



Victorian Certificate of Education 2003

MATHEMATICAL METHODS

Written examination 1 (Facts, skills and applications)

Friday 7 November 2003

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.45 am (1 hour 30 minutes)

PART I MULTIPLE-CHOICE QUESTION BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions). Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.

Part II consists of a separate question and answer book.

You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
27	27	27

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question book of 13 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination

- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
- You may retain this question book.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

This page is blank

Instructions for Part I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

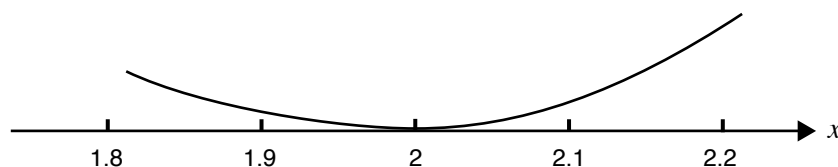
A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

A polynomial function p has degree three. A part of its graph, near the point on the graph with coordinates $(2, 0)$, is shown below.



Which one of the following could be the rule for the third degree polynomial p ?

- A. $p(x) = x(x + 2)^2$
- B. $p(x) = (x - 2)^3$
- C. $p(x) = x^2(x - 2)$
- D. $p(x) = (x - 1)(x - 2)^2$
- E. $p(x) = -x(x - 2)^2$

Question 2

Which one of the following is **not** true of the graph of the function $f: R^+ \rightarrow R, f(x) = \log_2(x)$?

- A. It has a vertical asymptote with equation $x = 0$.
- B. It passes through the point $(2, 0)$.
- C. The slope of the tangent at any point on the graph is positive.
- D. It has domain R^+ .
- E. It has range R .

Question 3

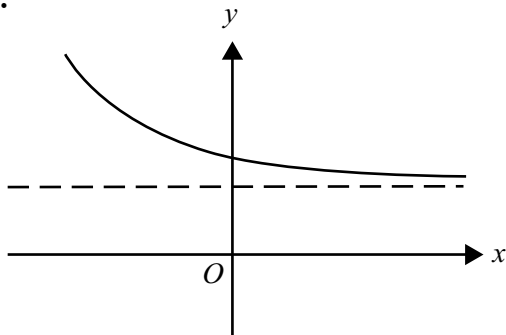
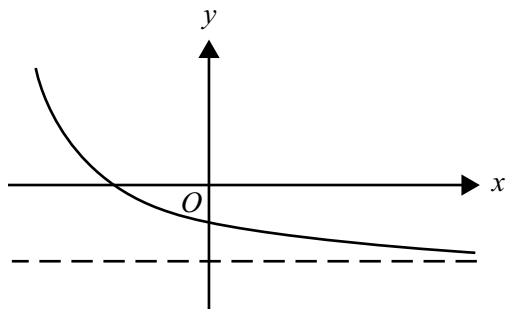
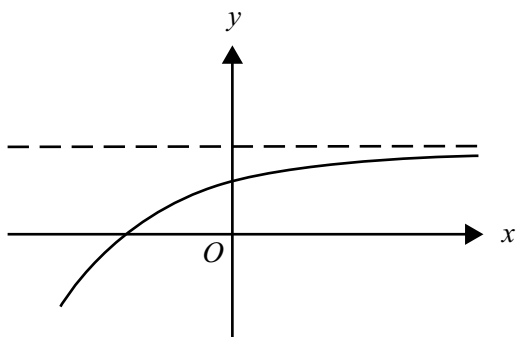
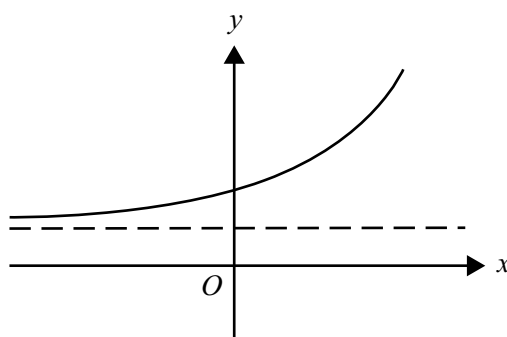
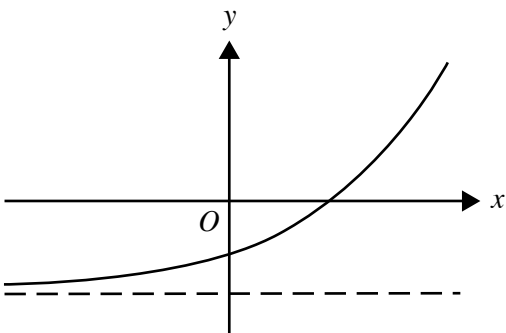
Dylan drew the graph of the function $f: R \rightarrow R, f(x) = \frac{x^3 + 1}{x}$ by adding the ordinates of the graphs of two functions g and h .

