

**THE
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GROUP**

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**MATHS METHODS 3 & 4
TRIAL EXAMINATION 1
SOLUTIONS
2004**

1. E	8. B	15. D	22. D
2. C	9. D	16. A	23. C
3. C	10. E	17. A	24. D
4. A	11. E	18. C	25. C
5. A	12. D	19. B	26. E
6. B	13. A	20. A	27. D
7. D	14. C	21. A	

Part I – Multiple-choice solutions

Question 1

The graph crosses the x -axis at $x = a$ so we have an $x - a$ factor.

The graph touches the x -axis at $x = b$ so we have a repeated $x - b$ factor, i.e., $(x - b)^2$.

The cubic graph comes down from the left so the x^3 term must be negative.

The rule is

$$y = -(x - a)(x - b)^2$$
$$= (a - x)(x - b)^2$$

The answer is E.

Question 2

The graph of $y = e^x$ is reflected in the y -axis to become the graph of $y = e^{-x}$. This graph is then translated 3 units up to become $y = e^{-x} + 3$.

The answer is C.

Question 3

The graph of $y = e^{ax}$ has the asymptote $y = 0$.

The graph of $\log_e(ax)$ has the asymptote of $x = 0$.

The graph of $y = \sqrt{ax}$ does not have an asymptote.

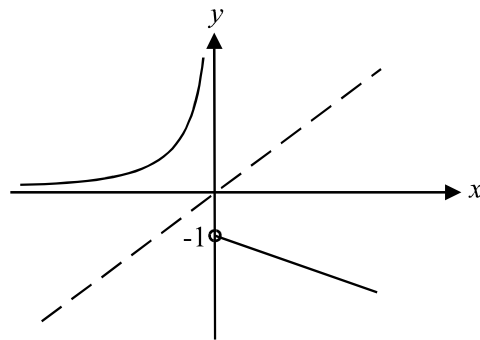
The graph of $y = \frac{1}{ax}$ has asymptotes of $x = 0$ and $y = 0$.

The graph of $y = \frac{1}{ax^2}$ has asymptotes of $x = 0$ and $y = 0$.

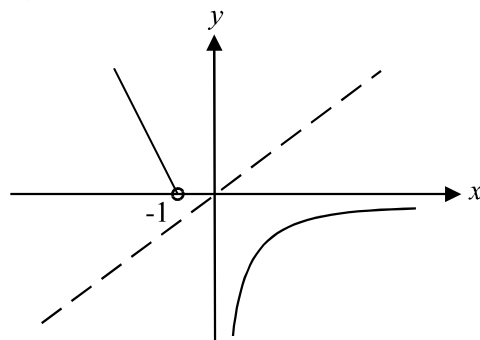
The answer is C.

Question 4

To obtain the graph of an inverse function you reflect the original graph in the line $y = x$.
So,



becomes

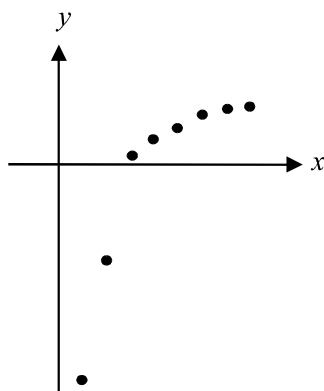


Note that the point $(0, -1)$ on the graph of $y = f(x)$ becomes $(-1, 0)$ on the graph of $y = f^{-1}(x)$.

The answer is A.

Question 5

Plot the data on your graphics calculator.



The data would be best modelled using a logarithmic function.

The answer is A.

Question 6

The shape of the graph is that of a sin graph that has been translated 1 unit down and has a period of π .

The graph of $y = \sin(nx)$ has a period of $\frac{2\pi}{n}$.

Since we require a period of π , we have $\pi = \frac{2\pi}{n}$ so $n = 2$.

The required equation is $y = \sin(2x) - 1$.

Note that an equation involving cos could be created but it is not one of the options offered here.

The answer is B.

Question 7Method 1

Use your graphics calculator to sketch the graphs of $y = 0.3 \tan\left(\frac{x}{2}\right)$ and $y = 1$ and find the point of intersection.

The x coordinate of this point of intersection is closest to 147° .

