

2004 Mathematical Methods

Written Examination 1 (Facts, skills and applications)

Suggested answers and solutions

Answers – Multiple Choice

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. B | 2. C | 3. A | 4. E | 5. C |
| 6. C | 7. E | 8. D | 9. E | 10. A |
| 11. A | 12. B | 13. D | 14. E | 15. C |
| 16. A | 17. D | 18. E | 19. C | 20. A |
| 21. C | 22. B | 23. C | 24. C | 25. D |
| 26. A | 27. E | | | |

Question 1

The graph is a negative quartic with solutions at $x = a$ and $x = b$.

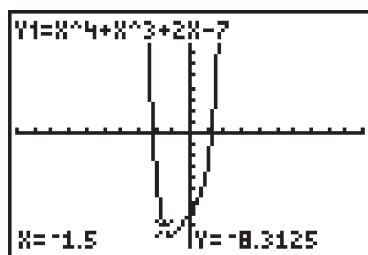
[B]

Question 2

$$2x + 3 = 0$$

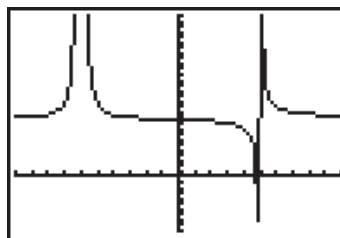
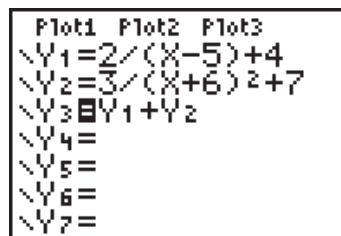
$$x = -\frac{3}{2}$$

$$\begin{aligned} f\left(-\frac{3}{2}\right) &= \left(-\frac{3}{2}\right)^4 + \left(-\frac{3}{2}\right)^3 + 2\left(-\frac{3}{2}\right) - 7 \\ &= -8\frac{5}{16} \end{aligned}$$



[C]

Question 3



The range is R

[A]

Question 4

The asymptotes of $y = \frac{-3}{(x-4)^2} + 2$ are

$x = 4$ and $y = 2$.

Hence the equations of the asymptotes of the inverse are $x = 2$ and $y = 4$.

[E]

Question 5

$y = -e^x$ reflected in the y -axis is $y = -e^{-x}$,

and dilated by a factor of $\frac{1}{2}$ from the y -axis is

$$y = -e^{-2x}.$$

[C]

Question 6

A dilation from the x -axis is $y = \frac{2}{\sin(x)}$.

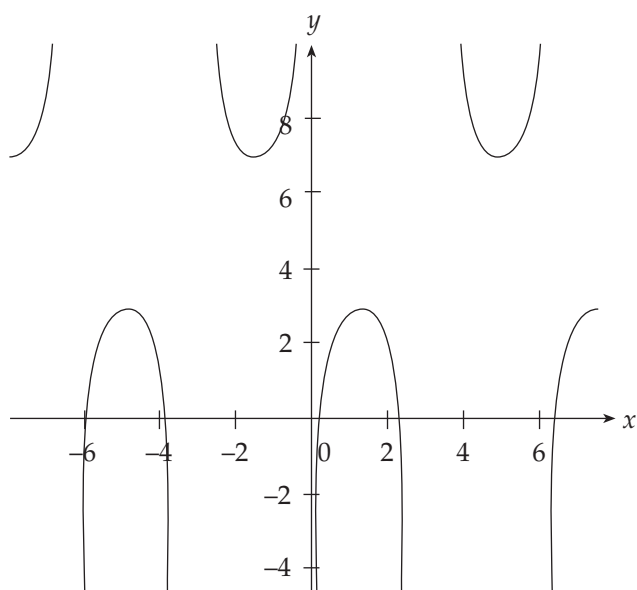
A reflection in both axes is $y = \frac{-2}{\sin(-x)} = \frac{2}{\sin(x)}$.

