the graph must be one – to – one. The only graph here, taking the domain of that graph into consideration, that is not one – to – one is E	Question 14 D $\log_a(xy) = \log_a(x) + \log_a(y)$ $\log_a(xy^4) = \log_a(x) + \log_a(y^4)$ $\log_a(xy^4) = \log_a(x) + 4\log_a(y) = p(1)$ $\log_a\left(\frac{y^3}{x}\right) = \log_a(y^3) - \log_a(x)$ $\log_a\left(\frac{y^3}{x}\right) = 3\log_a(y) - \log_a(x) = q(2)$ $(1) + (2) \rightarrow 7\log_a(y) = p + q$ $\rightarrow \log_a(y) = \frac{1}{7}(p + q)$ $4 \times (2) - 3 \times (1) \rightarrow -7\log_a(x) = 4q - 3p$ $\log_a(x) = \frac{1}{7}(3p - 4q)$ $\log_a(x) + \log_a(y) = \frac{1}{7}(3p - 4q + p + q)$ $\log_a(x) + \log_a(y) = \frac{4p - 3q}{7}$
Question 15 B $x = 8^t$ $\log_2 x = \log_2 8^t$ $\log_2 x = t \log_2 2^3$ $\log_2 x = 3t \log_2 2$ $\log_2 x = 3t$	Question 16 D When $x = -\frac{1}{2}, y = 0$ When $x = 1, y = 0$ $f(x) = (2x + 1)(x - 1) \therefore$ can be written as the product of two real linear factors. Completing the square gives $f(x) = 2\left(x^2 - \frac{1}{2}x - \frac{1}{2}\right) = 2\left(x^2 - \frac{1}{2}x + \frac{1}{16} - \frac{1}{2} - \frac{1}{16}\right)$ $f(x) = 2\left[\left(x - \frac{1}{4}\right)^2 - \frac{9}{16}\right] = 2\left(x - \frac{1}{4}\right)^2 - \frac{9}{8}$ Between $x = -\frac{1}{2}$ and $x = 1$, where this graph cuts the X axis, the graph is negative. $\therefore$ D is not true.

Question 17 E By dividing $x + 1$ into $2x + 5$ we get $\frac{2x + 5}{x + 1} = 2 + \frac{3}{x + 1}$ This gives the graph of $\frac{3}{x}$ translated 1 unit to the left and 2 units up. This means a vertical asymptote with equation $x = -1$ and a horizontal asymptote with equation $y = 2$	Question 18 B $\frac{dy}{dx} = -2e^{5-2x} = -8$ $e^{5-2x} = 4$ $5 - 2x = \log_e 4$ $2x = 5 - \log_e 4$ $x = \frac{5}{2} - \frac{1}{2} \log_e 4$ $x = \frac{5}{2} - \log_e 4^{\frac{1}{2}}$ $x = \frac{5}{2} - \log_e 2$
Question 19 A $\frac{dy}{dx} = \frac{1}{\cos(2x)} \times -2\sin(2x) = -2 \times \frac{\sin(2x)}{\cos(2x)}$ $= -2\tan(2x)$	Question 20 C $\frac{dy}{dx} = (x + 4)(-2e^{-2x}) + e^{-2x} \times 1$ $\frac{dy}{dx} = e^{-2x}(-2x - 8 + 1)$ $\frac{dy}{dx} = e^{-2x}(-2x - 7)$ Increasing function when $e^{-2x}(-2x - 7) > 0$ e^{-2x} is always > 0 $\therefore$ Increasing function when $(-2x - 7) > 0$ $\Rightarrow -2x > 7$ $\Rightarrow x < -3\frac{1}{2}$
Question 21 E $f'(x) = \frac{1}{2}(3x^2 - 4x + 18)^{-\frac{1}{2}} \times (6x - 4)$ $= \frac{3x - 2}{\sqrt{3x^2 - 4x + 18}}$	Question 22 B $f(1 - 0.02) \approx f(1) - 0.02 f'(1)$