Method 2

Put your calculator in degree mode.

$$0.3 \tan\left(\frac{x}{2}\right) = 1, \quad x \in (-180^\circ, 180^\circ)$$

$$\tan\left(\frac{x}{2}\right) = 3.333...$$

S	A
T	C

$$\frac{x}{2} = 73.3007...$$

$$x = 146.6015...$$

Note that the period of the graph of $y = 0.3 \tan\left(\frac{x}{2}\right)$ is $\pi \div \frac{1}{2} = 2\pi$.

So there is only one solution to the equation $0.3 \tan\left(\frac{x}{2}\right) = 1$.

Therefore the closest answer is 147° .

The answer is D.

Question 8

Some key points to consider on the graph of $y = f(x)$ are the maximum and minimum turning points which occur at $x = -2\pi, 0, 2\pi$.

For these values of x , the corresponding value of y on the graph of $y = f'(x)$ will be zero.

Instantly we can eliminate options A and E.

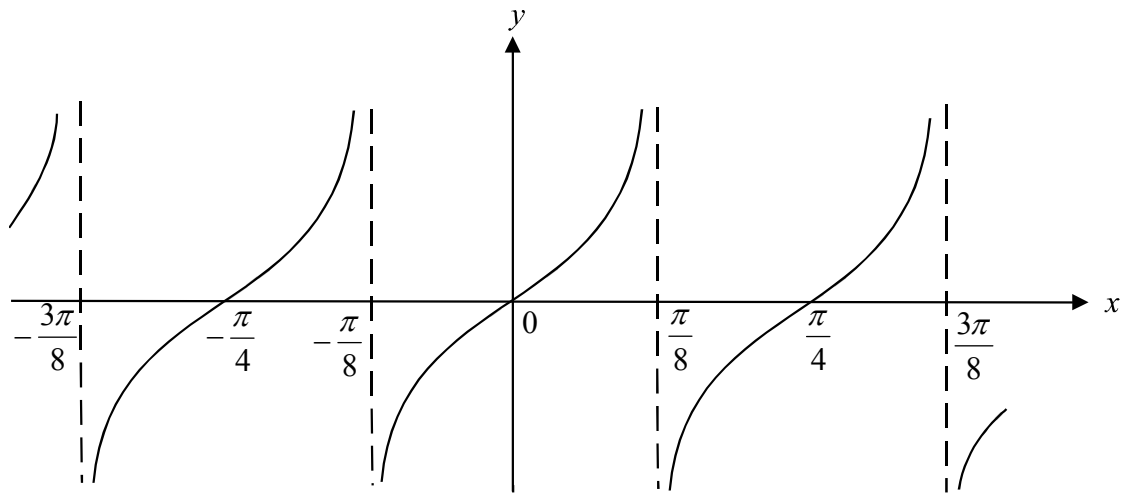
The gradient of the graph of $y = f(x)$ is positive for $x \in (-2\pi, 0)$ and for values $x \in (2\pi, 4\pi)$ if we follow the pattern of the graph. Hence the graph of $y = f'(x)$ must be positive for these values of x . We can eliminate options C and D.

The answer is B.

Question 9

The period of the graph of $y = \tan(4x)$ is $\frac{\pi}{4}$.

Sketch the graph.



We see that over the domains $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$, $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $[-4\pi, \pi]$, $\left[0, \frac{\pi}{4}\right)$ the function is not defined because these domains include value(s) of x where an asymptote exists.

Only over the domain $x \in [0, \frac{\pi}{8})$ is the function defined.

The answer is D.

Question 10

$$y = \sqrt{3x^2 - 4}$$

$$= (3x^2 - 4)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}(3x^2 - 4)^{-\frac{1}{2}} \times 6x \quad (\text{chain rule})$$

$$= \frac{3x}{\sqrt{3x^2 - 4}}$$

The answer is E.

Question 11

Let $y = \frac{\log_e(2x)}{x^3}$

So,
$$\begin{aligned}\frac{dy}{dx} &= \left(x^3 \times \frac{2}{2x} - 3x^2 \log_e(2x) \right) \div x^6 \\ &= (x^2 - 3x^2 \log_e(2x)) \div x^6 \\ &= x^2(1 - 3 \log_e(2x)) \div x^6 \\ &= \frac{1 - 3 \log_e(2x)}{x^4}\end{aligned}$$

The answer is E.

