2003 Mathematical Methods
Written Examination 1 (facts, skills and applications)
Suggested answers and solutions

Answers – Multiple Choice

1. **D**	2. **B**	3. **C**	4. **D**	5. **E**
6. **C**	7. **B**	8. **B**	9. **D**	10. **A**
11. **C**	12. **E**	13. **A**	14. **B**	15. **A**
16. **A**	17. **D**	18. **B**	19. **B**	20. **D**
21. **E**	22. **E**	23. **D**	24. **D**	25. **A**
26. **D**	27. **C**			

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1 Period $= \dfrac{2\pi}{n} = \dfrac{\pi}{2}$, amplitude = 3 **[D]**

2 Only 1 solution in the domain **[B]**

3 $\int_0^{\frac{\pi}{3}} (a\sin(\theta) + b\cos(\theta)d\theta =$

$$\left[-a\cos(\theta) + b\sin(\theta)\right]_0^{\frac{\pi}{3}}$$

$$= \left[-a\cos(\tfrac{\pi}{3}) + b\sin(\tfrac{\pi}{3})\right] - [-a + 0]$$

$$= -\frac{a}{2} + \frac{\sqrt{3}b}{2} + a$$

$$= \frac{a}{2} + \frac{\sqrt{3}b}{2}$$ **[C]**

4 $\cos\left(\pi + x\right) + \sin\left(\dfrac{\pi}{2} - x\right) = -\cos(x) + \cos(x)$

$= 0$ **[D]**

5 General term is ${}^5C_r\,(a)^{5-r}\left(-x^3\right)^r$

to find the term x^6, $r = 2$

$\therefore {}^5C_2\,(a)^{5-2}\left(-x^3\right)^2$

$= 10a^3x^6$

The coefficient of x^6 is $10a^3$ **[E]**

6 $\log_3(x-2) + \log_3(x) - 1 = 0$

$\log_3(x-2)x = 1$

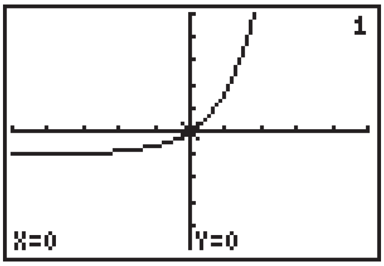
$3 = (x-2)x$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ since $x > 2$. **[C]**

7



$x = 0$

OR

Algebraic Solution

$2e^x - 1 = \dfrac{1}{e^x}$

$2e^{2x} - e^x - 1 = 0$

Let $a = e^x$

$2a^2 - a - 1 = 0$

$(2a+1)(a-1) = 0$

$a = 1$ or $a = -\dfrac{1}{2}$

$e^x = 1$, $e^x = -\dfrac{1}{2}$ no solution

$x = 0$ **[B]**

8 $2^{2x} + 2^x + b = 0$

Let $a = 2^x$

$a^2 + a + b = 0$

$a = \dfrac{-1 \pm \sqrt{1-4b}}{2}$

If $2^x = \dfrac{-1 - \sqrt{1-4b}}{2}$ no solution

If $2^x = \dfrac{-1 + \sqrt{1-4b}}{2}$ there will be one solution if

$\sqrt{1-4b} > 1$

$1 - 4b > 1$

$-4b > 0$

$b < 0$ **[B]**

9 The domain is $(3, \infty)$.

$f(3) = 2\sqrt{15-3} + 6$

$= 2\sqrt{12} + 6$

$= 4\sqrt{3} + 6$

Hence the range is

$(4\sqrt{3} + 6, \infty)$ **[D]**

10 $y = x^3e^x + 1$ translated -1 unit parallel to the x-axis becomes

$y = (x+1)^3\mathrm{e}^{(x+1)} + 1$.

