

Multiple Choice Questions

Question 1

The function $f(x) = 3 \cos(4(x - \pi))$ has a period and amplitude respectively of

- A $3, \frac{\pi}{4}$
- B $\frac{\pi}{4}, 3$
- C $3, \frac{\pi}{2}$
- D $\frac{\pi}{2}, 3$
- E $4\pi, 3$

Question 2

The solution(s) of $\sin(2x) = \cos(2 - x)$, $x \in [0, \pi]$ is/are closest to

- A 1.19, 3.28
- B 1.19
- C 1.19, 2.78
- D 1.19, 3.28, 5.38, 5.83
- E 2.78

Question 3

$\int_0^{\frac{\pi}{3}} (a \sin(\theta) + b \cos(\theta)) d\theta$ equals

- A $\frac{1}{2}(3a + \sqrt{3}b)$
- B $\frac{1}{2}(3a - \sqrt{3}b)$
- C $\frac{1}{2}(a + \sqrt{3}b)$
- D $\frac{1}{2}(a - \sqrt{3}b)$
- E $\frac{1}{2}(\sqrt{3}b - a)$

Question 4

The value of $\cos(\pi + x) + \sin\left(\frac{\pi}{2} - x\right)$ when $\cos(x) = 0.7$ is

- A $\pi + 0.7$
- B $\pi - 0.7$
- C -1.4
- D 0
- E 1.4

Question 5

The coefficient of the term containing x^6 in the expansion of $(a - x^3)^5$ is

- A $-a^3$
- B a^3
- C $10a^2$
- D $-10a^3$
- E $10a^3$

Question 6

If $\log_3(x - 2) + \log_3(x) - 1 = 0$ then x equals

- A -1
- B 1
- C 3
- D -1 or 3
- E -3 or 1

Question 7

If $2e^x - 1 = \frac{1}{e^x}$ then x equals

- A $-\frac{1}{2}$ or 1
- B 0
- C 1
- D $\log_e \frac{1}{2}$
- E $\log_e \frac{1}{2}$ or 1

Question 8

$2^{2x} + 2^x + b = 0$ has only one real solution if

- A $b > 0$
- B $b < 0$
- C $b \leq 0$
- D $b = \frac{1}{4}$ only
- E $b = -\frac{1}{4}$ only

Question 9

The range of $f: (3, \infty) \rightarrow \mathbb{R}$, where $f(x) = 2\sqrt{5x - 3} + 6$ is

- A $(6, \infty)$
- B $[\frac{3}{5}, \infty)$
- C $\mathbb{R}$
- D $(4\sqrt{3} + 6, \infty)$
- E $[4\sqrt{3} + 6, \infty)$

Question 10

The graph of $y = x^3e^x + 1$ is translated -1 unit parallel to the x -axis and then dilated by a factor of 2 from the x -axis. The equation of the new graph is

- A $y = 2(x + 1)^3e^{(x + 1)} + 2$
- B $y = 2(x + 1)^3e^{(x + 1)} + 1$
- C $y = (\frac{1}{2}x + 1)^3e^{(\frac{1}{2}x + 1)} + 1$
- D $y = 2(x - 1)^3e^{(x - 1)} + 2$
- E $y = \frac{1}{2}(x + 1)^3e^{(x + 1)} + \frac{1}{2}$

Question 11

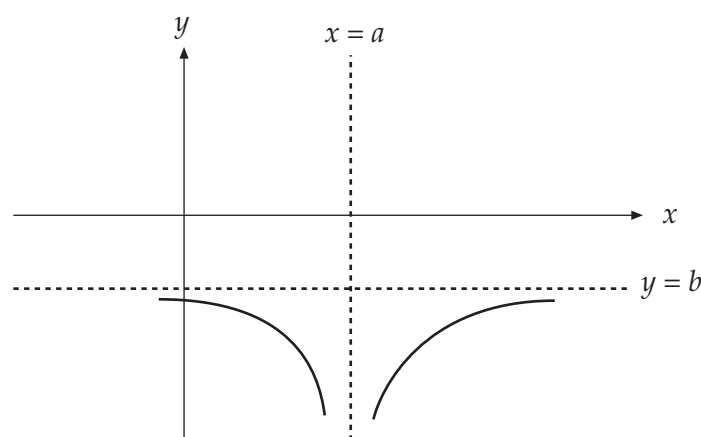
If $f: \mathbb{R} \setminus \{2\} \rightarrow \mathbb{R}$, $f(x) = \frac{5}{2 - x} + 3$ then the largest possible value of a where $g: (-\infty, a) \rightarrow \mathbb{R}$, $g(x) = f(2x)$ is a one to one function is