Question 12

Average rate of change between $t = 0$ and $t = 2$ is given by

$$\begin{aligned}&\frac{f(2) - f(0)}{2 - 0} \\ &= \frac{e^{\sqrt{4}} - e^0}{2} \\ &= \frac{e^2 - 1}{2}\end{aligned}$$

The answer is D.

Question 13

$$y = e^x \sin(x)$$

$$\frac{dy}{dx} = e^x \cos(x) + e^x \sin(x)$$

When $x = 0$,

$$\begin{aligned}\frac{dy}{dx} &= e^0 \cos(0) + e^0 \sin(0) \\ &= 1 \times 1 + 1 \times 0 \\ &= 1\end{aligned}$$

The gradient of the tangent is 1.

The gradient of the normal is $\frac{-1}{1} = -1$.

Now, when $x = 0$,

$$\begin{aligned}y &= e^0 \sin(0) \\ &= 0\end{aligned}$$

The equation of the normal to the curve at the point where $x = 0$, is given by

$$y - 0 = -1(x - 0)$$

$$y = -x$$

The answer is A.

Question 14

With $x = 4$, $f(x) = f(4)$

Now $x + h = 4.01$

So $4 + h = 4.01$

So $h = 0.01$

So $f(x + h) \approx f(4) + 0.01f'(4)$

The answer is C.

Question 15

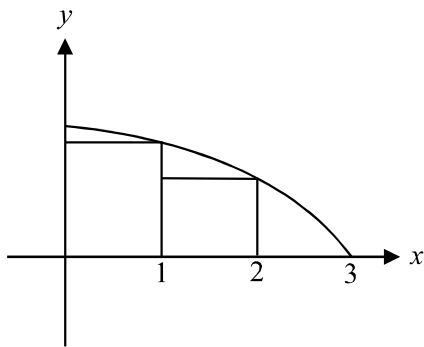
$y = g'(x)$ is the gradient function of the function g .

Hence the gradient of the graph of $y = g(x)$ will be positive when the graph of $y = g'(x)$ is positive. This occurs for $x \in (a, 0) \cup (d, f)$.

The answer is D.

Question 16

Sketch the graph and draw



the “right” rectangles.

The required approximation is given by

$$1 \times \log_e(4 - 1) + 1 \times \log_e(4 - 2)$$

$$= \log_e(3) + \log_e(2)$$

$$= \log_e(6)$$

The answer is A.

Question 17

$$\frac{dy}{dx} = e^{3x} + \cos(3x)$$

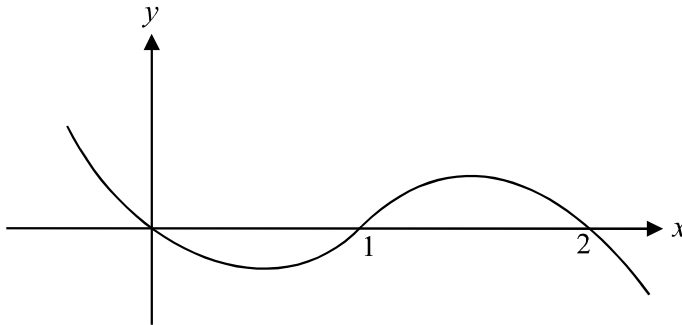
$$y = \int (e^{3x} + \cos(3x)) dx$$

$$= \frac{1}{3} e^{3x} + \frac{1}{3} \sin(3x) + c$$

The answer is A.

Question 18

Sketch a graph which is easy to do because the cubic function is in factorised form.

Method 1

Use a graphics calculator (2^{nd} Calc $\int f(x)dx$) to find the two areas.

The answer is $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$.

Method 2

$$\begin{aligned}\text{Now, } -x(x-1)(x-2) &= -x(x^2 - 3x + 2) \\ &= -x^3 + 3x^2 - 2x\end{aligned}$$

$$\begin{aligned}\text{Total area} &= \int_1^2 (-x^3 + 3x^2 - 2x) dx - \int_0^1 (-x^3 + 3x^2 - 2x) dx \\ &= \left[-\frac{x^4}{4} + \frac{3x^3}{3} - \frac{2x^2}{2} \right]_1^2 - \left[-\frac{x^4}{4} + \frac{3x^3}{3} - \frac{2x^2}{2} \right]_0^1 \\ &= \left\{ (-4 + 8 - 4) - \left(-\frac{1}{4} + 1 - 1 \right) \right\} - \left\{ \left(-\frac{1}{4} + 1 - 1 \right) - 0 \right\} \\ &= \frac{1}{4} + \frac{1}{4} \\ &= \frac{1}{2}\end{aligned}$$

The answer is C.

