



# Victorian Certificate of Education 2004

## MATHEMATICAL METHODS (CAS) PILOT STUDY

### Written examination 1 (Facts, skills and applications)

Friday 5 November 2004

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.45 am (1 hour 30 minutes)

### PART I MULTIPLE-CHOICE QUESTION BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions). Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.

Part II consists of a separate question and answer book.

You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

#### Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
27	27	27

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and one approved CAS calculator (memory may be retained) and/or one scientific calculator. For the TI-92, Voyage 200 or approved computer based CAS, their full functionality may be used, but other programs or files are not permitted.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

#### Materials supplied

- Question book of 13 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

#### Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

#### At the end of the examination

- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
- You may retain this question book.

**Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.**

**Working space**

### Instructions for Part I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

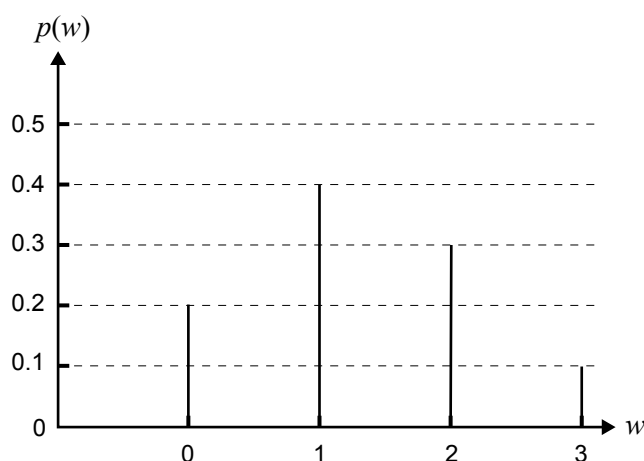
A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

#### Question 1

The graph of a probability distribution for the random variable  $W$  is shown below.



The expected value of  $W$  is equal to

- A. 0.25
- B. 1
- C. 1.3
- D. 1.5
- E. 2.5

#### Question 2

The continuous random variable  $X$  has a probability density function given by

$$f(x) = \begin{cases} 3x^2 & \text{if } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

The value of  $a$  such that  $\Pr(X < a) = 0.125$  is

- A. 0.125
- B. 0.204
- C. 0.500
- D. 0.750
- E. 0.794

**Question 3**

Christopher has five pairs of identical purple socks and three pairs of identical green socks. His socks are randomly mixed in his drawer. He takes two individual socks at random from the drawer in the dark.

The probability that he obtains a matching pair is

- A.  $\frac{1}{8}$
- B.  $\frac{5}{14}$
- C.  $\frac{3}{8}$
- D.  $\frac{13}{28}$
- E.  $\frac{1}{2}$

**Question 4**

A binomial random variable has mean 20 and variance 4.

The values of  $n$  and  $p$  respectively are

- A. 22 and 0.9
- B. 25 and 0.2
- C. 25 and 0.8
- D. 100 and 0.2
- E. 100 and 0.8

**Question 5**

The random variable  $X$  has a normal distribution with mean 12.2 and standard deviation 1.4.

If  $Z$  has the standard normal distribution, then the probability that  $X$  is greater than 15 is equal to

- A.  $\Pr(Z < 2)$
- B.  $\Pr(Z > 2)$
- C.  $\Pr(Z > -2)$
- D.  $1 - \Pr(Z > 2)$
- E.  $1 - \Pr(Z < -2)$

**Question 6**

A sine function  $f$  has an amplitude of 3 and a period of  $\frac{1}{10}$ .  
The rule for  $f$  could be

- A.  $f(t) = 3 \sin \left( \frac{\pi t}{5} \right)$
- B.  $f(t) = 3 \sin (20\pi t)$
- C.  $f(t) = 3 \sin (10t)$
- D.  $f(t) = 1.5 \sin \left( \frac{\pi t}{5} \right)$
- E.  $f(t) = 1.5 \sin (20\pi t)$

**Question 7**

The depth of water near the Lorne Pier changes with the tides according to the rule

$$h(t) = 7 - 2 \sin \left( \frac{4\pi t}{25} + \frac{\pi}{2} \right)$$

where  $t$  is the time in hours after low tide and  $h$  is the depth in metres. A low tide occurred at midnight.

The time of the **next high** tide is

- A. 6.00 am
- B. 6.15 am
- C. 12.30 pm
- D. 6.30 pm
- E. 1.00 am on the next day

**Question 8**

The **sum** of the solutions of  $\cos \left( \frac{x}{2} \right) = \frac{\sqrt{3}}{2}$ , for  $0 \leq x \leq 4\pi$ , is

- A.  $\frac{\pi}{6}$
- B.  $\frac{\pi}{3}$
- C.  $\frac{11\pi}{3}$
- D.  $4\pi$
- E.  $8\pi$

**Question 9**

Which one of the following is a **complete** set of **linear** factors of the third degree polynomial  $ax^3 - bx$ , where  $a$  and  $b$  are positive real numbers?