$y = (x+1)^3\mathrm{e}^{(x+1)} + 1$ dilated a factor of 2 from the x-axis

becomes $y = 2(x+1)^3\mathrm{e}^{(x+1)} + 2$. **[A]**

11 $g(x) = f(2x) = \dfrac{5}{2-2x} + 3$

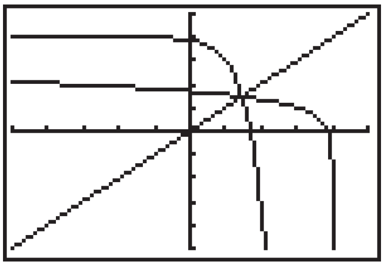
$2 - 2x \neq 0$

$x \neq 1$

The restricted domain is

$(-\infty, 1)$ **[C]**

12



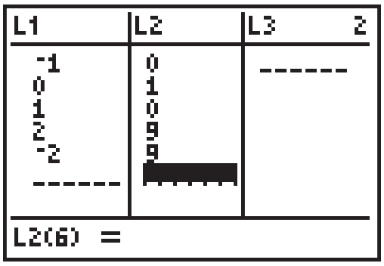
$f(x) = -e^{2(x-1)} + 4$ has one asymptote at $y = 4$ and crosses both axes.

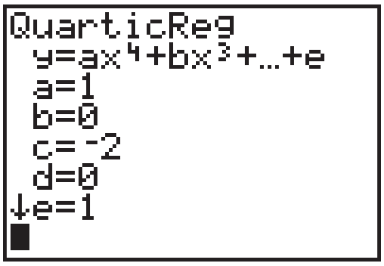
Hence $f^{-1}(x)$ has one asymptote at $x = 4$ and crosses both axes. **[E]**

13 $y = \dfrac{1}{x^2}$ has been reflected in the x-axis, then translated a units parallel to the x-axis and b units parallel to the y-axis.

$y = \dfrac{-1}{(x-a)^2} + b$ **[A]**

14



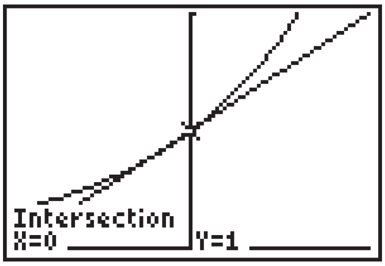


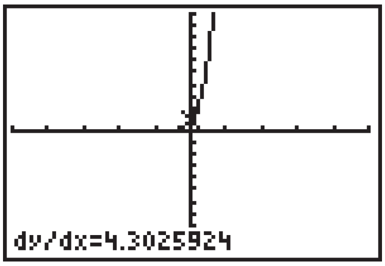
$y = x^4 - 2x^2 + 1$

$= (x^2 - 1)^2$

$= (x-1)^2(x+1)^2$ **[B]**

15





The largest rate of change is when $x = 0$.
The gradient at $x = 0$ is 4.303 **[A]**

16 $f(x) = x^2 + 4x - 5 = 0$

$(x + 5)(x - 1) = 0$

$x = -5$ or $x = 1$

$f'(x) = 2x + 4$

$f'(-5) = -6$

$f'(1) = 6$

at $x = -5$ tangent

$y - 0 = -6(x - -5)$

$y = -6x - 30$

at $x = 1$ tangent

$y - 0 = 6(x - 1)$

$y = 6x - 6$

Now $6x - 6 = -6x - 30$

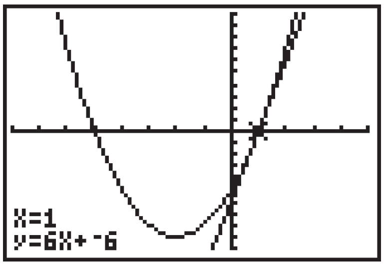
$12x = -24$

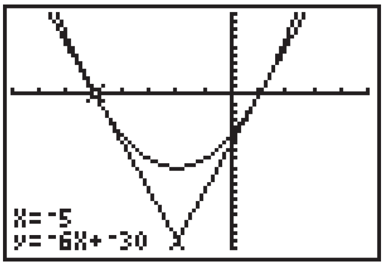
$x = -2$

$y = 6(-2) - 6 = 18$

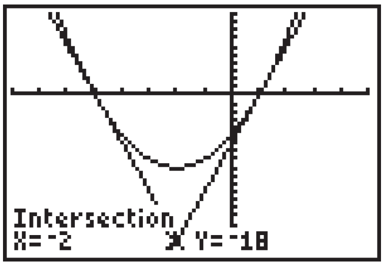
$(-2, -18)$

OR





Enter the equations for the tangents into $\boxed{y =}$



The intersection is $(-2, -18)$. **[A]**

17 $g(x) = \dfrac{\log_e(\cos(x))}{\tan(x)}$

$$g'(x) = \frac{\tan(x)\left(\frac{-\sin(x)}{\cos(x)}\right) - \log_e(\cos(x))\sec^2(x)}{\tan^2(x)}$$