Question 19

$$\text{Now } \int_1^2 g(x) dx = 5$$

$$\begin{aligned}\text{So } \int_{-1}^2 (1 - 2g(x)) dx &= \int_{-1}^2 1 dx - 2 \int_{-1}^2 g(x) dx \\ &= [x]_{-1}^2 - 2 \times 5 \\ &= 2 - (-1) - 10 \\ &= -7\end{aligned}$$

The answer is B.

Question 20Method 1

The line of symmetry of a parabola occurs when $x = -\frac{b}{2a}$

$$= \frac{-4}{2}$$

$$= -2$$

So, $a \leq -2$. The only value that a can be from the options is -5 .

Method 2

Sketch the parabola, which is upright, on your graphics calculator and then locate the minimum of the parabola (2nd Calc). This occurs at $(-2, 3)$.

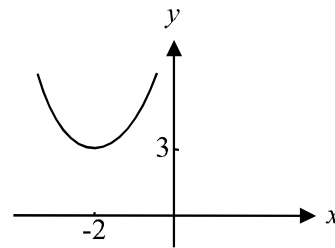
So, $a \leq -2$. The only value that a can be from the options is -5 .

Method 3

$$\begin{aligned} f(x) &= x^2 + 4x + 7 \\ &= (x^2 + 4x + 4) - 4 + 7 && \text{(completing the square)} \\ &= (x + 2)^2 + 3 && \text{(turning point form)} \end{aligned}$$

We have an upright parabola with a turning point of $(-2, 3)$.

If the inverse function f^{-1} is to exist then f must be 1:1. So, $a \leq -2$. The only value that a can be from the options is -5 .
The answer is A.

**Question 21**

The coefficient of x^3 in the expansion of $(2x - 1)^6$ is $20 \times 2^3 \times -1^3 = -160$.

The answer is A.

Question 22

$$\log_e \left(\frac{1}{x} \right) + 5 \log_e (x) = \log_e (1)$$

$$\log_e \left(\frac{1}{x} \right) + \log_e (x^5) = \log_e (1)$$

$$\log_e \left(\frac{1}{x} \times x^5 \right) = \log_e (1)$$

$$\log_e (x^4) = \log_e (1)$$

$$x^4 = 1$$

So $x = \pm 1$

But, $x > 0$ since we have $\log_e (x)$ above.

So $x = 1$

The answer is D.

Question 23

The expected value of X is given by

$$5 \times 0.2 + 10 \times 0.1 + 15 \times 0.4 + 20 \times 0.3 = 14.$$

The answer is C.

Question 24

We have a binomial distribution where $p = 0.85$, $n = 50$ and $x = 45$.

The required probability is given by

$${}^{50}C_{45} (0.85)^{45} (0.15)^5$$

The answer is D.

Question 25

In the class there are a total of 24 children. Six of these are in blue group and 18 are not in blue group. The principal selects without replacement so we have a hypergeometric distribution.

Where $D = 6$, $N = 24$, $x = 2$ and $n = 3$

$$\text{So, } \Pr(X = 2) = \frac{{}^6C_2 {}^{18}C_1}{{}^{24}C_3}$$

The answer is C.

Question 26

With 8 trials and a probability of success of 0.7 we would expect the graph to be negatively skewed.

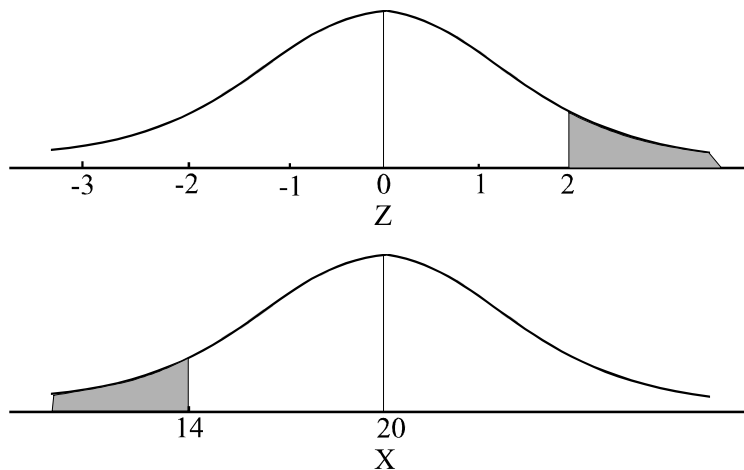
If the probability of success was 0.5 the graph would be symmetrical.