- A.  $x, ax^2 - b$
- B.  $x, ax - b, ax + b$
- C.  $x, \sqrt{ax - b}$
- D.  $x, \sqrt{a}x - b, \sqrt{a}x + b$
- E.  $x, \sqrt{a}x - \sqrt{b}, \sqrt{a}x + \sqrt{b}$

**Question 10**

Which one of the following functions does **not** have an inverse function?

- A.  $f: [2, 4) \rightarrow R, f(x) = \sqrt{x - 2}$
- B.  $g: R \setminus \{0\} \rightarrow R, g(x) = \frac{1}{x^2}$
- C.  $h: R^+ \rightarrow R, h(x) = x^3$
- D.  $k: (-\infty, 0] \rightarrow R, k(x) = x^2 + 1$
- E.  $m: R^+ \rightarrow R, m(x) = \frac{1}{x + 3}$

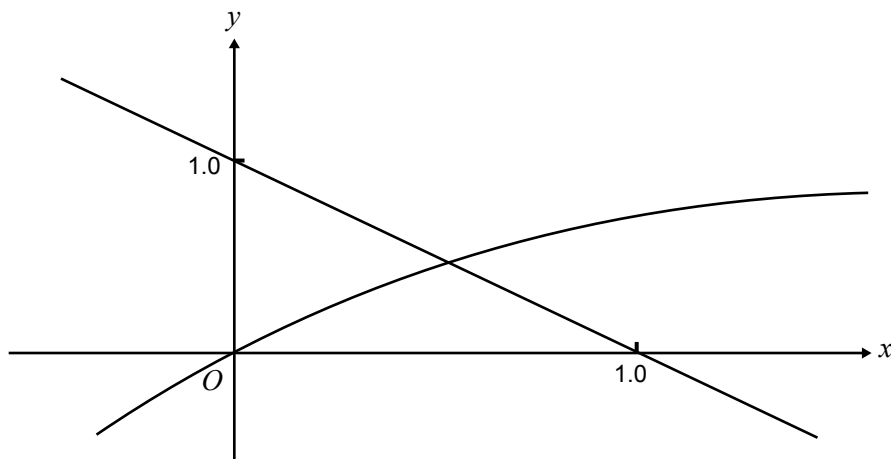
**Question 11**

If  $f(x) = e^{2x}$  for all  $x \in R$ , then  $[f(x)]^2 = f(y)$ , where  $y$  is equal to

- A.  $2x$
- B.  $4x$
- C.  $x^2$
- D.  $2x^2$
- E.  $4x^2$

**Question 12**

Parts of the graphs of the functions with equations  $y = \log_e(x + 1)$  and  $y = 1 - x$  are shown below.

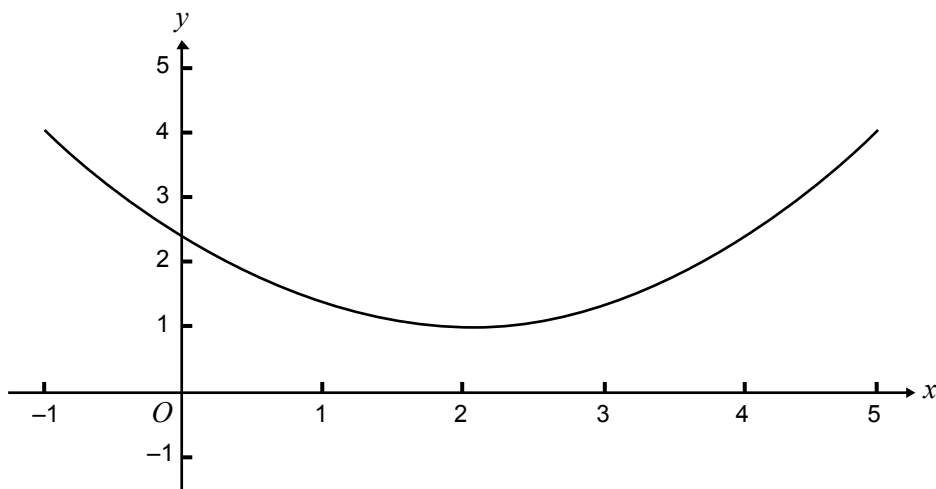


The solution of the equation  $\log_e(x + 1) = 1 - x$  is closest to

- A. 0.44
- B. 0.45
- C. 0.55
- D. 0.56
- E. 0.57

**Question 13**

The following shows part of the graph of the curve with equation  $y = P(x - Q)^2 + 1$ .