$$= \frac{-\tan^2(x) - \log_e(\cos(x))\sec^2(x)}{\tan^2(x)}$$

$$= \frac{-\tan^2(x)}{\tan^2(x)} - \frac{\log_e(\cos(x))\frac{1}{\cos^2(x)}}{\frac{\sin^2(x)}{\cos^2(x)}}$$

$$= -1 - \frac{\log_e(\cos x)}{\sin^2(x)}$$ **[D]**

18 $f(x+h) \approx f(x) + h\,f'(x)$

$f(x) = \dfrac{1}{\sqrt{x}}$

Note: $\dfrac{1}{\sqrt{99.96}} = \dfrac{1}{\sqrt{100-0.04}}$

$f(100) = \dfrac{1}{\sqrt{100}} = \dfrac{1}{10}$

$f'(x) = -\dfrac{1}{2x^{\frac{3}{2}}}$

$f'(100) = -\dfrac{1}{2 \times 100^{\frac{3}{2}}} = -\dfrac{1}{2000}$

$f(x+h) \approx f(x) + h\,f'(x)$

$\approx \dfrac{1}{10} - 0.04 \times -\dfrac{1}{2000}$ **[B]**

19 $f(x) = (x-3)^2 x$

$h'(x) = f(x)$

$f(x) < 0$ when $x < 0$.

$(-\infty, 0)$ **[B]**

20 For **A**, **B** and **C** the graphs are above the x-axis and as x increases y increases. Hence, the **right** rectangle rule will give an overestimate.
D and **E** are below the x-axis.
The **right** rectangle rule will give an underestimate if as x increases y increases. **[D]**

21 $\displaystyle\int \frac{4x^5}{x^6+7}\,dx$

$= \dfrac{4}{6}\displaystyle\int \frac{6x^5}{x^6+7}\,dx$

$= \dfrac{4}{6}\log_e(x^6+7) + c$

$= \dfrac{2}{3}\log_e(x^6+7) + c$

Thus $\dfrac{2}{3}\log_e(x^6+7) + 3$ is an

antiderivative. **[E]**

22 $\displaystyle\int_1^{\frac{3}{2}} a(2x-3)^4\,dx = 10$

$\left[\dfrac{a(2x-3)^5}{10}\right]_1^{\frac{3}{2}} = 10$

$0 - \dfrac{-a}{10} = 10$

$a = 100$ **[E]**

23 The area is below the x-axis between a and b.

$-\displaystyle\int_a^b f(x)dx = \int_b^a f(x)dx$

The area is above the x-axis between b and c.

The total area is $\displaystyle\int_b^a f(x)dx + \int_b^c f(x)dx$ **[D]**

24 InvNorm(0.734, 3, 1.6)
$= 4$ **[D]**

25 Pr(at least one fails) $= \Pr(X \geq 1)$
$= 1 - \Pr(X = 0)$
$= 1 - 0.95^{10}$ **[A]**

26 $N = 15$, $n = 3$, $D = 3$, $x = 1$

$\dfrac{{}^{12}C_2\,{}^{3}C_1}{{}^{15}C_3} = \dfrac{12 \times 11 \times 3}{5 \times 14 \times 13}$ **[D]**

27 $z = \dfrac{x-\mu}{\sigma} = \dfrac{1.2-5.6}{6.5}$

≈ -0.677 **[C]**

Solutions (short answer)

