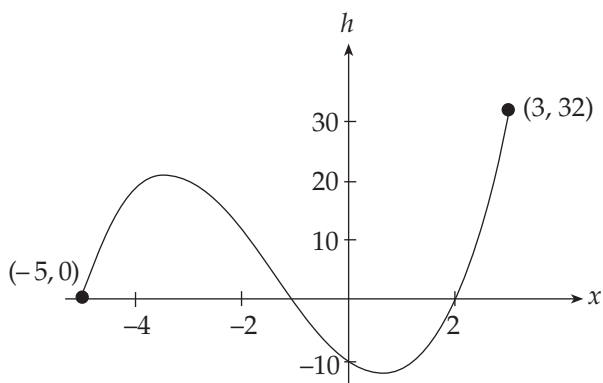
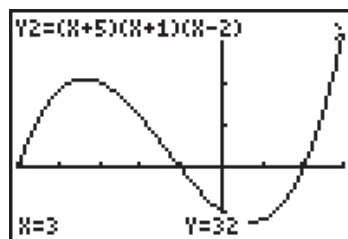


2004 Mathematical Methods Written Examination 2 (Analysis task) Suggested answers and solutions

Question 1

- a The end points are $(-5, 0)$ and $(3, 32)$. [2A]



- b $f\left(\frac{1}{2}\right) = -\frac{99}{8}$ and $f(-5) = 0$

$$m = \frac{-\frac{99}{8} - 0}{\frac{1}{2} + 5} = -\frac{9}{4} \quad [1M]$$

$$y - 0 = -\frac{9}{4}(x + 5)$$

$$4y = -9x - 45 \quad [1A]$$

- c $h_1(x) = (x + 5)(x + 1)(x - 2)$
 $= x^3 + 4x^2 - 7x - 10$

$$h'_1(x) = 3x^2 + 8x - 7 \quad [1M]$$

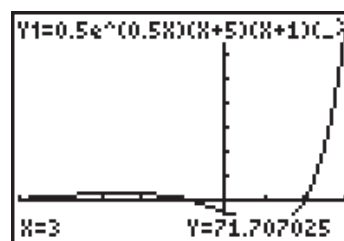
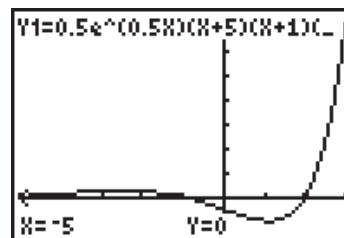
$$h'_1\left(\frac{1}{2}\right) = \frac{3}{4} + 4 - 7 = -\frac{9}{4} \quad [1M]$$

Since the gradient of the tangent to

$h_1(x)$ at $x = \frac{1}{2}$ is $-\frac{9}{4}$ then $4y = -9x - 45$

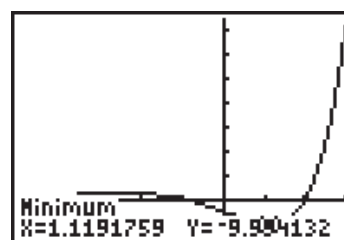
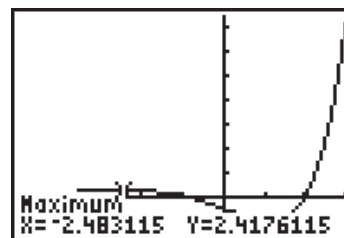
is the equation of the tangent. [1A]

- d The end points are $(-5, 0)$ and $(3, 71.71)$. [1A]



The turning points are $(-2.48, 2.42)$
and $(1.12, -9.99)$.