A translation 5 units parallel to the y -axis is

$$y = \frac{2}{\sin(x)} + 5.$$

A translation 3 units parallel to the x -axis is

$$y = \frac{2}{\sin(x-3)} + 5.$$



[C]

Question 7

$$f(x) = A - B \sin(Cx + D)$$

The amplitude is $|B| = B$

The period is $\frac{2\pi}{C}$

Question 8

Via the chain rule

$$\begin{aligned} \frac{dy}{dx} &= 2 \times 0.5 \sin(2x) 2 \cos(2x) \\ &= 2 \sin(2x) \cos(2x) \end{aligned}$$

[D]

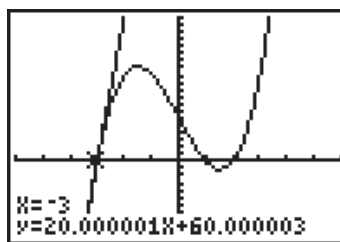
Question 9

$$\begin{aligned} \text{Area} &= 2 \int_0^{\frac{\pi}{4}} \sin(x) \cdot dx = 2[-\cos(\frac{\pi}{4}) + \cos(0)] \\ &= 2[-\frac{1}{\sqrt{2}} + 1] \\ &= 2 - \sqrt{2} \end{aligned}$$

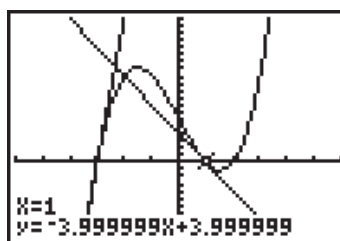
[E]

Question 10

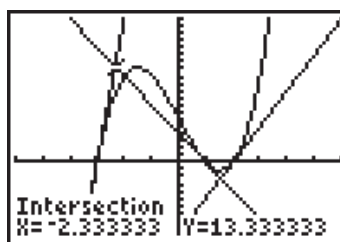
The equation of the tangent at $x = -3$ is $y = 20x + 60$,



and at $x = 1$, $y = -4x + 4$



$y = -4x + 4$ and $y = 20x + 60$ intersect at $(-2\frac{1}{3}, 13\frac{1}{3})$.



[E]

The gradients are -4 and 20 .

[A]

Question 11

$$f(x+h) \approx f(x) + h f'(x)$$

$$f(x+h) - f(x) \approx h f'(x)$$

$$= 0.001 \times \frac{-2}{(x+4)^2}$$

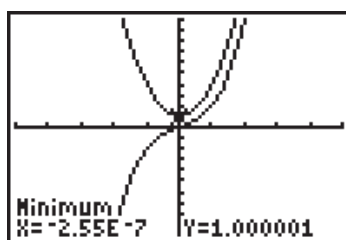
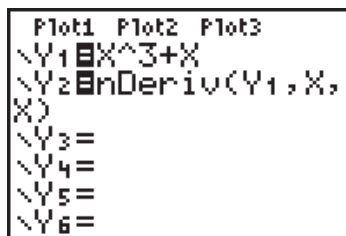
[A]

Question 12

$f(x) = x^3 + x$ has a point of inflection at $x = 0$.

$$f'(x) = 3x^2 + 1$$

$$f'(0) = 1$$

**Question 13**

$$\frac{d}{dx} \log_e(1 - \sin^2(x))$$

$$= \frac{d}{dx} \log_e(\cos^2(x))$$

$$= \frac{-2\cos(x)\sin(x)}{\cos^2(x)}$$

$$= \frac{-2\sin(x)}{\cos(x)}$$

$$= -2\tan(x)$$

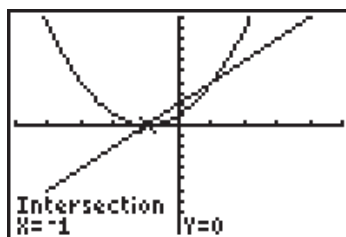
[B]

Question 14

$$g'(x) = 2x + 2$$

$g'(x)$ intersects $g(x)$ at $x = -1$.

$g'(-1) = 0$ and the gradient of $g'(x) = 2$.



