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Student Name……

MATHEMATICAL METHODS UNITS 3 & 4

TRIAL EXAMINATION 1

(FACTS, SKILLS AND APPLICATIONS TASK)

2003

Reading Time: 15 minutes
Writing time: 90 minutes

Instructions to students

This exam consists of Part I and Part II.
Part I consists of 27 multiple-choice questions, which should be answered on the detachable answer sheet which can be found on page 21 of this exam.
Part II consists of 8 short-answer questions which should be answered in the spaces provided.
Part I begins on page 2 of this exam and is worth 27 marks.
Part II begins on page 14 of this exam and is worth 23 marks.
There is a total of 50 marks available.
All questions in Part I and Part II should be answered.
Students may bring up to two A4 pages of pre-written notes into the exam.

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PART I

Question 1

The graph of the function $y = 2\sin 3\left(x + \frac{\pi}{2}\right)$ over the interval $[0,7\pi]$ has a range given by

A. $[0,2]$
B. $\left[\frac{-2\pi}{3}, \frac{2\pi}{3}\right]$
C. $[-2,2]$
D. $[0,7\pi]$
E. R

Question 2

The sum of the solutions to the equation $2\cos(2x) = 1$, in the interval $[0,2\pi]$ is

A. $\frac{\pi}{3}$
B. $\frac{2\pi}{3}$
C. π
D. 2π
E. 4π

Question 3

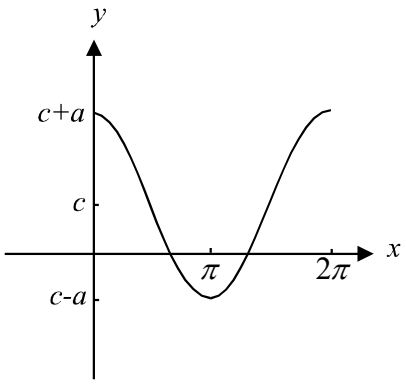
The dose *d*, (in mg/ml) of a drug administered intravenously, changes over the course of a cycle according to the rule $d = 0.05\sin\left(\frac{\pi t}{16}\right) + 0 \cdot 04$, $t \geq 0$ where *t* is measured in hours.
After how many hours would the patient have had one complete cycle of the drug administered?

A. 0.05
B. 0.44
C. 16
D. 24
E. 32

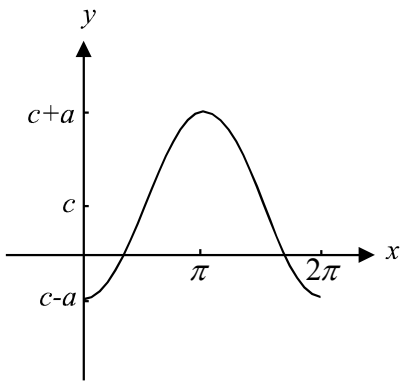
Question 4

A possible graph of the function $y = -a\cos(x) + c$ over the interval $[0,2\pi]$ where $a > 0,\ c > 0$ and $a > c,$ could be:

A.



B.



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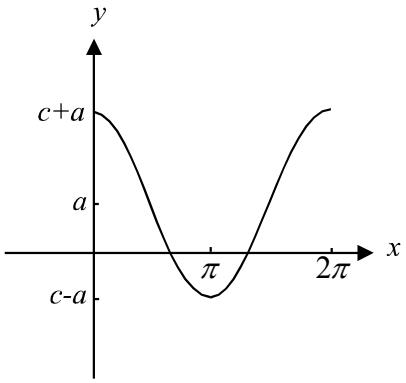
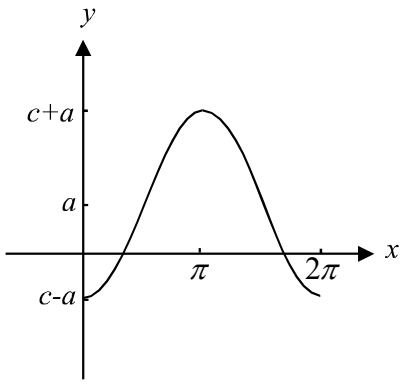
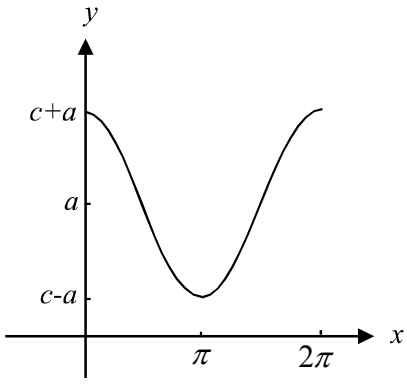