The rules for g and h that Dylan could have used are

- A. $g(x) = x^3$ and $h(x) = \frac{1}{x}$
- B. $g(x) = x^2$ and $h(x) = \frac{1}{x}$
- C. $g(x) = x^3 + 1$ and $h(x) = x$
- D. $g(x) = x^3 + 1$ and $h(x) = \frac{1}{x}$
- E. $g(x) = x^2$ and $h(x) = 1$

Question 4

If k and P are positive real numbers, which one of the following graphs is most likely to be the graph of the function with equation $y = e^{kx} + P$?

A.**B.****C.****D.****E.****Question 5**

The graph of the function f is obtained from the graph of the function with equation $y = \sqrt{x}$ by a reflection in the y -axis followed by a dilation of 2 units from the x -axis.

The rule for f is

A. $f(x) = -2\sqrt{x}$

B. $f(x) = \sqrt{-2x}$

C. $f(x) = \sqrt{-0.5x}$

D. $f(x) = -0.5\sqrt{x}$

E. $f(x) = 2\sqrt{-x}$

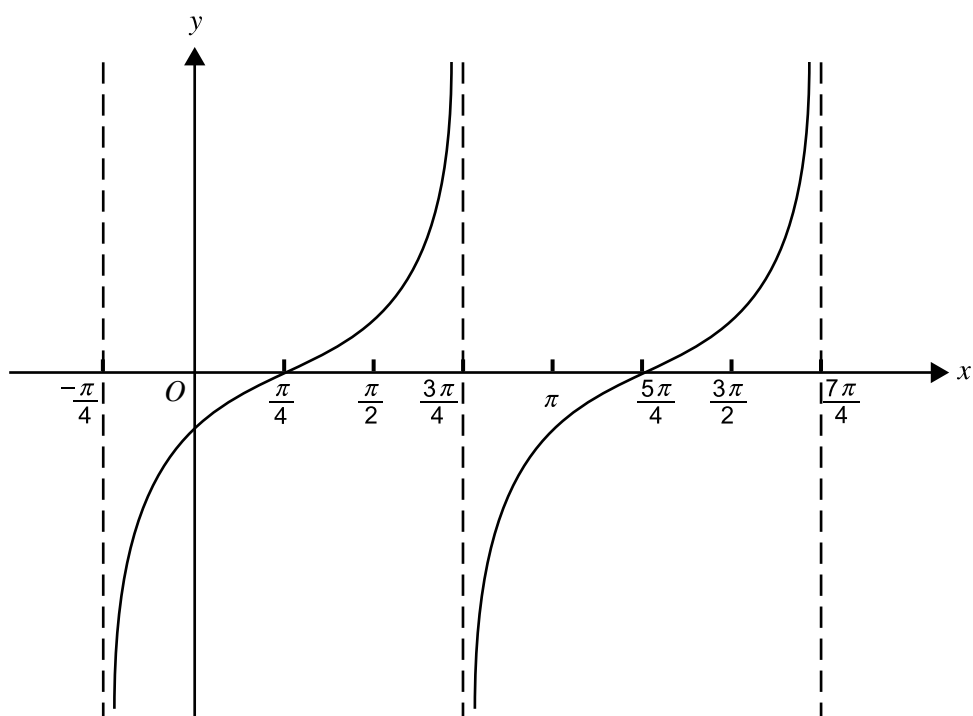
Question 6

The number of solutions of the equation $0.5 \cos(2x) = 1$, for $x \in [-\pi, \pi]$, is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

Question 7

The diagram shows two cycles of the graph of a circular function.