The values of  $P$  and  $Q$  respectively could be

- |    | $P$           | $Q$ |
|----|---------------|-----|
| A. | $\frac{1}{3}$ | -2  |
| B. | $\frac{1}{3}$ | 2   |
| C. | 1             | 2   |
| D. | 3             | -2  |
| E. | 3             | 2   |

**Question 14**

Let  $f$  be a polynomial function of degree 3. The graph of the curve with rule  $y = f(x)$  either intersects the  $x$ -axis or touches the  $x$ -axis at exactly two points  $(a, 0)$  and  $(b, 0)$ .

A possible rule for  $f$  could be

- A.  $f(x) = (x - a)(x - b)$
- B.  $f(x) = (x - a)(x + b)^2$
- C.  $f(x) = (x - a)(x - b)^2$
- D.  $f(x) = (x + a)^2(x - b)$
- E.  $f(x) = (x + a)^2(x + b)$

**Question 15**

The maximal domain of the function  $f$  with rule  $f(x) = \log_e(|x|) + 1$  is

- A.  $\mathbb{R} \setminus \{0\}$
- B.  $(-1, \infty)$
- C.  $\mathbb{R}$
- D.  $(0, \infty)$
- E.  $(-\infty, 0)$

**Question 16**

The graph of the curve with rule  $y = f(x)$ , where  $f$  is a one-to-one function, has exactly one asymptote whose equation is  $x = 3$ .

The graph of the curve with rule  $y = f^{-1}(x)$ , where  $f^{-1}$  is the inverse function of  $f$ , will have a

- A. horizontal asymptote at  $y = -3$
- B. horizontal asymptote at  $y = \frac{1}{3}$
- C. horizontal asymptote at  $y = 3$
- D. vertical asymptote at  $x = \frac{1}{3}$
- E. vertical asymptote at  $x = 3$

**Question 17**

The graph of the function  $f$  with rule  $f(x) = \cos(x)$  is transformed to the graph of the function  $g$  with rule  $g(x) = 5 \cos(3x)$  by

- A. a dilation from the  $x$ -axis by a scale factor of 5 and a dilation from the  $y$ -axis by a scale factor of 3
- B. a dilation from the  $x$ -axis by a scale factor of 3 and a dilation from the  $y$ -axis by a scale factor of 5
- C. a dilation from the  $x$ -axis by a scale factor of  $\frac{1}{3}$  and a dilation from the  $y$ -axis by a scale factor of 5
- D. a dilation from the  $x$ -axis by a scale factor of  $\frac{1}{5}$  and a dilation from the  $y$ -axis by a scale factor of 3
- E. a dilation from the  $x$ -axis by a scale factor of 5 and a dilation from the  $y$ -axis by a scale factor of  $\frac{1}{3}$



**Question 18**

At  $x = -1$ , the graph of the function  $f$ , with rule  $f(x) = (x + 1)^3(x - 1) + 4$  has

- A. an  $x$ -axis intercept.
- B. a  $y$ -axis intercept.
- C. a local minimum.
- D. a local maximum.
- E. a point of inflection with zero gradient.

**Question 19**

If  $y = |\sin(x)|$ , the rate of change of  $y$  with respect to  $x$  at  $x = k$ ,  $\pi < k < 2\pi$ , is

- A.  $-\cos(k)$
- B.  $\cos(k)$
- C.  $-\sin(k)$
- D.  $\sin(k)$
- E.  $k \cos(1)$

**Question 20**

The number of ants,  $N$ , in a colony varies with time according to the rule  $N(t) = 1000e^{0.1t}$ , where  $t$  is the time measured in days, and  $t \geq 0$ .

The **average** rate of change in the number of ants over the first 10 days is closest to

- A. 172
- B. 183
- C. 272
- D. 1718
- E. 2718

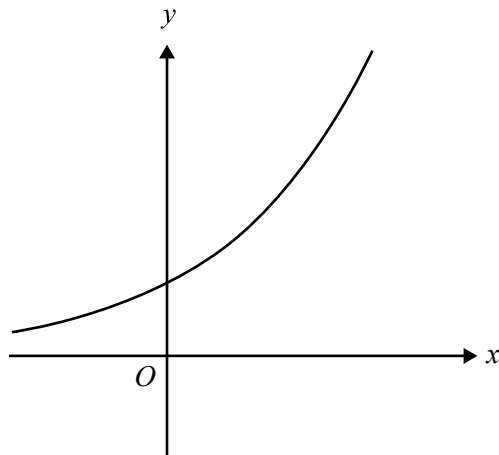
**Question 21**

Using the approximation formula  $f(x + h) \approx f(x) + hf'(x)$ , with  $f(x) = x^3$  and  $x = 2$ , an approximate value of  $(1.8)^3$  is given by