Question 23 C The given graph starts with a positive gradient, then has a gradient of zero, then a negative gradient then a zero gradient, and then a positive gradient. In the gradient graphs drawn for the solutions, a positive gradient lies above the X axis and a negative gradient lies below the X axis and a gradient of zero lies on the X axis. So the solution will start above the X axis, then go below the X axis and then above the X axis. $\therefore$ C	Question 24 E $A = \int_a^b f(x)dx - \int_b^c f(x)dx + \int_c^d f(x)dx$ $A = \int_a^b f(x)dx + \int_b^c f(x)dx + \int_c^d f(x)dx$ $\therefore$ E
Question 25 A $\sin(1-x) = 0$ $1-x = -\pi, 0, \pi, \dots$ $x = \pi+1, 1, 1-\pi, \dots$ But $0 < a < \frac{\pi}{2}$ $\therefore a = 1$ $A = \int_0^1 \sin(1-x)dx$ $= -\cos(1-x) \times -1 \Big _0^1$ $= \cos(1-x) \Big _0^1$ $= \cos(0) - \cos(1)$ $= 1 - \cos(1)$	Question 26 E $A = \int_0^2 \frac{kx}{1+x^2} dx$ $A = k \int_0^2 \frac{x}{1+x^2} dx$ $A = \frac{k}{2} \int_0^2 \frac{2x}{1+x^2} dx$ $A = \frac{k}{2} [\log_e(1+x^2)]_0^2$ $A = \frac{k}{2} [\log_e(5) - \log_e(1)]$ $A = \frac{k}{2} \log_e(5)$
Question 27 A $\int_0^3 [f(x) - 2]dx + \int_3^6 [f(x) + 1]dx = \int_0^3 f(x)dx - \int_0^3 2dx + \int_3^6 f(x)dx + \int_3^6 1dx$ $= \int_0^6 f(x)dx - \int_0^3 2dx + \int_3^6 1dx$ $= 5 - [2x]_0^3 + [x]_3^6$ $= 5 - 6 + 6 - 3 = 2$	

Question 1

a.

$$\begin{array}{r} 1 \\ x-3 \overline{)x+1} \\ \underline{x-3} \\ 4 \end{array}$$

$$f(x) = 1 + \frac{4}{x-3}$$

$$A = 1, B = 4, C = -3 \quad (1 \text{ mark})$$

b.

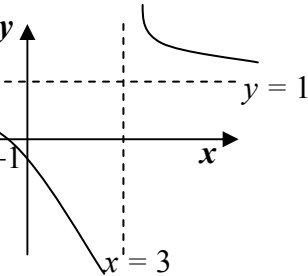
$$1 + \frac{4}{x-3} = 0$$

$$\frac{4}{x-3} = -1$$

$$-x + 3 = 4$$

$$-x = 1$$

$$x = -1$$



$$(1 \text{ mark})$$

$$\{x : x < -1\} \cup \{x : x > 3\} \quad (1 \text{ mark})$$

Question 2

$$\Pr(X < 50) = 0.35$$

$$\Pr(Z < -a) = 0.35$$

$$\Pr(Z > a) = 0.35$$

$$\Pr(Z < a) = 1 - 0.35 = 0.65$$

$$a = 0.385$$

$$-a = -0.385$$

(1 mark)

$$-0.385 = \frac{50 - 60}{\sigma}$$

$$-0.385\sigma = -10$$

$$\sigma = 26 \text{ to the nearest whole number} \quad (1 \text{ mark})$$

Question 3

a.

$$\text{Let } e^x = A$$

$$A^2 + 4A - 12 = 0$$

$$(A + 6)(A - 2) = 0$$

$$A = -6, \text{ or } A = 2 \quad (1 \text{ mark})$$

$$\Rightarrow e^x = -6, \text{ or } e^x = 2$$

$$\text{But } e^x > 0$$

$$\therefore e^x = 2 \Rightarrow x = \log_e(2) \quad (1 \text{ mark})$$

Question 3

b.