D.

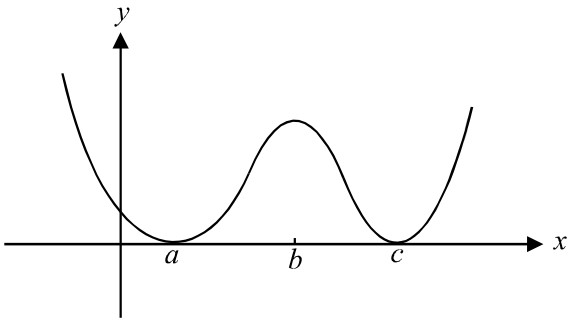


E.



Question 5

The graph of a polynomial function is shown below.



The rule of the function could be

A. $y = (x-b)^4$

B. $y = (x-a)(x-c)$

C. $y = (x-a)^2(x-c)^2$

D. $y = (x-a)(x-b)(x-c)$

E. $y = (x-a)(x-b)^2(x-c)$

Question 6

Which one of the following functions has a graph with a vertical asymptote of $x = a$?

A. $y = \log_e(x+a)$

B. $y = \dfrac{1}{x+a}$

C. $y = \dfrac{1}{x-a} - a$

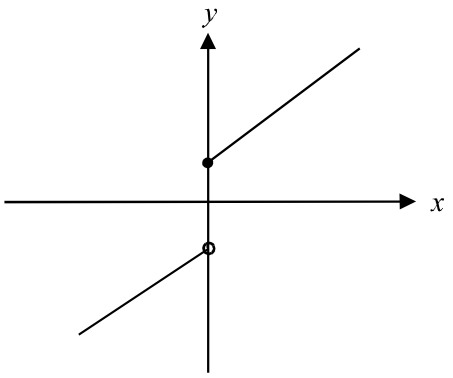
D. $y = e^x + a$

E. $y = e^{(x-a)}$

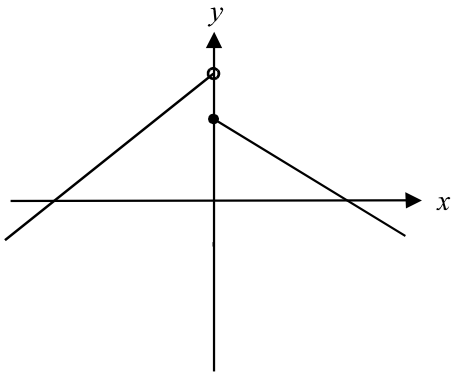
Question 7

Which one of the following graphs shows a many-one function?

A.



B.