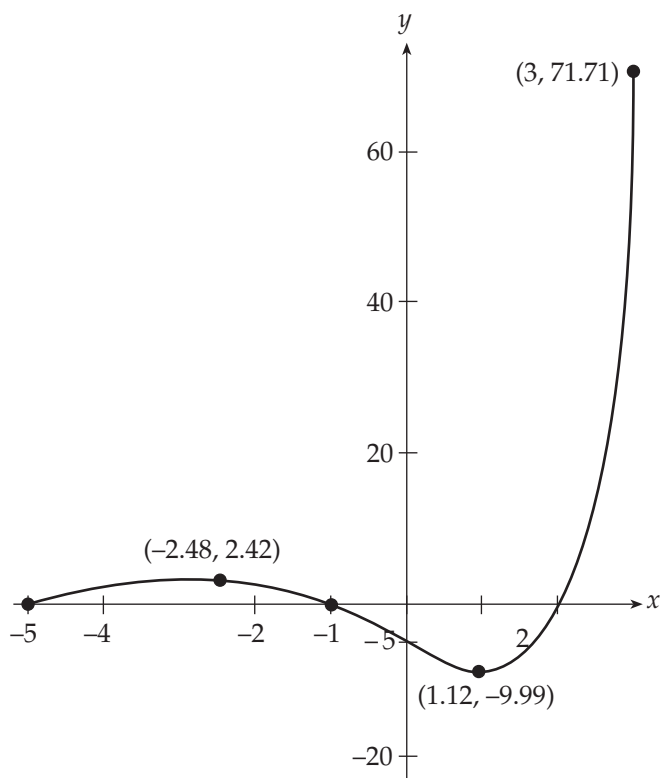
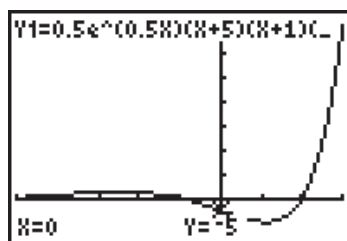
[1A]



The x -intercepts are -5 , -1 and 2 .

The y -intercept is -5 .

[1A]



Correct shape.

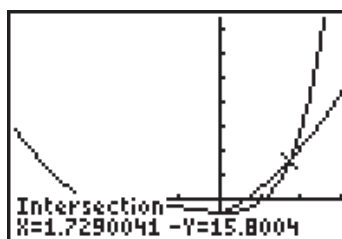
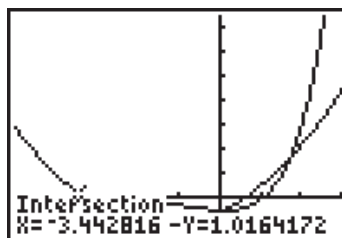
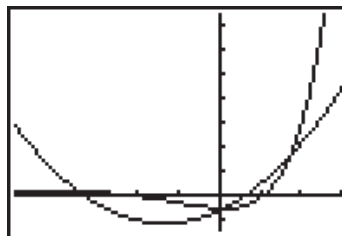
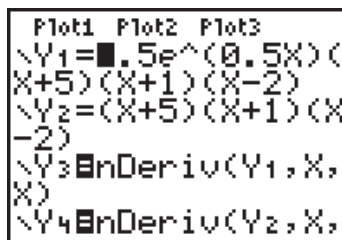
[1A]

e $h_2(3) - h_1(3) = 71.707 - 32 = 39.71$

[1A]

f $\{x: -3.44 < x < 0.11\} \cup \{x: 1.73 \leq x \leq 3\}$

[2A]



g $h'_1(x) = \frac{1}{4}e^{\frac{1}{2}x}(x^3 + 4x^2 - 7x - 10)$

$$+ \frac{1}{2}e^{\frac{1}{2}x}(3x^2 + 8x - 7)$$

via the product rule

[1M]