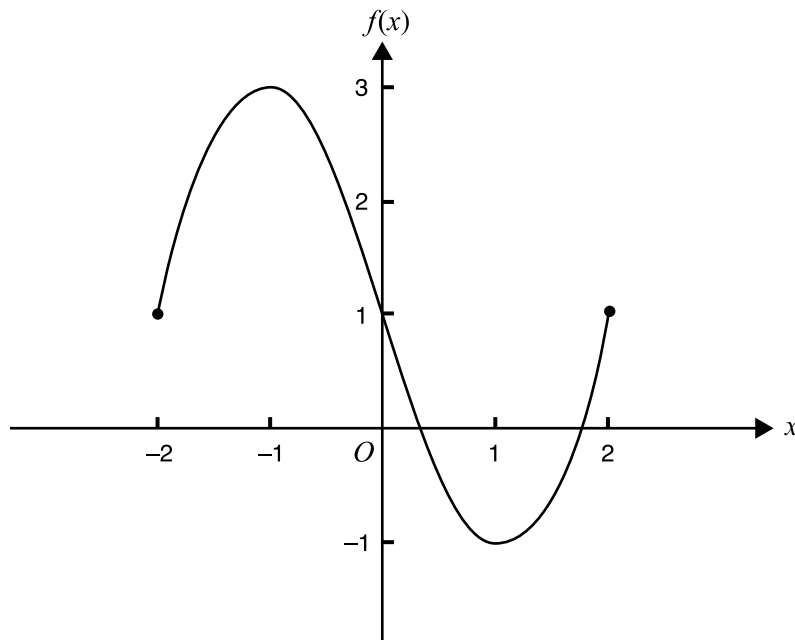


The period of the circular function is

- A. $\frac{\pi}{2}$
- B. $\frac{3\pi}{4}$
- C. π
- D. $\frac{7\pi}{4}$
- E. 2π

Question 8

The graph of the function $f: [-2, 2] \rightarrow \mathbb{R}, f(x) = P \sin(k\pi x) + Q$ is shown below.



The values of P , k and Q respectively are

- | | P | k | Q |
|----|-----|-----|-----|
| A. | 2 | 0.5 | 1 |
| B. | 2 | 2 | 1 |
| C. | -2 | 2 | -1 |
| D. | -2 | 0.5 | 1 |
| E. | -2 | 0.5 | -1 |

Question 9

If $y = 2 \tan(2x)$, then $\frac{dy}{dx}$ is equal to

- A. $\frac{1}{\cos^2(2x)}$
- B. $\frac{2}{\cos^2(2x)}$
- C. $\frac{4}{\cos^2(x)}$
- D. $\frac{4}{\cos^2(2x)}$
- E. $\frac{4}{\cos^2(4x)}$

Question 10

If u is a function of x and if $y = u(x) \log_e(x)$, then the rate of change of y with respect to x when $x = 2$ is equal to

- A. $u(2) \log_e(2)$
- B. $u'(2) \log_e(2) - \frac{u(2)}{2}$
- C. $u'(2) \log_e(2) + \frac{u(2)}{2}$
- D. $\frac{u'(2)}{2}$
- E. $u'(2) \log_e(2)$

Question 11

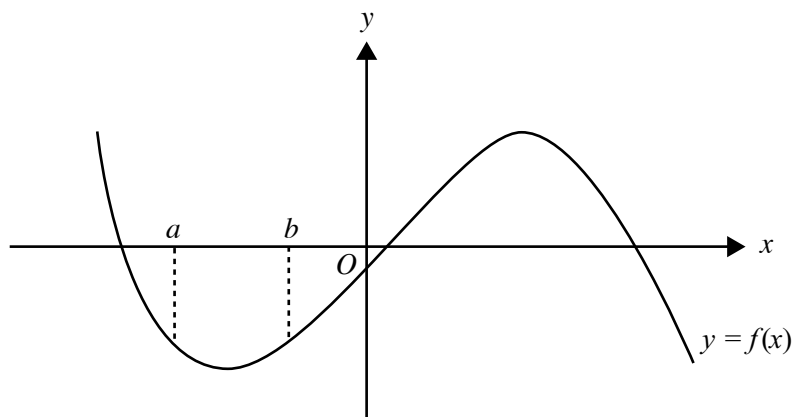
Let $f: R \rightarrow R$ be a function such that $f'(-1) = 0$
 and $f'(x) > 0$ when $x < -1$
 and $f'(x) > 0$ when $x > -1$.