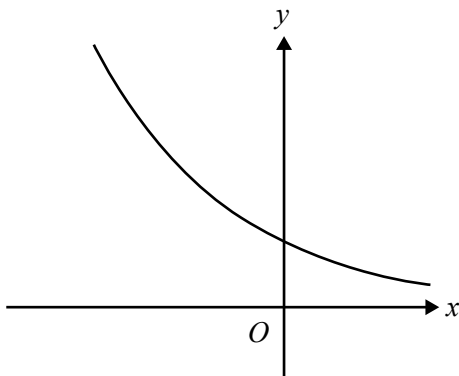
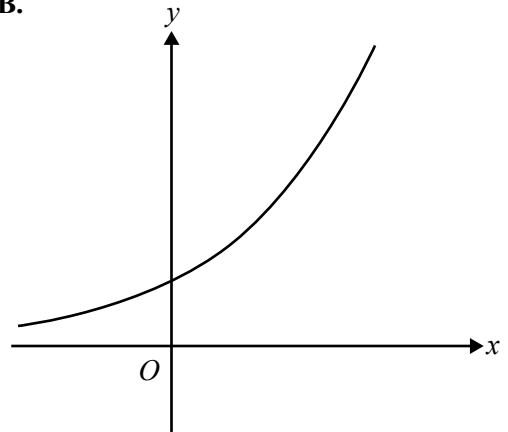
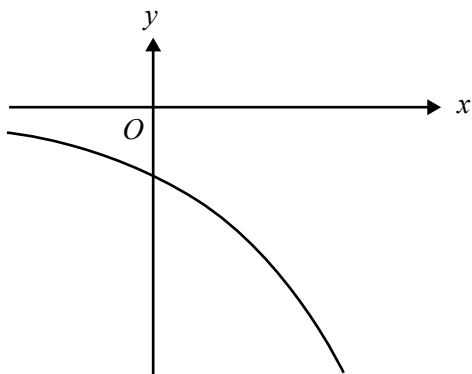
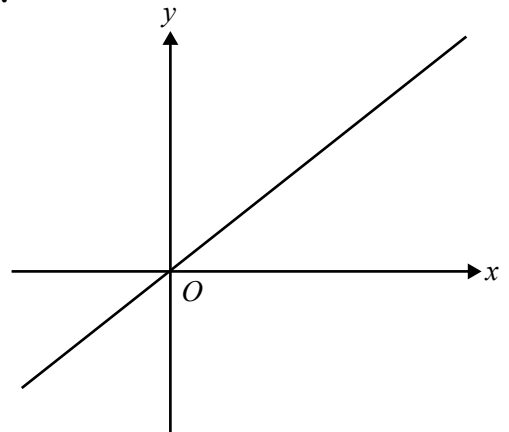
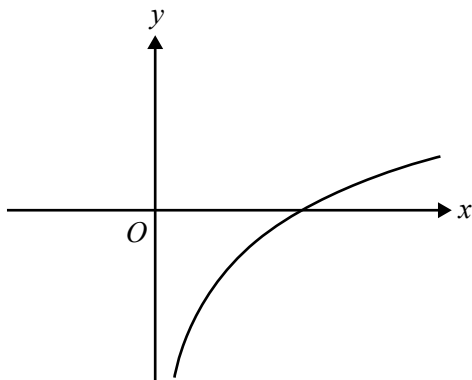
- A.  $f(2) + 0.2f'(2)$
- B.  $f(8) + 0.2f'(8)$
- C.  $f(2) - f(1.8)$
- D.  $f(2) - 0.2f'(2)$
- E.  $f(8) - 0.2f'(8)$

**Question 22**

The graph of the function  $f$ , with rule  $y = f(x)$ , is shown below.

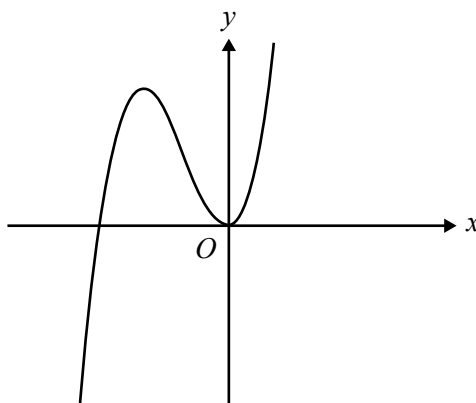


Which one of the following could be the graph of the curve with equation  $y = f'(x)$ ?