Also $\mu = np$

$$= 8 \times 0.7 = 5.6 \text{ so the highest part of the graph should be between 5 and 6.}$$

The only feasible graph is E.

The answer is E.

Question 27

If $\Pr(X < 14) = \Pr(z > 2)$, then because of the symmetry of the normal curve, 14 is two standard deviations from the mean.

$$\text{So } 20 - 14 = 6$$

$$\text{So one standard deviation is } 6 \div 2 = 3.$$

Note also that standard deviations can't be negative.

The answer is D.

PART II**Question 1**

$$\begin{aligned}
 \text{a.} \quad & e^x(e^{-x} - 1)^2 \\
 &= e^x(e^{-2x} - 2e^{-x} + 1) \\
 &= e^{-x} - 2e^0 + e^x \\
 &= e^{-x} - 2 + e^x
 \end{aligned}$$

(1 mark)

$$\text{b.} \quad \text{Let } P(x) = x^4 + 6x^3 + ax^2 - 11x + 3$$

If $P(x)$ is exactly divisible by $(x + 3)$, then $P(-3) = 0$

(1 mark)

$$\text{So, } P(-3) = 81 - 162 + 9a + 33 + 3 = 0$$

$$9a - 45 = 0$$

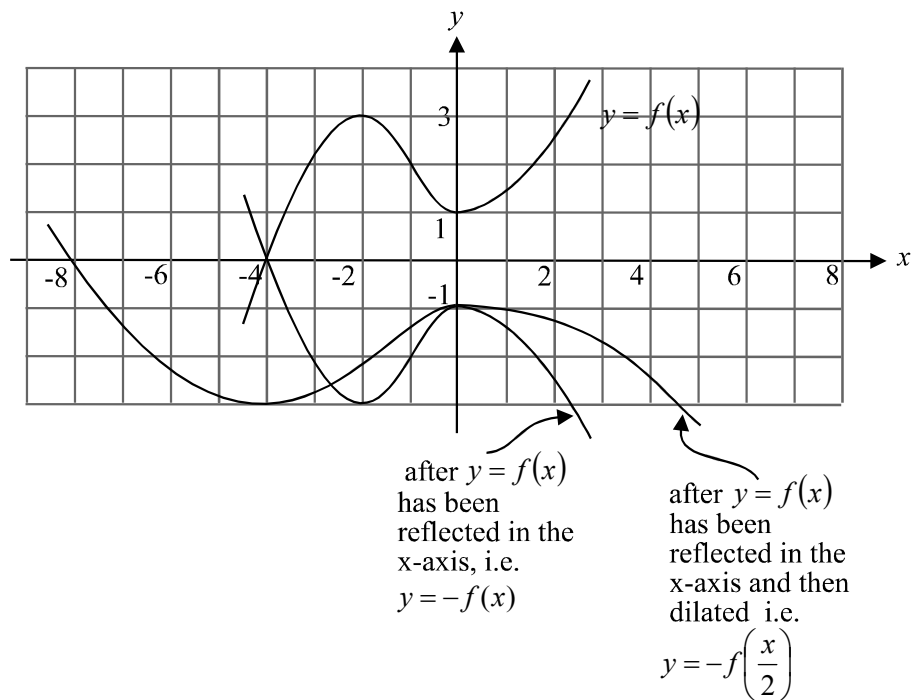
$$a = 5$$

(1 mark)**Question 2**

First sketch the graph of $y = f(x)$ after it has been reflected in the x -axis.

(1 mark) for graph of reflection

Then dilate this graph, i.e. stretch this graph parallel to the x -axis by a factor of two. This means that effectively each x -coordinate is multiplied by 2.