At $x = -1$, the graph of f has a

- A. local minimum.
- B. local maximum.
- C. stationary point of inflection.
- D. point of discontinuity.
- E. gradient of -1 .

Question 12

Part of the graph of the function f is shown below.



Let g be a function such that $g'(x) = f(x)$.

On the interval (a, b) , the graph of g will have a

- A. negative gradient.
- B. positive gradient.
- C. local minimum value.
- D. local maximum value.
- E. zero gradient.

Question 13

If $f'(x) = 4e^{2x}$, then $f(x)$ could be equal to

- A. $2e^{2x} + 3$
- B. $4e^{2x} + 5$
- C. $8e^{2x} + 2$
- D. $4\log_e(2x) - 4$
- E. $\log_e(8x) + 5$

Question 14

If $\int_1^4 f(x)dx = 2$, then $\int_1^4 (2f(x) + 3)dx$ is equal to

- A. 2
- B. 4
- C. 7
- D. 10
- E. 13

Question 15

Let g be any continuous function on the interval $[0, 5]$, and f a function such that $f'(x) = g(x)$, for all $x \in [0, 5]$.

Then $\int_0^5 g(x)dx$ is equal to

- A. $g'(5) - g'(0)$
- B. $f(5)$
- C. $f'(5) - f'(0)$
- D. $g(5) - g(0)$
- E. $f(5) - f(0)$

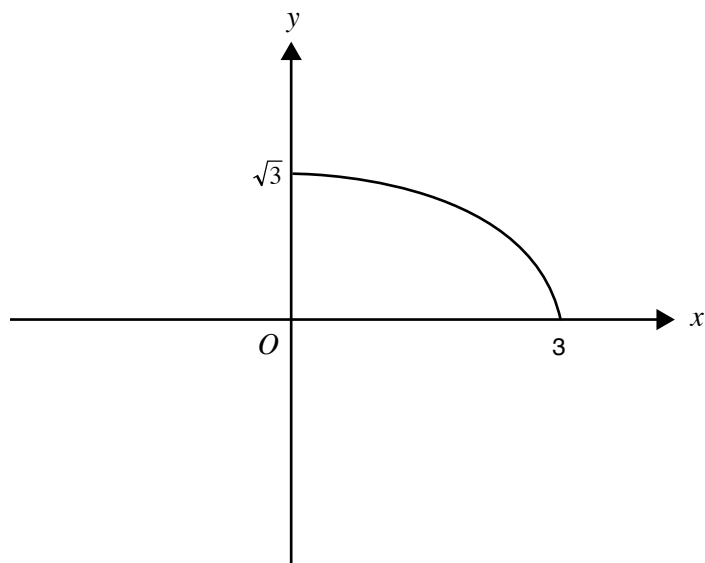
Question 16

The total area of the regions enclosed by the graph of the function with equation $y = \sin(2x)$, and the x -axis between $x = 0$ and $x = 2\pi$, is equal to

- A. 1
- B. 2
- C. 4
- D. 8
- E. 16

Question 17

The diagram below shows part of the graph with equation $y = \sqrt{3-x}$.



Using the left-rectangle approximation with rectangles of width 1 unit, an approximation to $\int_0^3 \sqrt{3-x} \, dx$ is

- A. $\frac{3\sqrt{3}}{2}$
- B. $\sqrt{2} + \sqrt{3}$
- C. π
- D. $2\sqrt{3}$
- E. $1 + \sqrt{2} + \sqrt{3}$

Question 18

Given that $f(x) = 2[(x+3)^2 - 4]$, the coordinates of the turning point of the graph of f are

- A. $(-3, -8)$
- B. $(-3, -4)$
- C. $(3, -8)$
- D. $(3, -4)$
- E. $(-3, 8)$

Question 19

The first seven lines of Pascal's Triangle are given below.