- A -2
- B -1
- C 1
- D 2
- E 3

Question 12

The function f defined by $f: R \rightarrow R$, where $f(x) = -e^{2(x-1)} + 4$ will have an inverse function with

- A two asymptotes.
- B only one asymptote, $y = 4$ and only one intercept, $(0, 4 - \frac{1}{e^2})$.
- C only one asymptote, $x = 4$ and only one intercept, $(4 - \frac{1}{e^2}, 0)$.
- D only one asymptote, $y = 4$ and intercepts at $(0, 4 - \frac{1}{e^2})$ and $(\frac{1}{2} \log_e(4) + 1, 0)$.
- E only one asymptote, $x = 4$ and intercepts at $(4 - \frac{1}{e^2}, 0)$ and $(0, \frac{1}{2} \log_e(4) + 1)$.

Question 13

The rule for the above graph could be

- A $y = \frac{-1}{(x-a)^2} + b$
- B $y = \frac{-1}{(x-a)^2} - b$
- C $y = \frac{1}{(x-a)^2} + b$
- D $y = \frac{-1}{(x+a)^2} + b$
- E $y = \frac{-1}{(x+a)^2} - b$

Question 14

The equation of the graph of the quartic function which passes through the points with coordinates $(-1, 0)$, $(0, 1)$, $(1, 0)$, $(2, 9)$ and $(-2, 9)$ is

- A $y = x^4 - 2x^3 + 1$
- B $y = (x - 1)^2(x + 1)^2$
- C $y = -x^4 + x^2 + 2x - 2$
- D $y = -(x + 1)(x - 1)^3$
- E $y = -(x - 1)(x + 1)^3$

Question 15

The **largest** instantaneous rate of change, correct to three decimal places, of the function $f(x) = (2x + 1)10^x$ with respect to x where $2x + 1 = 10^x$ is

- A 4.303
- B 4.302
- C 2.719
- D 0
- E -0.125

Question 16

If $h(x)$ and $g(x)$ are the tangents to the curve $f(x) = x^2 + 4x - 5$ where $f(x) = 0$, then $h(x) = g(x)$ at

- A $(-2, -18)$
- B $(-2, -9)$
- C $(-5, 1)$
- D $(-5, -6)$
- E $(-6, 6)$

Question 17

If $g(x) = \frac{\log_e(\cos x)}{\tan x}$ then $g'(x)$ equals

- A $\frac{\tan^2(x) - \sec^2(x) \log_e(\cos x)}{\tan^2(x)}$
- B $\frac{\tan^2(x) + \sec^2(x) \log_e(\cos x)}{\tan^2(x)}$
- C $\frac{-\tan^2(x) + \sec^2(x) \log_e(\cos x)}{\tan^2(x)}$
- D $-1 - \frac{\log_e(\cos x)}{\sin^2(x)}$
- E $-1 - \frac{\log_e(\cos x)}{\tan^2(x)}$

Question 18

An approximate value for $\frac{1}{\sqrt{99.96}}$ is

- A $\frac{1}{10} + 0.04 \times \frac{-1}{2000}$
- B $\frac{1}{10} - 0.04 \times \frac{-1}{2000}$
- C $\frac{1}{10} - 0.04 \times \frac{-1}{20}$
- D $\frac{1}{10} + 0.04 \times \frac{-1}{20}$
- E $\frac{1}{10} - 0.4 \times \frac{-1}{2000}$

Question 19

If $f(x) = (x-3)^2x$ and h is a function such that $h'(x) = f(x)$, then the largest subset of R for which the gradient of $h(x)$ is negative is

- A $(-\infty, 0) \cup (3, \infty)$
- B $(-\infty, 0)$
- C $(1, 3)$
- D $(-\infty, 1) \cup (3, \infty)$
- E $(0, 3) \cup (3, \infty)$

Question 20

Which of the following rules for $f(x)$ will not give an overestimate of the area bounded by the graph of $f(x)$ and the x -axis, if the **right** rectangle rule is used between $x = 0$ and $x = 3$ using strips of width 0.5?