**(1 mark)** for graph of dilation

Question 3**a.**

Number of sixes (X)	Probability $\Pr(X = x)$
0	$\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$
1	$\frac{5}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{5}{6} = \frac{10}{36}$
2	$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

(2 marks)(1 or 2 correct = **1 mark**, 3 correct = **2 marks**)**b.** We have a binomial distribution where $n = 5$ and $p = 0.3$.

$$\begin{aligned}
 \Pr(X > 3) &= \Pr(X = 4) + \Pr(X = 5) \\
 &= {}^5C_4 (0.3)^4 (0.7)^1 + {}^5C_5 (0.3)^5 (0.7)^0 \\
 &= 0.02835 + 0.00243 \\
 &= 0.0308 \text{ correct to 4 decimal places}
 \end{aligned}$$

(1 mark)**Question 4**

a.

$$\begin{aligned}
 f(x) &= 0 & x &\in [0, \pi] \\
 2 \cos(2x) + \sqrt{3} &= 0 & 2x &\in [0, 2\pi] \\
 \cos(2x) &= \frac{-\sqrt{3}}{2} \\
 2x &= \frac{5\pi}{6}, \frac{7\pi}{6} \\
 x &= \frac{5\pi}{12}, \frac{7\pi}{12}
 \end{aligned}$$

S	A
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(1 mark)

b.

$$\begin{aligned}
 f(x) &= 2 \cos(2x) + \sqrt{3} \\
 f'(x) &= -4 \sin(2x)
 \end{aligned}$$

(1 mark)

c. The range of the graph of $y = f'(x)$ is $-4 \leq y \leq 4$.
Hence $f'(x) \leq 4$ for all x .

(1 mark)

Question 5

a.
$$\begin{aligned}\frac{d}{dx}(5x \log_e(2x)) &= 5 \log_e(2x) + 5x \times \frac{2}{2x} \\ &= 5 \log_e(2x) + 5\end{aligned}$$

(1 mark)

b. Method 1
From a.,

$$\begin{aligned}5 \log_e(2x) &= \frac{d}{dx}(5x \log_e(2x)) - 5 \\ \log_e(2x) &= \frac{d}{dx}(x \log_e(2x)) - 1\end{aligned}$$

(1 mark)

$$\begin{aligned}\int \log_e(2x) dx &= \int \frac{d}{dx}(x \log_e(2x)) dx - \int 1 dx \\ &= x \log_e(2x) - x + c\end{aligned}$$

(1 mark)

Method 2
From a.,

$$\begin{aligned}\int (5 \log_e(2x) + 5) dx &= 5x \log_e(2x) \quad \textbf{(1 mark)} \\ 5 \int \log_e(2x) dx + \int 5 dx &= 5x \log_e(2x) \\ 5 \int \log_e(2x) dx &= 5x \log_e(2x) - \int 5 dx \\ 5 \int \log_e(2x) dx &= 5x \log_e(2x) - 5x + c \\ \int \log_e(2x) dx &= x \log_e(2x) - x + c \quad \textbf{(1 mark)}\end{aligned}$$

Question 6

a. The maximal domain of $g(x)$
where $g(x) = \frac{1}{\sqrt{x-1}}$ is $x-1 > 0$
 $x > 1$

So $a = 1$.

(1 mark)

b. Method 1

Find the x -intercept on the graph of $y = g(x)$.

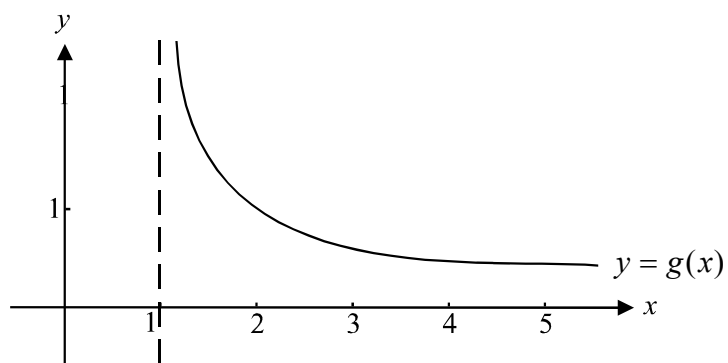
We have $0 = \frac{1}{\sqrt{x-1}}$. Since $0 \neq 1$ there is no solution and hence there is no x -

intercept on the graph of $y = g(x)$. Hence there is no y -intercept on the graph of $y = g^{-1}(x)$