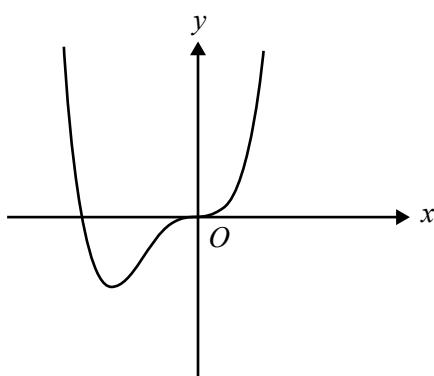
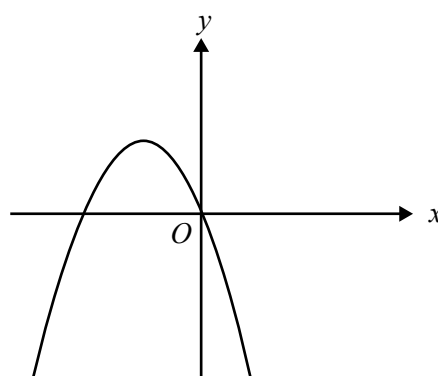
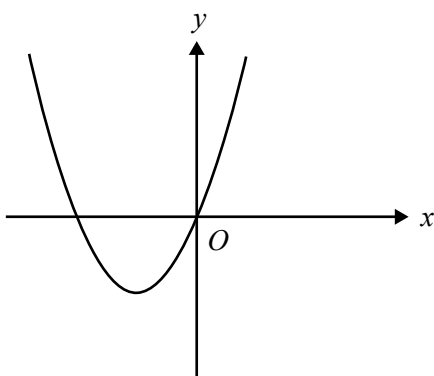
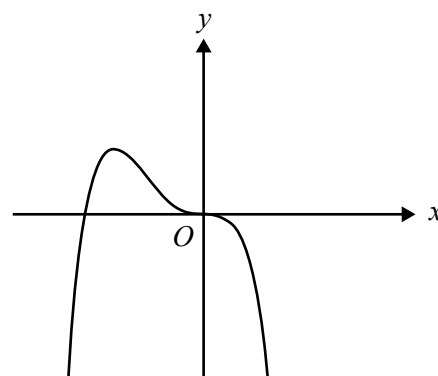
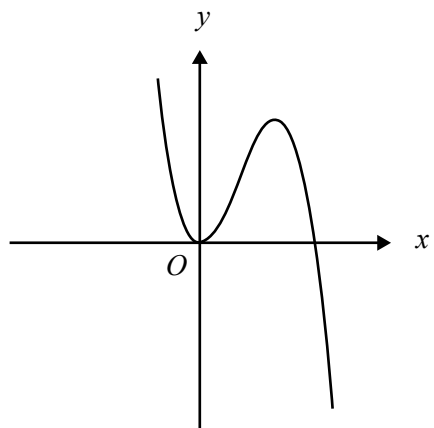
**A.****B.****C.****D.****E.**

**Question 23**

The graph of the function  $f$ , with rule  $y = f(x)$ , is shown below.



Which one of the following is most likely to be the graph of an antiderivative function of  $f$ ?

**A.****B.****C.****D.****E.**

**Question 24**

The interval  $[1, 2]$  is divided into  $n$  equal subintervals by the points  $x_0, x_1, \dots, x_{n-1}, x_n$  where  $1 = x_0 < x_1 < \dots < x_{n-1} < x_n = 2$ .

Let  $\delta x = x_i - x_{i-1}$  for  $i = 1, 2, \dots, n$ .

Then  $\lim_{\delta x \rightarrow 0} \sum_{i=1}^n (x_i^2 \delta x)$  is equal to

A. 0

B. 3

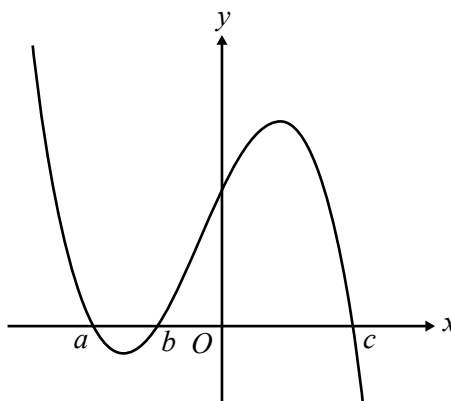
C.  $\int_1^2 2x dx$

D.  $\int_1^2 x^2 dx$

E.  $\int_1^2 \frac{x^3}{3} dx$

**Question 25**

The graph of the function  $f$ , with rule  $y = f(x)$ , is shown below.



The total area bounded by the curve with equation  $y = f(x)$  and the  $x$ -axis on the interval  $[a, c]$  is given by

A.  $\int_a^c f(x) dx$

B.  $\int_a^b f(x) dx + \int_b^c f(x) dx$

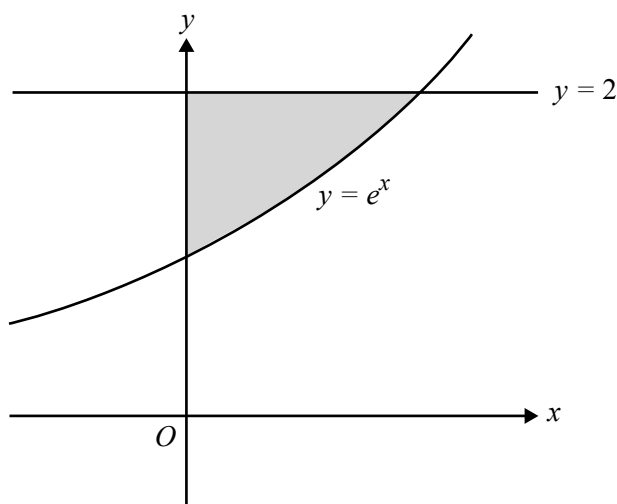
C.  $-\int_a^b f(x) dx + \int_b^c f(x) dx$