Hence $g'(x)$ is not a tangent to $g(x)$ at $x = -1$. [E]

Question 15

$$\int (2x^2 + 4)^3 dx$$

$$= \int (8x^6 + 48x^4 + 96x^2 + 64) dx$$

by the binomial expansion or otherwise

$$= \frac{8x^7}{7} + \frac{48x^5}{5} + 32x^3 + 64x + c,$$

where c is a constant.

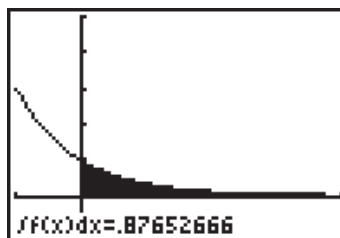
$$\text{Hence } \frac{8x^7}{7} + \frac{48x^5}{5} + 32x^3 + 64x + 8$$

is an antiderivative

[C]

Question 16

$\frac{13}{27}$ is less than the exact answer.



Hence the right rectangle rule must have been used.

Try width 1 unit.

$$\text{Area} \approx f(1) + f(2) + f(3)$$

$$= \frac{1}{3} + \frac{1}{9} + \frac{1}{27}$$

$$= \frac{13}{27}$$

[A]

Question 17

Top curve minus the bottom curve.

$$\int_a^b (g(x) - f(x)) dx + \int_b^c ((f(x) - g(x))) dx$$

[D]

Question 18

$${}^9C_5(2x)^4(-1)^5 = -126 \times 16x^4$$

$$= -2016x^4$$

The term is $-2016x^4$.

[E]

Question 19

$$f(x) = 2\sqrt{1-3x}$$

The range is $(0, \infty)$.

$$\text{Let } y = 2\sqrt{1-3x}$$

Then the inverse is

$$x = 2\sqrt{1-3y}$$

$$\left(\frac{x}{2}\right)^2 = 1-3y$$

$$y = \frac{1 - \frac{x^2}{4}}{3}$$

$$= \frac{4-x^2}{12}$$

The domain of the inverse is $(0, \infty)$.

$$f^{-1}: (0, \infty) \rightarrow R, \text{ where } f^{-1}(x) = -\frac{x^2}{12} + \frac{1}{3}$$

[C]

Question 20

$$y = \log_e(2x+1) + 3$$

x -intercept

$$\text{Let } y = 0$$

$$0 = \log_e(2x+1) + 3$$

$$e^{-3} = 2x+1$$

$$x = \frac{e^{-3}-1}{2}$$

$$= \frac{1-e^3}{2e^3}$$

y -intercept

$$\text{Let } x = 0$$

$$y = \log_e(1) + 3$$

$$= 3$$

[A]

Question 21

$$3e^{2x} - 17e^x + 10 = 0$$

$$\text{Let } a = e^x$$

$$3a^2 - 17a + 10 = 0$$

$$(3a-2)(a-5) = 0$$

$$a = \frac{2}{3} \text{ or } 5$$

$$e^x = \frac{2}{3} \text{ or } e^x = 5$$

$$x = \log_e\left(\frac{2}{3}\right) = \log_e(2) - \log_e(3) \text{ or } x = \log_e(5) \quad [C]$$

Question 22

$$\log_2(2x) - 5\log_2(x-1) - \log_2(y) = 2$$

$$\log_2\left(\frac{2x}{y(x-1)^5}\right) = 2$$

$$\frac{2x}{y(x-1)^5} = 4$$

$$y = \frac{2x}{4(x-1)^5} = \frac{x}{2(x-1)^5} \quad [B]$$

Question 23

$$\Pr(X > 15) = \text{normalcdf}(15, 1E99, 10, 3)$$

$$= 0.0478 \quad [C]$$

Question 24

Hypergeometric distribution with $N=16$,
 $n=10$, $D=4$, $x=0$, 1

[C]