1 $1 + \sin(x) = 2\cos^2(x)$ **[1M]**

$1 + \sin(x) = 2(1 - \sin^2(x))$

$2\sin^2(x) + \sin(x) - 1 = 0$

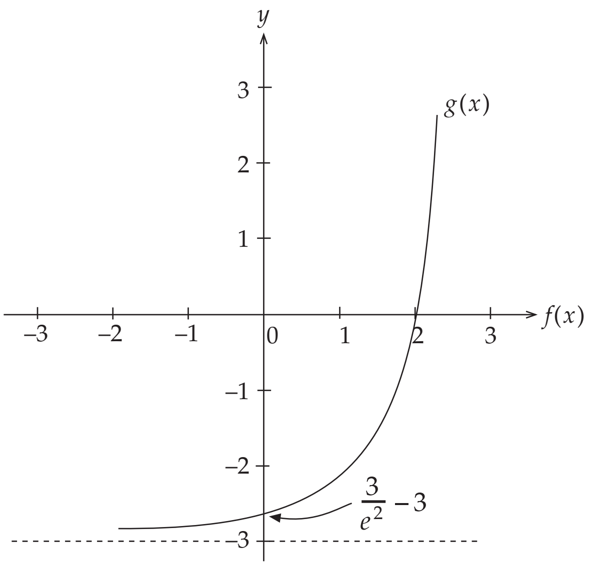
$(2\sin(x) - 1)(\sin(x) + 1) = 0$ **[1M]**

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **[2A]**

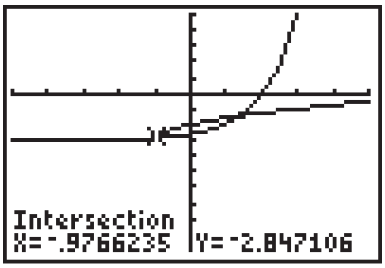
2 **a** Asymptote $y = -3$ **[1A]**

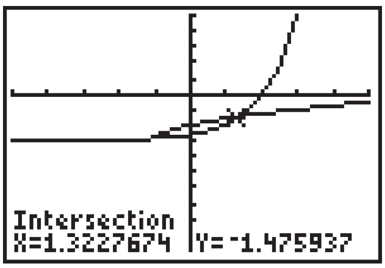
Intercepts$(2, 0)$ $(0, \frac{3}{e^2} - 3)$ **[1A]**

Shape **[1A]**



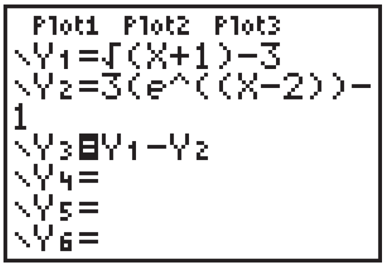
b $(-0.98, -2.85)$ and $(1.32, -1.48)$ **[2A]**

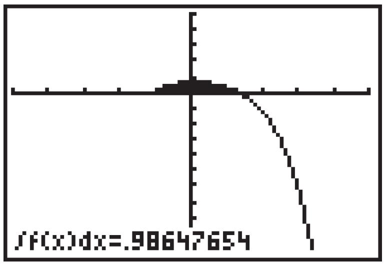




c $\int_{-0.9766235}^{1.3227674} (f(x) - g(x))dx$ **[1M]**

≈ 0.99 units squared **[1A]**





3 **a** $f(x) = (2x - 1)^3(x + 2)$

$f'(x) = 6(2x - 1)^2(x + 2) + (2x - 1)^3$,

by the product rule. **[1A]**

b $f'(x) = (2x - 1)^2(6(x + 2) + (2x - 1))$

$= (2x - 1)^2(8x + 11)$ **[1M]**

$= (2x - 1)^2(8x + 11) = 0$

at a stationary point. **[1M]**

$x = \frac{1}{2}$ or $x = -\frac{11}{8}$

c $f(\frac{1}{2}) = 0$ and $f(-\frac{11}{8}) = -\frac{16875}{512}$ **[2M]**

Average rate of change $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-\frac{16875}{512} - 0}{-\frac{11}{8} - \frac{1}{2}}$ **[1A]**

$= \frac{1125}{64}$

4 **a** $y = x^2\log_e x$

$\frac{dy}{dx} = 2x\log_e(x) + \frac{x^2}{x}$, by the product rule

$= 2x\log_e(x) + x$ **[1 A]**

b $\int(2x\log_e(x) + x)dx = x^2\log_e x + c$ **[1M]**

$\int(2x\log_e(x))dx + \int(x)dx = x^2\log_e x + c$,

where

$\int(2x\log_e(x))dx = x^2\log_e x - \frac{x^2}{2} + k$,

where k is a constant. **[1A]**

5 Company X: Pr(block ≥ 750) = 0.9234 **[1A]**

Company Y: Pr(block ≥ 750) = 0.9007 **[1A]**

More likely Company X **[1A]**