Year 2003

VCE

Mathematical Methods

Trial Examination 2

KILBAHA PTY LTD (Publishers in Education) **ABN 47 065 111 373** **PO BOX 2227 KEW VIC 3101** **AUSTRALIA**	**TEL: (03) 9817 5374** **FAX: (03) 9817 4334** **chemas@chemas.com** www.chemas.com

STUDENT NUMBER									Letter
Figures									
Words									

VICTORIAN CERTIFICATE OF EDUCATION 2003

MATHEMATICAL METHODS

Trial Written Examination 2 (Analysis task)

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*
4	4

Directions to students

Materials
Question and answer book of 12 pages.
Working space is provided throughout the book.
There is a detachable sheet of miscellaneous formula supplied.
You may bring to the examination up to four pages (two A4 sheets) of pre-written notes.
You may use an approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve-sketching.

The task
Detach the formula sheet from the book during reading time.
Please ensure that your **student number** in the space provided on the front cover of this book.
Answer **all** questions
The marks allotted to each part of each question are indicated at the end of the part.
There is a total of 55 marks available for the examination.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
Where an exact answer is required to a question, appropriate working must be shown and calculus must be used to evaluate derivatives and definite integrals.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
All written responses should be in English.

MATHEMATICAL METHODS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$

area of a triangle: $\frac{1}{2}bc\sin A$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax}dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e x\right) = \frac{1}{x} \qquad \int \frac{1}{x}dx = \log_e x + c, \text{for } x > 0$$

$$\frac{d}{dx}(\sin ax) = a\cos ax \qquad \int \sin ax\ dx = -\frac{1}{a}\cos ax + c$$

$$\frac{d}{dx}(\cos ax) = -a\sin ax \qquad \int \cos ax\ dx = \frac{1}{a}\sin ax + c$$

$$\frac{d}{dx}(\tan ax) = \frac{a}{\cos^2 ax} = a\sec^2 ax$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Statistics and Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Discrete distributions			
	$\Pr(X = x)$	mean	variance
general	$p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\,(x-\mu)^2\, p(x)$ $= \Sigma\, x^2 p(x) - \mu^2$
binomial	${}^nC_x\, p^x (1-p)^{n-x}$	np	$np(1-p)$
hypergeometric	$\frac{{}^DC_x\ {}^{N-D}C_{n-x}}{{}^NC_n}$	$n\frac{D}{N}$	$n\frac{D}{N}\left(1-\frac{D}{N}\right)\frac{N-n}{N-1}$
Continuous distributions			
normal	If X is distributed $\mathrm{N}(\mu, \sigma^2)$ and $Z = \frac{X-\mu}{\sigma}$, then Z is distributed $\mathrm{N}(0, 1)$.		

Table 1 Normal distribution – cdf

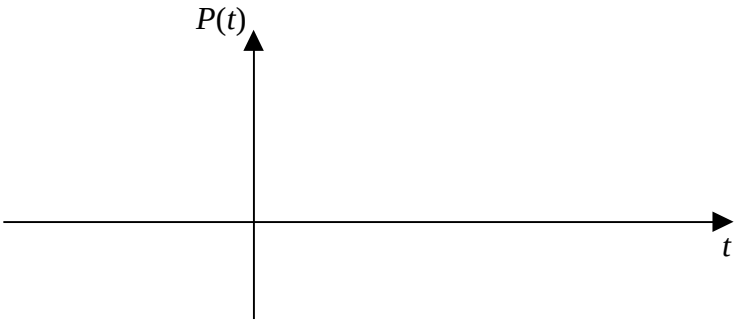
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

END OF FORMULA SHEET

Question 1

a. The size of the population of a country can be modeled by the equation $P(t) = 50e^{0.05t}$ where P is the population measured in millions of people, and t is the time in years measured from 2003.

i. Sketch the graph of $y = P(t)$ on the given axes, showing all intercepts with the axes.



2 marks

ii. What is the predicted population for 2008? Give your answer to the nearest million.

1 mark

iii At what average rate is it expected that the population will increase between 2003 and 2008? Give your answer to one decimal place.

3 marks

Question 1
a. (continued)

iv At what instantaneous rate is the population expected to grow in 2023? Give an exact answer.

2 marks

b. In the same country earthquakes are common. The intensity of an earthquake is rated by its measurement on the Richter scale, R, where $R = \log_{10}\left(\frac{a}{T}\right) + B$. a is the amplitude (in micrometres) of the vertical ground motion at the receiving station, T is the period of the seismic wave (in seconds) and B is a factor that accounts for the weakening of the seismic wave with increasing distance from the epicentre of the earthquake.

i. What would the Richter scale reading be for an earthquake with an amplitude of 3×10^6 micrometres, a period of 3×10^{12} seconds and where $B = 12$?

2 marks

ii. In terms of a what would the Richter scale reading be for an earthquake with an amplitude of a hundred times a micrometres, and the same values of T and B as in **part i**.?

1 mark

Question 1
b. (continued)

iii. Show that for any earthquake where the amplitude, a , at the beginning of the earthquake becomes one hundred times greater, but where B and T are constants, then the change in the intensity of the earthquake is 2.

4 marks

(Total = 15 marks)

Question 2.

The annual rainfall in the country of *Aradulia* is normally distributed and the mean annual rainfall over a large number of years has been determined to be 310cm, with a standard deviation of 80cm. Drought conditions occur in *Aradulia* when the annual rainfall is less than 150cm.

a What is the probability in a year selected at random that drought conditions will occur? Give your answer to one significant figure.

2 marks

b. What is the probability that in any 10 years, drought conditions will occur in *Aradulia* in exactly 4 of those years? Give your answer to four decimal places.