Question 25

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$= 4k - 4k^2$$

$$\text{Sd}(X) = \sqrt{\text{Var}(X)} = 2\sqrt{k(1-k)} \quad [D]$$

Question 26

Recognise the symmetry; $0.7 + 0.15 = 0.85$

$$\text{InvNorm}(0.85, 1, 1.5) = 2.555$$

$$M + k = 2.555, k = 1.555 \quad [A]$$

Question 27

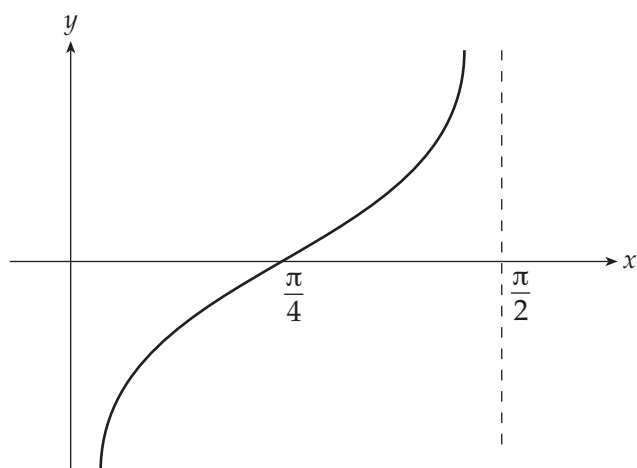
For binomial

$$E(X) = np, \text{ Var}(X) = np(1-p)$$

For hypergeometric $E(X) = \frac{nD}{N}$,

$$\text{Var}(X) = \frac{nD(N-D)(N-n)}{N^2(N-1)}$$

$$< np(1-p)$$

[E]**Solutions Short Answer 2004****Question 1**

Asymptotes and intercept shown

[1A]

Correct shape

[1A]