- A $f(x) = 10^x$
- B $f(x) = e^x$
- C $f(x) = \sqrt{x}$
- D $f(x) = (x - 3)(3 - x)$
- E $f(x) = -x^2(x + 1)(x + 2)$

Question 21

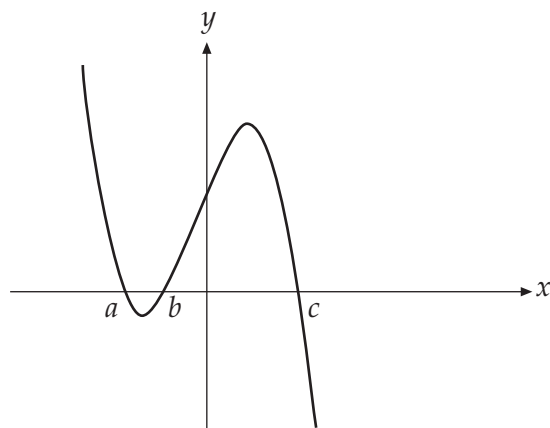
An antiderivative of $\frac{4x^5}{x^6 + 7}$ could be

- A $\log_e(x^6 + 7)$
- B $4\log_e(x^6 + 7)$
- C $\frac{2}{3}\log_e(x^6)$
- D $\frac{3}{2}\log_e(x^6 + 7)$
- E $\frac{2}{3}\log_e(x^6 + 7) + 3$

Question 22

If $\int_1^{\frac{3}{2}} a(2x - 3)^4 dx = 10$, where a is a constant, then a equals

- A -100
- B -50
- C 1
- D 50
- E 100

Question 23

The area enclosed by the graph $y = f(x)$ shown above and the x -axis can be determined by evaluating

- A $\int_a^b f(x)dx + \int_b^c f(x)dx$
- B $\int_a^b f(x)dx + \int_c^b f(x)dx$
- C $\int_b^a f(x)dx - \int_b^c f(x)dx$
- D $\int_b^a f(x)dx + \int_b^c f(x)dx$
- E $\int_0^a f(x)dx + \int_0^b f(x)dx - \int_b^c f(x)dx$

Question 24

X is a normally distributed variable with $\mu = 3$ and $\sigma^2 = 2.56$.

If $\Pr(X < k) = 0.734$, then k equals

- A 0.2
- B 1.4
- C 2.0
- D 4.0
- E 4.6

Question 25

A fire alarm has a probability of failure of 0.05. In an apartment block where there are 10 such alarms. The probability that at least one fails is given by:

- A $1 - (0.95)^{10}$
- B $1 - {}^{10}C_1 (0.05)^1 (0.95)^9$
- C $1 - [{}^{10}C_1 (0.05)^1 (0.95)^9 + (0.95)^{10}]$
- D $1 - [{}^{10}C_9 (0.05)^9 (0.95)^1 + (0.05)^{10}]$
- E $1 - (0.05)^{10}$

Question 26

In a container of tulip bulbs there are 5 that have red flowers, 7 yellow and 3 orange. A group of three bulbs is chosen at random. The probability that exactly one of the flowers is orange is:

- A $\frac{1}{5}$
- B $\frac{2}{5 \times 14 \times 13}$
- C $\frac{2 \times 3}{5 \times 14 \times 13}$
- D $\frac{12 \times 11 \times 3}{5 \times 14 \times 13}$
- E $\frac{12 \times 11}{5 \times 14 \times 13}$

Question 27

A normal distribution has $\mu = 5.6$ and $\sigma = 6.5$. If the variable x , has a value of 1.2, the value of the normal variable z , is closest to:

- A 0.338
- B -0.786
- C -0.677
- D 0.677
- E -4.40

Part II: Short Answer

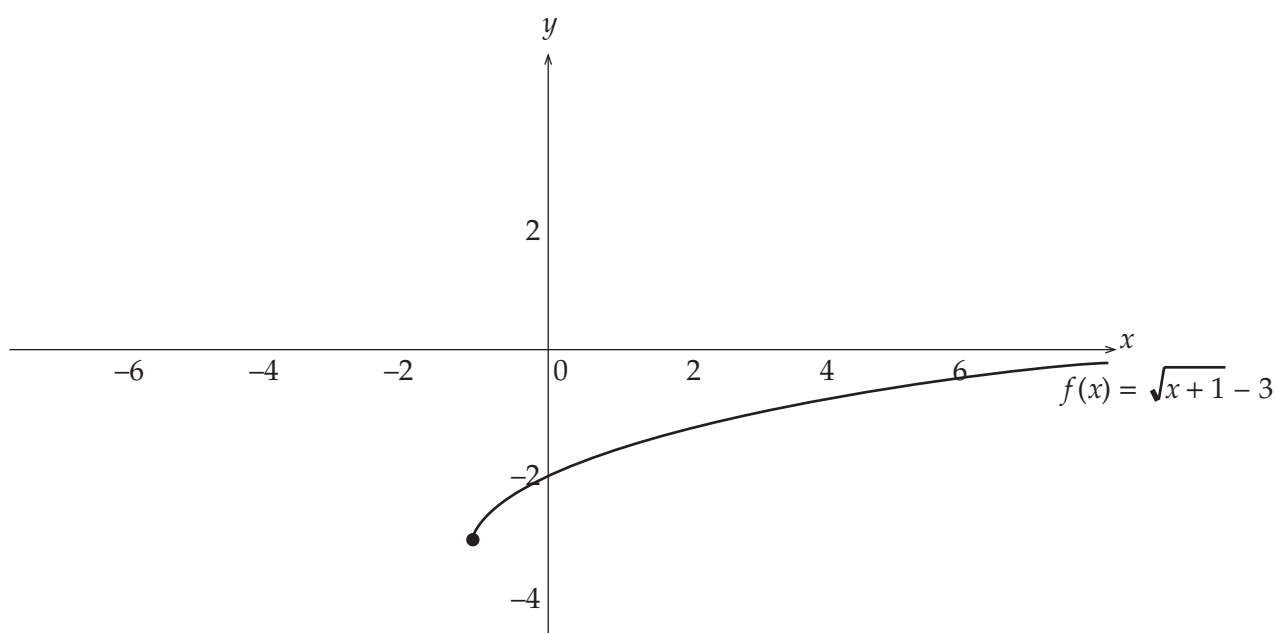
Question 1

Find exact solutions of $1 + \sin(x) = 2\cos^2(x)$ where $x \in [0, 2\pi]$.

4 marks

Question 2

The graph of $f: [-1, \infty) \rightarrow \mathbb{R}, f(x) = \sqrt{x+1} - 3$ is shown below.



- Sketch $g: (-\infty, 3) \rightarrow \mathbb{R}, g(x) = 3e^{(x-2)} - 3$ on the above axes, labelling all relevant features of the graph.
- Find the coordinates of the points where $f(x) = g(x)$, correct to two decimal places.
- Find the area bounded by the two curves, correct to two decimal places.

3 + 2 + 2 = 7 marks

Question 3

Let $f(x) = (2x - 1)^3(x + 2)$.

a Find $f'(x)$.

b Hence show that the stationary points occur at $x = \frac{1}{2}$ and $x = -\frac{11}{8}$.

c Find the average rate of change of $f(x)$ between the two stationary points.

1 + 2 + 3 = 6 marks

Question 4

a If $y = x^2 \log_e x$ find $\frac{dy}{dx}$.

b Hence find $\int 2x \log_e(x) dx$.

1 + 2 = 3 marks

Question 5

Two chocolate companies make 750 gram blocks of milk chocolate. Company X's blocks have a mean of 760 and a standard deviation of 7, while Company Y's blocks have a mean of 768 and a standard deviation of 14.

Determine which company has the higher probability of producing a block that weighs at least 750 grams?

3 marks