2 marks

c. If it is known that that the rainfall was less than 310 cm in one particular year in *Aradulia*, what is the probability that drought conditions occurred in that year. Give your answer to four decimal places.

3 marks

d. Because of the increasing population in *Aradulia*, the demand for water is predicted to increase linearly. This demand can be modeled by the equation $r(t) = 30t + 220$, where $r(t)$ is the annual rainfall needed to meet the demand for water t years after 2003.
Find the probability that the annual rainfall will satisfy the demand in 2010.

4 marks

e. The probability that less than a certain amount of rainfall fell in *Aradulia* in 2002 was known to be 0.346. How much rainfall was there in *Aradulia* during that year?

3 marks

(Total = 14 marks)

Question 3

The cost of producing computer chips can be modeled by the formula $C = a + \frac{b}{x^2}$ where C is the cost per chip and x is the number of chips produced.
The Outel Chip company can produce 1000 chips at a cost of $200 per chip and 5000 chips at a cost of $140 per chip.

a. Show that the value of b is 62,500,000

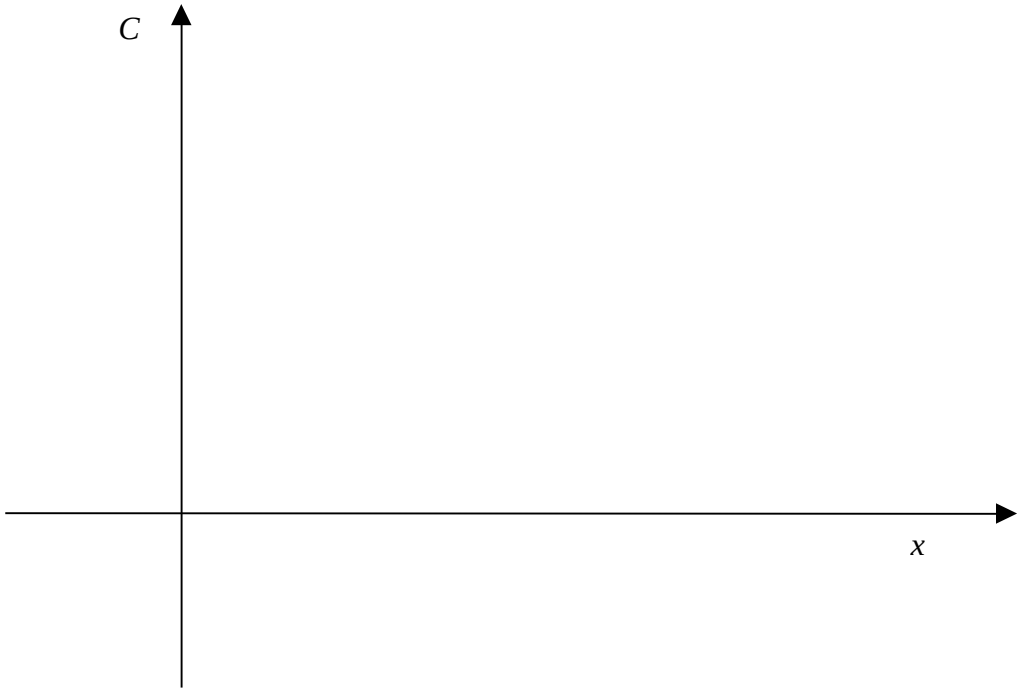
1 mark

b. Find the value of a to one decimal place.

1 mark

Question 3 (continued)

c. Sketch the graph of C against x on the axes below for $0 < x \leq 10{,}000$.
Show the point on the graph where x = 10,000.



2 marks

Question 3 (continued)

d. The revenue produced by the chips is the **product** of the number of chips made and C, the cost of one chip.
How many chips should be produced so that the revenue is maximised?

3 marks

e. Find $C^{-1}(x)$, the inverse of $C(x)$

2 marks

Question 3 (continued)

f. Using the result from part **e** or otherwise, find the number of chips that must be produced for the cost to be \$300 per chip.

1 mark

(Total = 10 marks)

Question 4

Consider the function $f:\Re \rightarrow \Re,\ f(t) = \log_e \sin[2\pi(t + 0.5)]$

a. Identify one of the largest possible continuous domain that contains $t = -0.25$ for the function $f(t)$

3 marks

b. Explain why the domain of $f(t)$ cannot be continuous over $\Re$

1 mark

Question 4 (continued)

c. Find the value of t when $f(t) = 0$

2 marks

d. Find the area between the graph of $f(t)$, the t axis and the ordinates $t = -0.25$ and $t = -0.04$. Give your answer to four decimal places.

1 mark

e. Show that the slope of the tangent to the graph $y = f(t)$ at $t = -\frac{3}{8}$ is 2π

2 marks

Question 4 (continued)

f. If $f(t) = \log_e \sin[2\pi(t + 0.5)]$ where $-0.25 < t < 0$ then find the rule $g(t)$ for the reflection of this graph in the line $y = t$

3 marks

g. Explain why the domain of $g(t)$ is $-\infty < t \leq 0$.

2 marks

Question 4 (continued)

h. Explain what happens to $g(t)$ as $t \to -\infty$

_____

_____

_____

_____

_____

2 marks

(Total = 16 marks)

End of 2003 Mathematical Methods Trial Examination 2
Question and Answer Book

KILBAHA PTY LTD (Publishers in Education) **ABN 47 065 111 373** **PO BOX 2227 KEW VIC 3101** **AUSTRALIA**	**TEL: (03) 9817 5374** **FAX: (03) 9817 4334** **chemas@chemas.com** www.chemas.com