$$= \frac{1}{4}e^{\frac{1}{2}x}[(x^3 + 4x^2 - 7x - 10) + 2(3x^2 + 8x - 7)]$$

$$= \frac{1}{4}e^{\frac{1}{2}x}(x^3 + 10x^2 + 9x - 24)$$

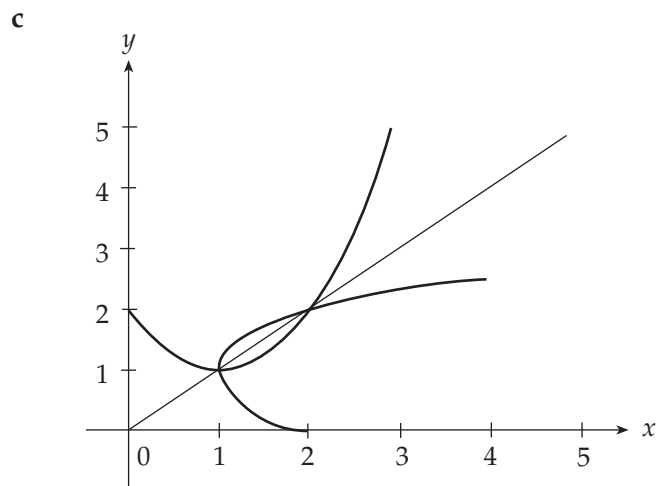
$a = 1, b = 10, c = 9$ and $d = -24$

[2A]

Question 2

a $x = -\frac{b}{2a}$ [1A]

b From the graph x is positive.
 a is positive as there is a minimum. [1A]
 Hence b has to be negative.



Correct endpoints [1A]

Correct shape [1A]

d $x = ay^2 + by + 2$ [1M]

$$0 = ay^2 + by + 2 - x$$

$$y = \frac{-b \pm \sqrt{b^2 - 4a(2-x)}}{2a} \quad [1M]$$

$$= \frac{-b \pm \sqrt{b^2 + 4a(x-2)}}{2a}$$

$$= \frac{-b \pm \sqrt{4a(x-2) + b^2}}{2a} \text{ as required} \quad [1A]$$

e $x = ax^2 + bx + 2$ where $a > 0$ and $b < 0$

$$x = -\frac{b}{2a} \text{ at the turning point.}$$

$$-\frac{b}{2a} = a\left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) + 2 \quad [1M]$$

$$-2b = b^2 - 2b^2 + 8a \quad [1M]$$

$$-8a = -b^2 + 2b$$

$$a = \frac{b^2 - 2b}{8} \text{ as required}$$

f $1 = \frac{b^2 - 2b}{8}$

$$b^2 - 2b - 8 = 0$$

$$(b-4)(b+2) = 0$$

$$\text{Hence } b = -2 \quad [1A]$$

g $f(x) = x^2 - 2x + 2$

$x = x^2 - 2x + 2$ at the points of intersection.

$$0 = x^2 - 3x + 2$$

$$0 = (x-2)(x-1)$$

$$x = 1 \text{ or } 2 \quad [1M]$$

$$\text{Area} = 2 \int_1^2 \left(x - (x^2 - 2x + 2) \right) dx \quad [1M]$$

$$= 2 \int_1^2 (-x^2 + 3x - 2) dx$$

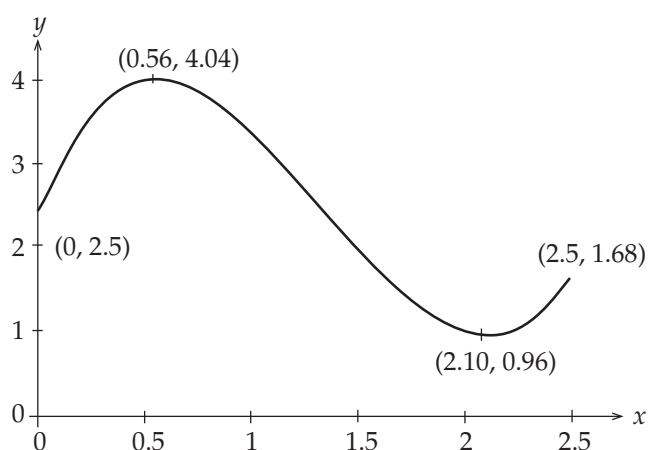
$$= 2 \left[-\frac{x^3}{3} + \frac{3x^2}{2} - 2x \right]_1^2 \quad [1M]$$

$$= 2 \left(\left(-\frac{8}{3} + 6 - 4 \right) - \left(-\frac{1}{3} + \frac{3}{2} - 2 \right) \right)$$