D.  $-\int_a^0 f(x) dx + \int_0^c f(x) dx$

E.  $-\int_a^b f(x) dx - \int_b^0 f(x) dx - \int_0^c f(x) dx$

**Question 26**

Parts of the graphs with equations  $y = e^x$  and  $y = 2$  are shown below.



The total area of the region bounded by the  $y$ -axis, the line  $y = 2$  and the curve with equation  $y = e^x$  is given by

- A.  $\int_0^{e^2} (e^x - 2) dx$
- B.  $\int_0^{e^2} (2 - e^x) dx$
- C.  $\int_0^{\log_e(2)} (e^x - 2) dx$
- D.  $\int_0^{\log_e(2)} (2 - e^x) dx$
- E.  $\int_0^2 (e^x - 2) dx$

**Question 27**

If  $\int_a^b f(x) dx = 3$ , then  $\int_a^b (4 - 2f(x)) dx$  is equal to

- A.  $4(b - a) - 6$
- B.  $4(b - a) + 6$
- C.  $4(a - b) + 6$
- D.  $-2$
- E.  $10$



# Victorian Certificate of Education 2004

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Letter

Figures

Words


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## MATHEMATICAL METHODS (CAS) PILOT STUDY Written examination 1 (Facts, skills and applications)

Friday 5 November 2004

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.45 am (1 hour 30 minutes)

### PART II QUESTION AND ANSWER BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions). Part I consists of a separate question book and must be answered on the answer sheet provided for multiple-choice questions.

Part II consists of this question and answer book.

You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

#### Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
6	6	23

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and one approved CAS calculator (memory may be retained) and/or one scientific calculator. For the TI-92, Voyage 200 or approved computer based CAS, their full functionality may be used, but other programs or files are not permitted.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

#### Materials supplied

- Question and answer book of 7 pages.

#### Instructions

- Detach the formula sheet from the centre of the Part I book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

#### At the end of the examination

- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book.

**Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.**

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**Instructions for Part II**

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

In questions where more than 1 mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1**

The diameters of circular mats produced by a machine are normally distributed, with mean 12 cm and standard deviation 1.5 cm.

- a. Sketch the probability density curve for the diameters of the circular mats produced by the machine.
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- b. It is known that 16.00 % of mats produced by the machine have a diameter less than  $K$  cm. Find the value of  $K$ , correct to one decimal place.

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2 + 1 = 3 marks



**Question 2**

Find the relationship between  $m$  and  $k$  such that the line with equation  $y = mx$  intersects the curve with equation  $y = x^2(x - k)$  at the point  $(0, 0)$  and one other point only.

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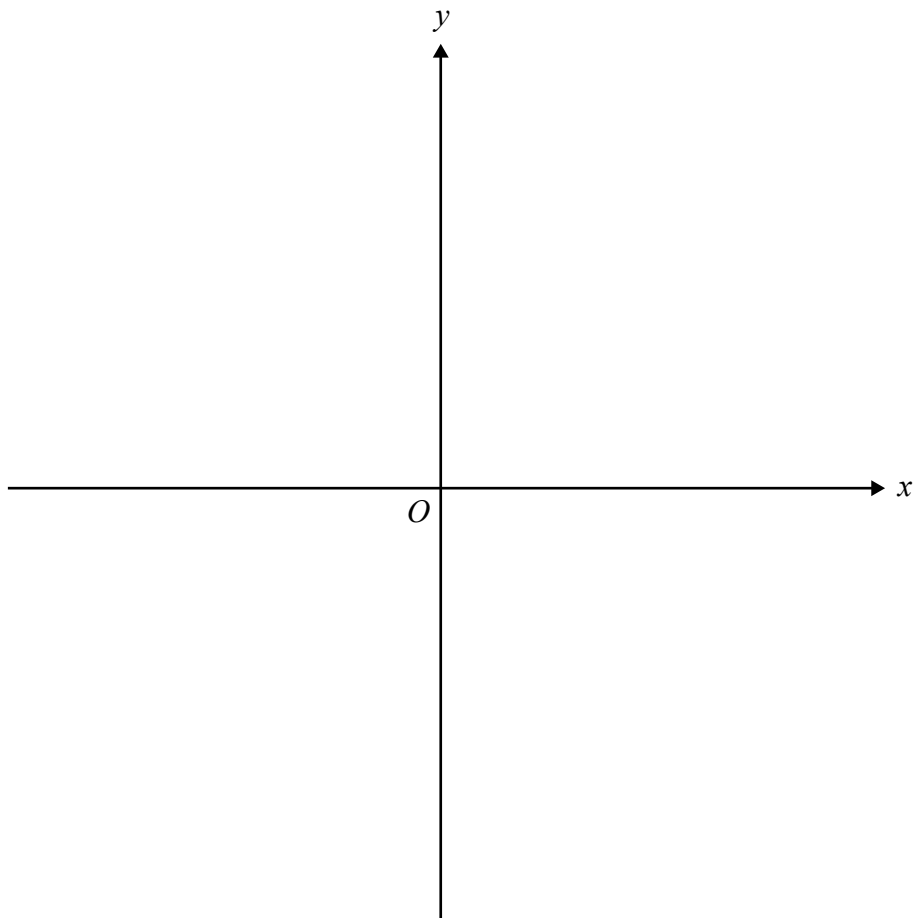
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3 marks