						1							
						1		1					
					1		2		1				
			1			3		3		1			
		1		4			6		4		1		
	1		5		10			10		5		1	
1		6		15		20			15		6		1

The coefficient of x^2 in the expansion of $(2x - 3)^5$ is equal to

- A. 1080
- B. 540
- C. -10
- D. -540
- E. -1080

Question 20

Let $p(x) = (x^2 + a)(x + b)(x - c)$, where a , b and c are three distinct positive real numbers.

The number of real solutions to the equation $p(x) = 0$ is exactly

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

Question 21

Let $f(x) = \frac{2}{x-3} + 1$.

The equations of the asymptotes of the graph of the inverse function f^{-1} are

- A. $x = 1$ and $y = 3$
- B. $x = 1$ and $y = -3$
- C. $x = 3$ and $y = 1$
- D. $x = -3$ and $y = -1$
- E. $x = -1$ and $y = -3$

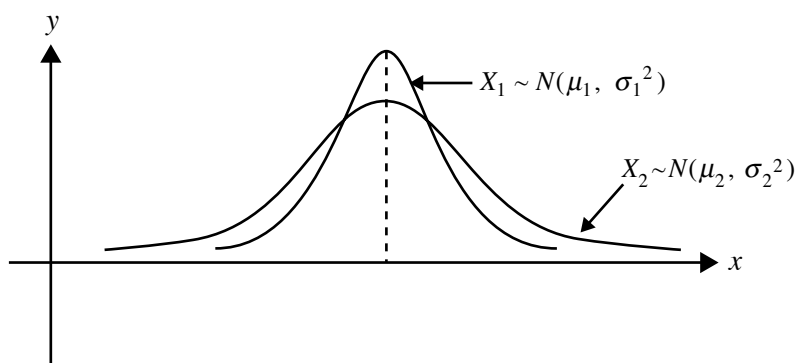
Question 22

If $2 \log_e(x) - \log_e(x+2) = 1 + \log_e(y)$, then y is equal to

- A. $\frac{x^2}{10(x+2)}$
 B. $\frac{2x}{x+2} - 1$
 C. $\frac{x^2}{x+2}$
 D. $\frac{2x}{x+2}$
 E. $\frac{x^2}{e(x+2)}$

Question 23

The diagram below shows the graphs of two normal distribution curves with means μ_1 and μ_2 and standard deviations σ_1 and σ_2 respectively.



Which one of the following statements is true?

- A. $\mu_1 > \mu_2$ and $\sigma_1 = \sigma_2$
 B. $\mu_1 > \mu_2$ and $\sigma_1 > \sigma_2$
 C. $\mu_1 = \mu_2$ and $\sigma_1 > \sigma_2$
 D. $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$
 E. $\mu_1 < \mu_2$ and $\sigma_1 = \sigma_2$

Question 24

Which of the following tables could represent the probability distribution of a discrete random variable?

I	v	2	3	4	5
	$\Pr(V = v)$	0.1	0.2	0.4	0.5

II	w	-2	-1	0	1
	$\Pr(W = w)$	0.2	0.3	0.3	0.2

III	x	10	20	30	40
	$\Pr(X = x)$	0.4	0.3	0.2	0.1

IV	y	0	1	2	3
	$\Pr(Y = y)$	-0.1	0.4	0.5	0.2

V	z	1	2	3	4
	$\Pr(Z = z)$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$

- A.** I and III
B. I and IV
C. II and IV
D. II, III and V
E. I, II and V

Question 25

The random variable X has the following probability distribution, where $0 < p < 1$.

x	0	1
$\Pr(X = x)$	p	$1 - p$

The standard deviation of X is

- A.** $1 - p$
B. $\sqrt{p(1 - p)}$
C. $p(1 - p)$
D. $\sqrt{p^2 + (1 - p)^2}$
E. $p^2 + (1 - p)^2$

Question 26

60 per cent of all tickets sold at a racecourse are Adult tickets and the remaining 40 per cent are Concession tickets. A random sample of 20 tickets is taken.