$$n = 2, k = \frac{\pi}{2}$$

[2A]**Question 2****a**

L1	L2	L3	2
-5	1	-----	
2	8	-----	
5	6	-----	
4	2	-----	
8	7	-----	
-----	-----	-----	
L2(1)=1			

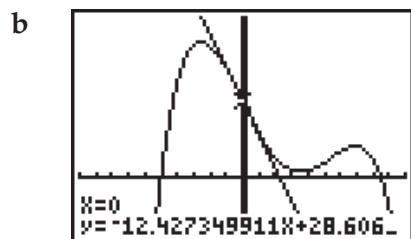
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QuarticReg
y=ax^4+bx^3+...+e
a=-.0491452991
b=.5170940171
c=.2243589744
d=-12.42735043
e=28.60683761

```

$$y = -0.049x^4 + 0.517x^3 + 0.224x^2 - 12.427x + 28.607$$

[1A]



The gradient of the tangent is ≈ -12.42735 .

The gradient of the normal is $\approx \frac{1}{12.42735} \approx 0.08047$

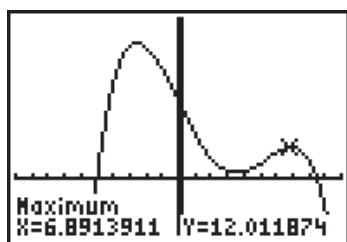
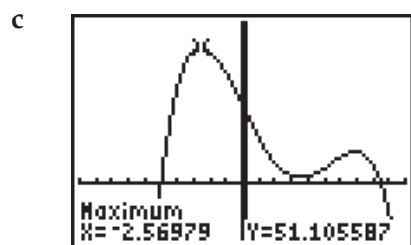
The y -intercept is ≈ 28.607 .

The equation of the normal is

$$y = 0.08x + 28.61$$

correct to two decimal places.

[2A]

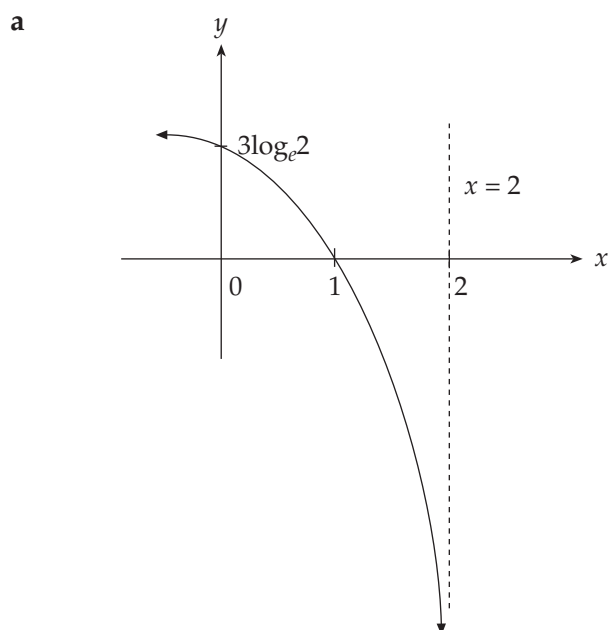


Average rate of change $\approx \frac{12.01187 - 51.10559}{6.89139 - -2.56979}$ [1M]

$$\approx -4.13$$

[1A]

Question 3



Asymptotes and cuts on axes

[3A]

b $f'(x) = \frac{-3}{2-x} = \frac{3}{x-2}$

[1A]

Question 4

a $x = 2\sqrt{3x+2}$

$$x^2 = 4(3x+2)$$

$$x^2 - 12x - 8 = 0$$

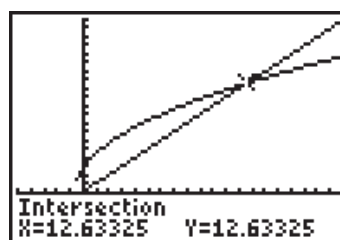
[1M]

$$x = \frac{12 \pm \sqrt{144 - (4 \times 1 \times -8)}}{2}$$

$$= \frac{12 \pm \sqrt{176}}{2}$$

$$= 6 + 2\sqrt{11} \text{ as } 6 - 2\sqrt{11} \text{ is outside the domain.}$$

[1A]



b Let $y = 2\sqrt{3x+2}$

Inverse $x = 2\sqrt{3y+2}$ [1M]

$$\frac{x^2}{4} = 3y + 2$$

$$y = \frac{x^2}{12} - \frac{2}{3}$$

$f^{-1}: [0, \infty) \rightarrow \mathbb{R}$, where $f^{-1}(x) = \frac{x^2}{12} - \frac{2}{3}$ [1A]

c $\int_0^{6+2\sqrt{11}} \left(x - \left(\frac{x^2}{12} - \frac{2}{3} \right) \right) dx$ [1M]

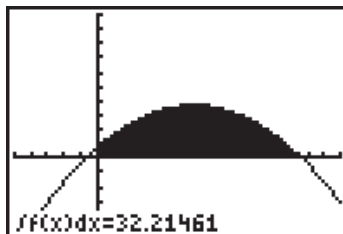
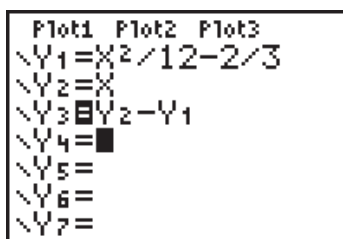
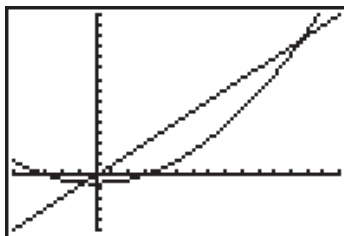
$$= \left[\frac{x^2}{2} - \frac{x^3}{36} + \frac{2}{3}x \right]_0^{6+2\sqrt{11}}$$
 [1M]

$$= \frac{(6+2\sqrt{11})^2}{2} - \frac{(6+2\sqrt{11})^3}{36} + \frac{2(6+2\sqrt{11})}{3}$$

$$= 16 + \frac{44\sqrt{11}}{9}$$

$$\approx 32.2146$$
 [1A]

Calculator check



Question 5

a $\Pr(B \geq 1) = 1 - \Pr(B = 0)$
 $= 1 - 0.49^3$
 $= 0.8824$ [1A]

b $\Pr(B = 3 | B \geq 1) = \frac{\Pr(B = 3)}{\Pr(B \geq 1)}$ [1M]
 $= \frac{0.51^3}{0.8824}$
 $= 0.1503$ [1A]