**Question 3**

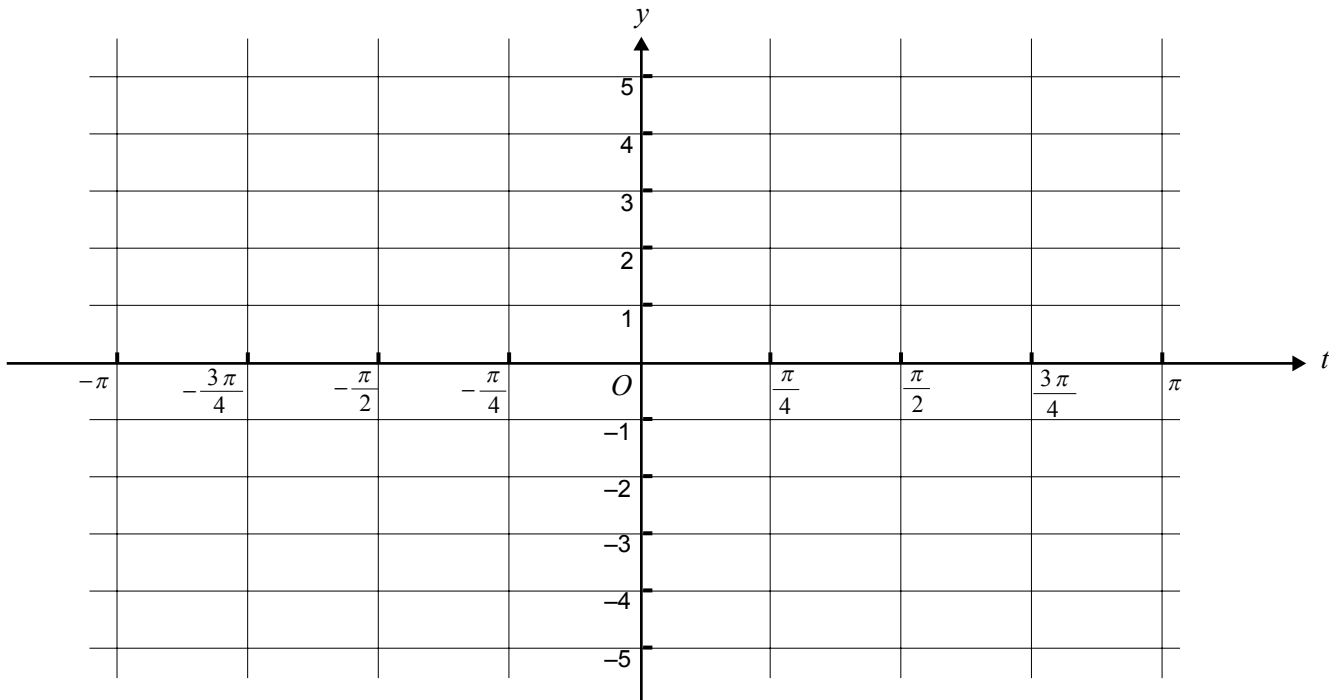
On the set of axes below, sketch the graph of the function  $f$  with rule  $f(x) = 2 - \frac{1}{|x+1|}$ .  
Label each asymptote with its equation.



2 marks

**Question 4**

- a. On the set of axes below, sketch the graph of the function with rule  $y = 3 \sin \left( 2 \left( t - \frac{\pi}{4} \right) \right)$ ,  $-\pi \leq t \leq \pi$ .



- b. Write down the general solution of the equation  $3 \sin \left( 2 \left( t - \frac{\pi}{4} \right) \right) = \frac{3}{2}$  where  $t \in R$ .

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2 + 3 = 5 marks

**Question 5**

Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a continuous function with the following properties.