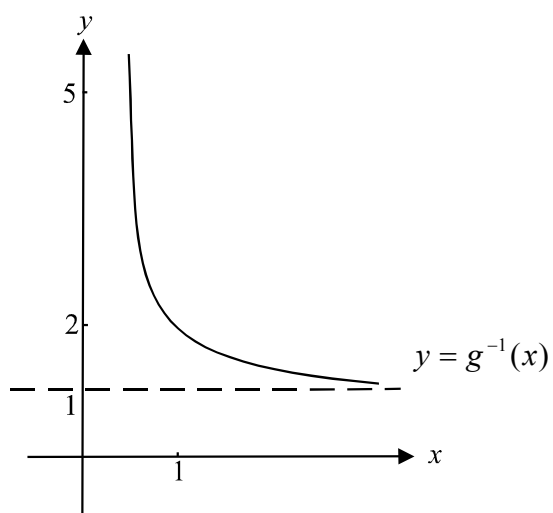
(1 mark)

Method 2

The graph of $y = g(x)$ is shown.



The graph of $y = g^{-1}(x)$ is a reflection of this graph in the line $y = x$.



Hence the graph of $y = g^{-1}(x)$ never crosses the y -axis.

(1 mark)

c. $\int_n^5 g(x) dx = 2$

So, $\int_n^5 \frac{1}{\sqrt{x-1}} dx = 2$

$$\int_n^5 (x-1)^{-\frac{1}{2}} dx = 2$$

$$\left[\frac{1}{-\frac{1}{2} + 1} (x-1)^{\frac{1}{2}} \right]_n^5 = 2 \quad (1 \text{ mark})$$

$$2 \left\{ 4^{\frac{1}{2}} - (n-1)^{\frac{1}{2}} \right\} = 2$$

$$2 - (n-1)^{\frac{1}{2}} = 1$$

$$-(n-1)^{\frac{1}{2}} = -1$$

$$(n-1)^{\frac{1}{2}} = 1$$

$$n-1 = 1$$

$$n = 2$$

(1 mark)

Question 7

- a. Find the equation of the tangent first.
Now, $y = \tan(2x)$

$$\frac{dy}{dx} = 2 \sec^2(2x)$$

At $x = \frac{\pi}{8}$, $\frac{dy}{dx} = 2 \sec^2\left(\frac{\pi}{4}\right)$

$$= \frac{2}{\cos^2\left(\frac{\pi}{4}\right)}$$

$$= \frac{2}{\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= 2 \div \frac{1}{2}$$

$$= 4$$

(1 mark)

The equation of the tangent is therefore

$$y - 1 = 4\left(x - \frac{\pi}{8}\right)$$

$$y = 4x - \frac{\pi}{2} + 1$$

(1 mark)

This tangent crosses the x -axis when $y = 0$.

$$\text{So, } 0 = 4x - \frac{\pi}{2} + 1$$

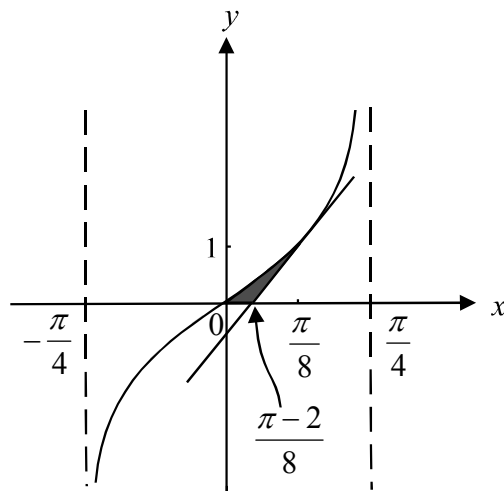
$$4x = \frac{\pi}{2} - 1$$

$$4x = \frac{\pi - 2}{2}$$

$$x = \frac{\pi - 2}{8} \text{ as required.}$$

(1 mark)

b. Sketch a quick graph.



The area required is the shaded area.

$$\text{Area required} = \int_0^{\frac{\pi}{8}} \tan(2x) dx - \int_{\frac{\pi-2}{8}}^{\frac{\pi}{8}} \left(4x - \frac{\pi}{2} + 1\right) dx$$

Alternatively,

$$\text{area required} = \int_0^{\frac{\pi-2}{8}} \tan(2x) dx - \int_{\frac{\pi-2}{8}}^{\frac{\pi}{8}} \left\{ \tan(2x) - \left(4x - \frac{\pi}{2} + 1\right) \right\} dx$$

(1 mark) first integral

(1 mark) second integral

Total 23 marks