$$\log_e(y^2) = \log_e \frac{2x-8}{6}$$

$$(y^2) = \frac{2x-8}{6} \quad (1 \text{ mark})$$

$$(y^2) = \frac{x-4}{3}$$

$$y = \pm \sqrt{\frac{x-4}{3}}$$

$$\text{But } y > 0$$

$$y = \sqrt{\frac{x-4}{3}} \quad (1 \text{ mark})$$

c.

$$x - 4 > 0$$

$$\therefore x > 4 \quad (1 \text{ mark})$$

Question 4

a.

$$y = mx + c$$

$$m = \frac{dy}{dx} = 2x - 5$$

$$\text{When } x = 3, m = 1 \quad (1 \text{ mark})$$

$$y = x + c$$

$$\text{When } x = 3, y = 0$$

$$\therefore 0 = 3 + c$$

$$\Rightarrow c = -3$$

$$\therefore y = x - 3 \quad (1 \text{ mark})$$

b.

$$y = -x + c$$

$$0 = -3 + c$$

$$3 = c$$

$$y = -x + 3 \quad (1 \text{ mark})$$

c.

$$\text{At point of interscn. } x^2 - 5x + 6 = -x + 3$$

$$x^2 - 4x + 3 = 0$$

$$(x - 3)(x - 1) = 0$$

$$x = 3, \text{ or } x = 1$$

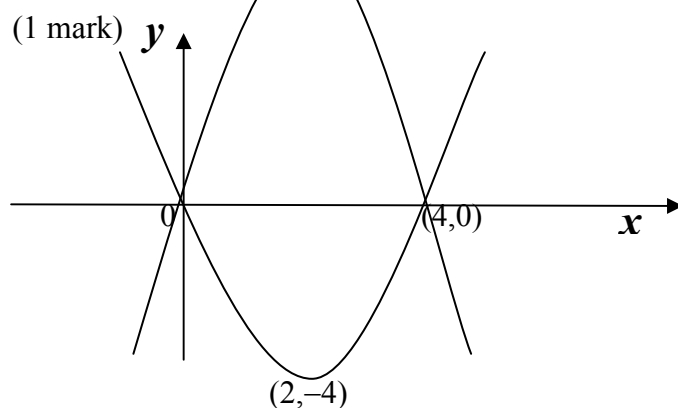
A is point where $x = 1$

$$\text{When } x = 1, y = 2$$

$$(1, 2) \quad (1 \text{ mark})$$

Question 5

a.



b.

$$g(x) = ax(x - 4)$$

$$\text{When } x = 2, g(x) = 8$$

$$8 = a \times 2 \times -2$$

$$-4a = 8$$

$$a = -2$$

$$g(x) = -2x^2 + 8x \quad (1 \text{ mark})$$

c.

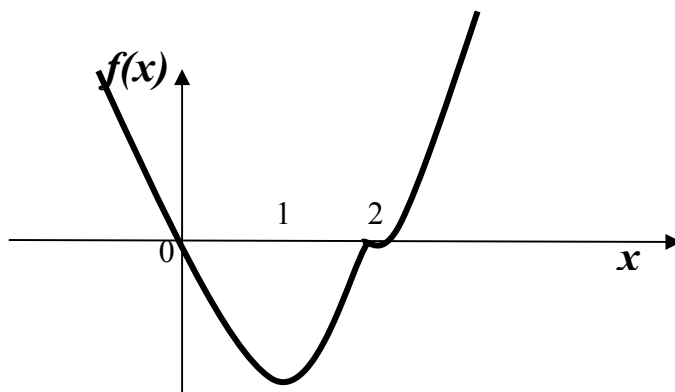
Translate $g(x)$ 2 units to left parallel to X axis and 8 units down parallel to Y axis. (2 marks)

Reflect in X axis (1 mark)

Dilate by a factor of $\frac{1}{2}$ in the Y direction (1 mark)

Question 6

- 1 mark for shape, with turning point at (1,0)
- 1 mark for (0,0)
- 1 mark for point of inflexion at (2,0)



END OF SUGGESTED SOLUTIONS
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