The probability that this sample contains exactly twelve Adult tickets is equal to

- A. $\frac{{}^{60}C_{12} \times {}^{40}C_8}{{}^{100}C_{20}}$
- B. ${}^{20}C_{12} (0.4)^8 \times (0.6)^{12}$
- C. ${}^{20}C_{12} (0.4)^{12} \times (0.6)^8$
- D. $(0.4)^8 \times (0.6)^{12}$
- E. $(0.4)^{12} \times (0.6)^8$

Question 27

A bag contains 12 bread rolls, of which 8 are white and the remainder multigrain. Tony takes 2 bread rolls at random from the bag to eat.

The probability that at least one is a multigrain roll is

- A. $1 - \frac{2^{12}}{3^{12}}$
- B. $1 - \frac{{}^8C_2}{{}^{12}C_2}$
- C. $1 - \frac{2^{12}}{3^{12}} - 12 \times \frac{1}{3} \times \frac{2^{11}}{3^{11}}$
- D. $1 - \frac{{}^8C_2}{{}^{12}C_2} - \frac{{}^8C_1 \times {}^4C_1}{{}^{12}C_2}$
- E. $\frac{{}^8C_1 \times {}^4C_1}{{}^{12}C_2}$

Victorian Certificate of Education 2003

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Letter

Figures

Words

--

MATHEMATICAL METHODS

Written examination 1 (Facts, skills and applications)

Friday 7 November 2003

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.45 am (1 hour 30 minutes)

PART II QUESTION AND ANSWER BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions). Part I consists of a separate question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of this question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
6	6	23

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphic calculator (memory must be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 7 pages.

Instructions

- Detach the formula sheet from the centre of the Part I book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II).

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

This page is blank

Instructions for Part II

- Answer **all** questions in the spaces provided.
- A decimal approximation will not be accepted if an **exact** answer is required to a question.
- Where an **exact** answer is required to a question, appropriate working must be shown.
- In questions where more than 1 mark is available, appropriate working must be shown.
- Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Use the remainder theorem to determine if $2x^4 - 3x^3 + 7x + 11$ is exactly divisible by $(x + 1)$.

2 marks

Question 2

- a. Find the coordinates of the point P on the curve with equation $y = x^2 - 2x - 1$ at which the tangent is parallel to the line $y = 3x - 5$.

- b. Find the equation of the normal to the curve at the point P .

3 + 2 = 5 marks

Question 3

Find the **exact** solutions of the equation $\sin(2\pi x) = -\sqrt{3} \cos(2\pi x)$, $0 \leq x \leq 1$.