$$= \frac{1}{3} \text{ square unit} \quad [1A]$$

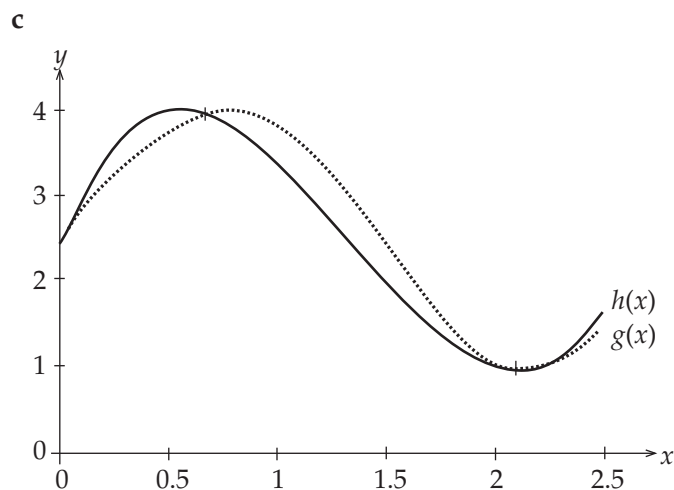
Question 3

a



Four correct coordinates [4A]

b Amplitude = $0.5(4 - 1)$
 $A = 1.5$ [1A]
Period = $2(2.25 - 0.75)$
 $= 3$
Period = $\frac{2\pi}{n\pi}$; $n = \frac{2}{3}$ [1A]
 $B = 2.5$
 $h(x) = 1.5\sin\left(\frac{2}{3}\pi x\right) + 2.5$ [1A]



Correct shape [1A]
Labelling $g(x)$ and $h(x)$ [1A]
Endpoints at $x = 2.5$ [1A]

d $g'(x) = \pi \cos\left(\frac{2\pi}{3}x\right)$ [1A]
Maximum occurs where $\cos\left(\frac{2\pi}{3}x\right) = 1$
 $\frac{2\pi}{3}x = 0, 2\pi$ [1A]
 $x = 0$ or 3 (outside domain)
therefore rate of change is greatest at $x = 0$. [1A]

e $\int_0^{2.5} h(x) \cdot dx = 6.323 \text{ km}^2$ [1A]
 $\int_0^{2.5} g(x) dx = 6.608 \text{ km}^2$ [1A]

Question 4

a D represents a damaged item and P a perfect team

1 st item	2 nd item	3 rd item	Outcome	Probability
	D		Rejected	$\frac{3 \times 2}{10 \times 9} = \frac{1}{15}$
D	P	D	Rejected	$\frac{3 \times 7 \times 2}{10 \times 9 \times 8} = \frac{7}{120}$
	P	P	Accepted	$\frac{3 \times 7 \times 6}{10 \times 9 \times 8} = \frac{7}{40}$
P	D	D	Rejected	$\frac{7 \times 3 \times 2}{10 \times 9 \times 8} = \frac{7}{120}$
	P	P	Accepted	$\frac{7 \times 3 \times 6}{10 \times 9 \times 8} = \frac{7}{40}$
		P	Accepted	$\frac{7 \times 6}{10 \times 9} = \frac{7}{15}$

Partially correct branches [1M]
All branches correct [1A]
Correct outcomes [1A]

b $\Pr(\text{accepted}) = \frac{7}{40} + \frac{7}{40} + \frac{7}{15} = \frac{49}{60}$ [1M]
[1A]

c $\Pr(\text{only 2 items checked} \mid \text{accepted})$
 $= \frac{7}{15} \div \frac{49}{60}$ [1M]
 $= \frac{4}{7}$ [1A]

d Binomial $n = 8, p = \frac{49}{60}$

Pr (6 boxes accepted)

$$= {}^8C_6 \times \left(\frac{49}{60}\right)^6 \times \left(\frac{11}{60}\right)^2 \quad [1M]$$

$$= 0.2792 \quad [1A]$$

e $\mu = np = 500 \times 0.03$

$$= 15 \quad [1A]$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{500 \times 0.03 \times 0.97}$$

$$= 3.8144 \quad [1A]$$

$$\text{normalcdf}(6, 16, 15, 3.8144)$$

$$= 0.59 \quad [1H]$$

$$\text{or normalcdf}(5.5, 16.5, 15, 3.8144)$$

$$= 0.65$$