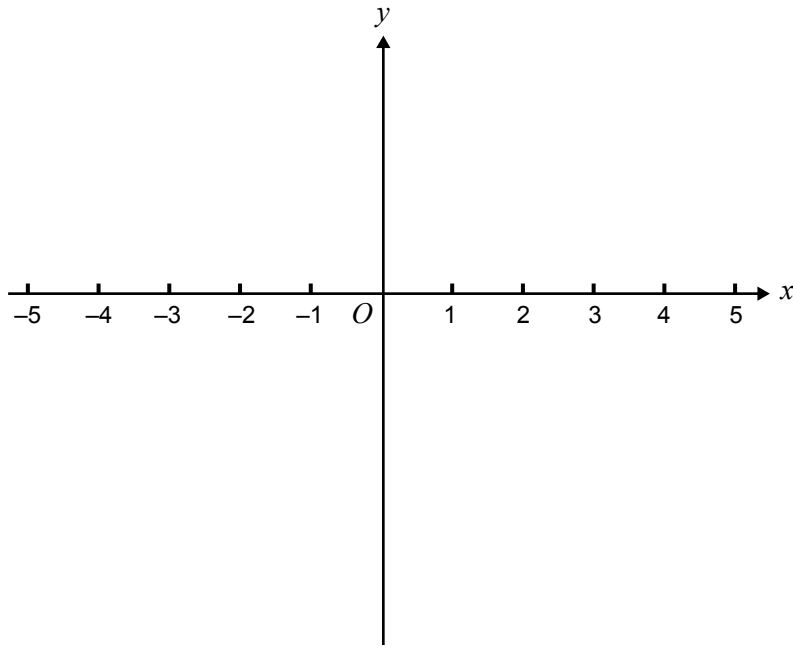
$$f(0) = 0 \quad f'(0) = 0$$

$$f(4) = 0 \quad f'(3) = 0$$

$$f'(x) < 0 \text{ for } x \in (-\infty, 0) \cup (0, 3)$$

$$f'(x) > 0 \text{ for } x \in (3, \infty)$$

- a. On the set of axes provided below, sketch a possible graph of  $f$ .



- b. If  $f(x) = ax^3(x - b)$  and the point  $(2, -4)$  also lies on the graph of  $f$ , find the values of  $a$  and  $b$ .

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- c. Find the equation of the tangent to the graph of  $f$  at the point where  $x = 4$ .

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2 + 2 + 3 = 7 marks

**Question 6**

A barrel contains 100 balls, some of which are rainbow-coloured. Four balls are randomly selected from the barrel, **with replacement**. This means that a ball is selected, its colour noted, and the ball replaced **before** the next ball is selected.

Let  $p$  be the proportion of rainbow-coloured balls in the barrel.

- a. Write down an expression for the probability that **exactly** one of the four balls selected is rainbow-coloured.

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- b. Find the exact value of  $p$  for which this probability will be a maximum.

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1 + 2 = 3 marks

# **MATHEMATICAL METHODS (CAS) PILOT STUDY**

## **Written examinations 1 and 2**

### **FORMULA SHEET**

#### **Directions to students**

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

## Mathematical Methods CAS Formulas

### Mensuration

area of a trapezium:  $\frac{1}{2}(a+b)h$

curved surface area of a cylinder:  $2\pi rh$

volume of a cylinder:  $\pi r^2 h$

volume of a cone:  $\frac{1}{3}\pi r^2 h$

volume of a pyramid:  $\frac{1}{3}Ah$

volume of a sphere:  $\frac{4}{3}\pi r^3$

area of a triangle:  $\frac{1}{2}bc \sin A$

### Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$$

approximation:  $f(x+h) \approx f(x) + hf'(x)$

average value:  $\frac{1}{b-a} \int_a^b f(x) dx$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

product rule:  $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

quotient rule:  $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

### Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

mean:  $\mu = E(X)$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

transition matrices:  $S_n = T^n \times S_0$

variance:  $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

Discrete distributions			
	$\Pr(X=x)$	mean	variance
general	$p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$ $= \sum x^2 p(x) - \mu^2$
binomial	${}^n C_x p^x (1-p)^{n-x}$	$np$	$np(1-p)$
hypergeometric	$\frac{{}^D C_x {}^{N-D} C_{n-x}}{{}^N C_n}$	$n \frac{D}{N}$	$n \frac{D}{N} \left(1 - \frac{D}{N}\right) \left(\frac{N-n}{N-1}\right)$
Continuous distributions			
	$\Pr(a < X < b)$	mean	variance
general	$\int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$ $= \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2$
normal	If $X$ is distributed $N(\mu, \sigma^2)$ and $Z = \frac{X - \mu}{\sigma}$ , then $Z$ is distributed $N(0, 1)$ , $f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$		

Table 1 Normal distribution – cdf

$x$	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

END OF FORMULA SHEET