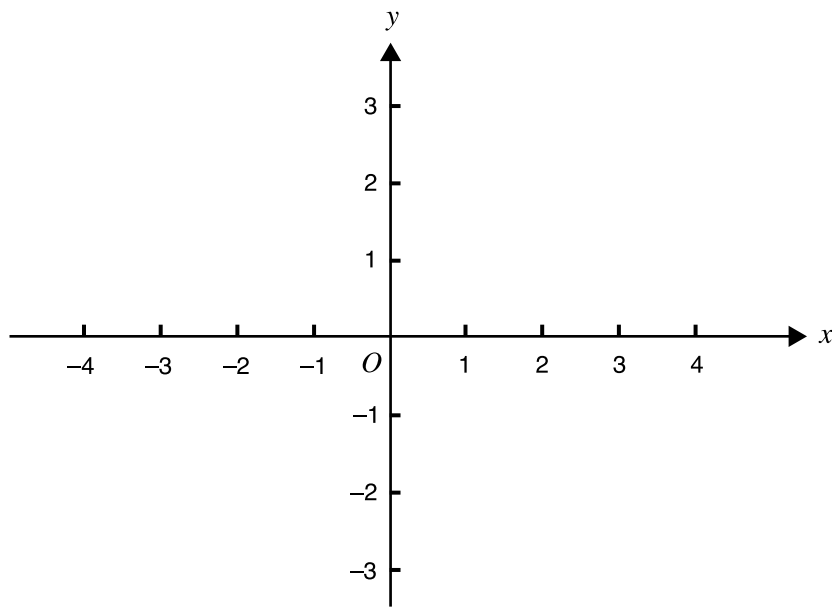
2 marks

Question 4

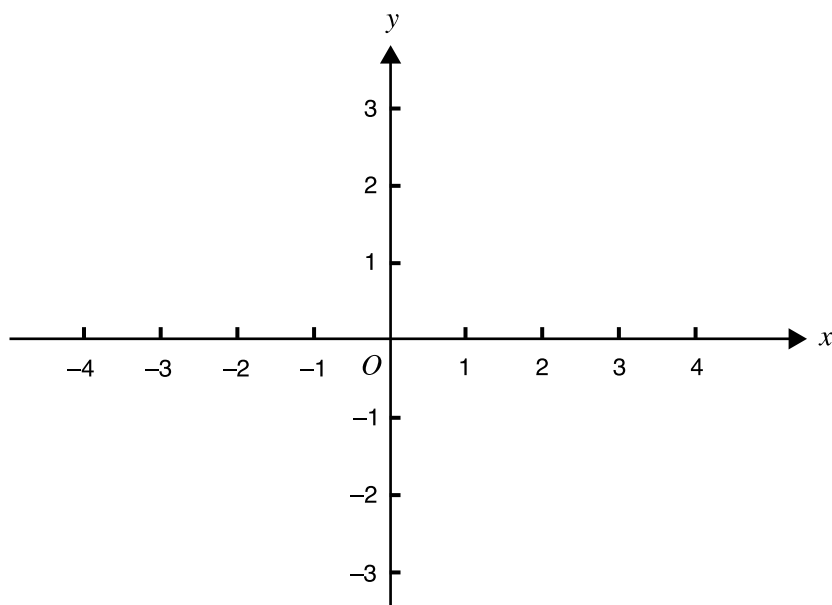
- a. The graph of the function f with rule $f(x) = 2 \log_e(x + 3) + 1$ intersects the axes at the points $(a, 0)$ and $(0, b)$.

Find the **exact** values of a and b .

- b. Hence sketch the graph of the function with rule $f(x) = 2 \log_e(x + 3) + 1$ on the axes below. Label any asymptote with its equation.



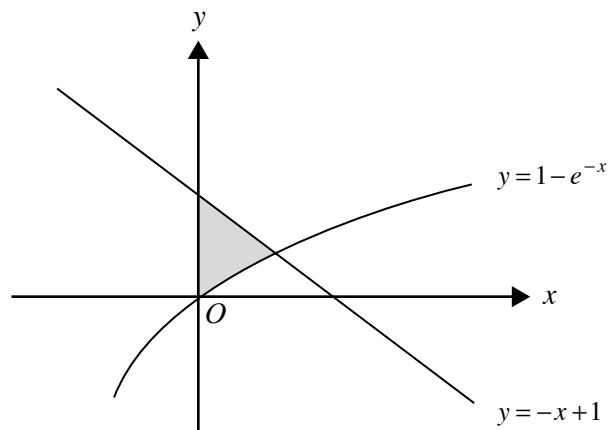
- c. On the axes below, sketch the graph of f' , the derivative of f . Label any point(s) of intersection with the axes with its coordinates. Label any asymptote with its equation.



2 + 2 + 2 = 6 marks

Question 5

The graphs with equations $y = -x + 1$ and $y = 1 - e^{-x}$ are shown below.



- a. Find the solution to the equation $-x + 1 = 1 - e^{-x}$, correct to three decimal places.

- b. Use calculus to find the area of the shaded region, correct to two decimal places.

1 + 3 = 4 marks

Question 6

The height of plants sold by a garden nursery supplier are normally distributed with a mean of 20 cm and a standard deviation of 5 cm.

- a. Complete the following table by finding the proportions for each of the three plant sizes, correct to four decimal places.

Description of plants	Size of plants (cm)	Cost (\$)	Proportion of plants
Small	less than 10	1.50	
Medium	10–30	2.50	
Large	above 30	4.00	

- b. Find the expected cost, to the nearest dollar, for 100 plants chosen at random from the garden nursery supplier.

2 + 2 = 4 marks