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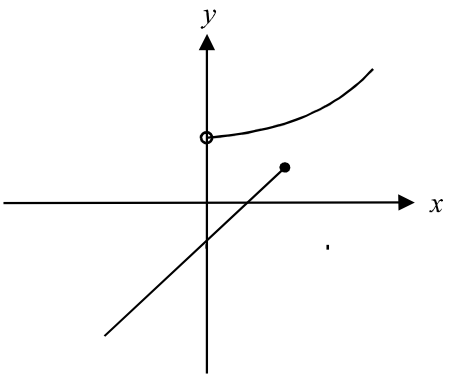

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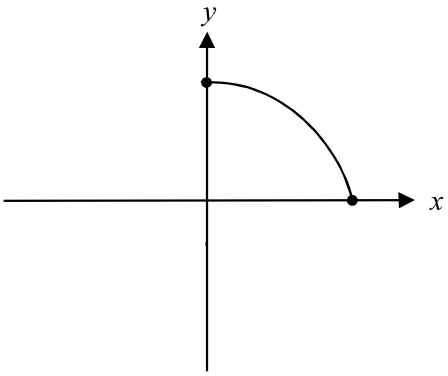
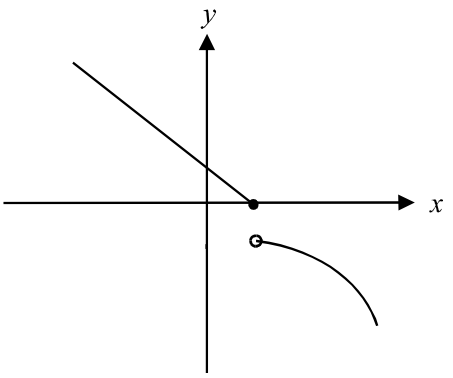
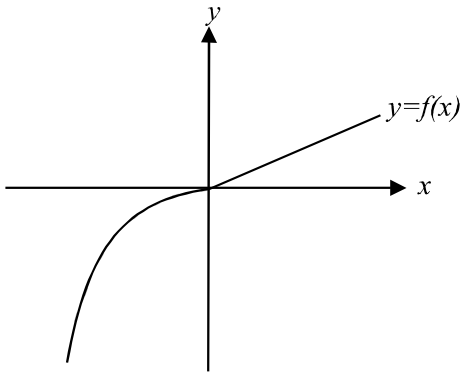


E.



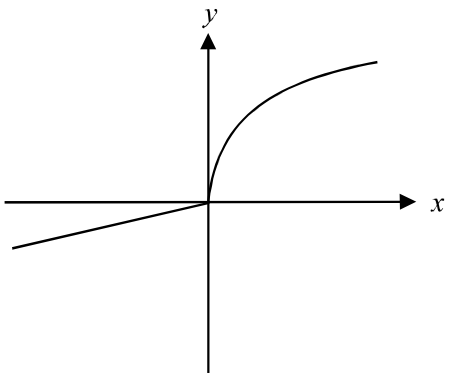
Question 8

The graph of the function $y = f(x)$ is shown below. The scale on the *x*-axis is the same as that on the *y*-axis.

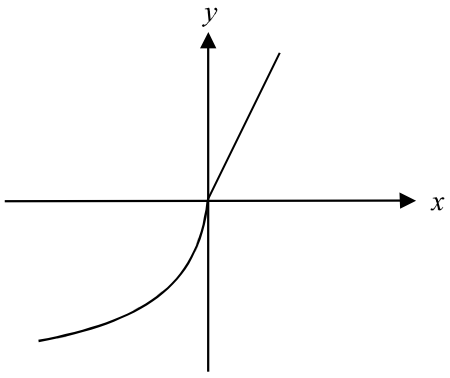


Which one of the following could be the graph of the inverse function?

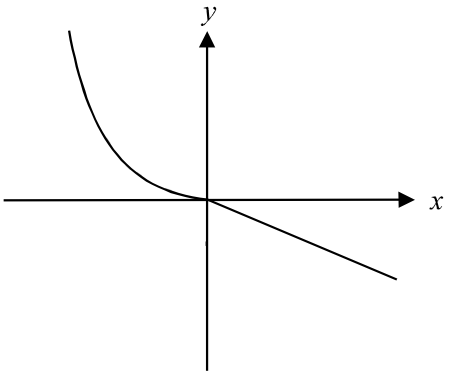
A.



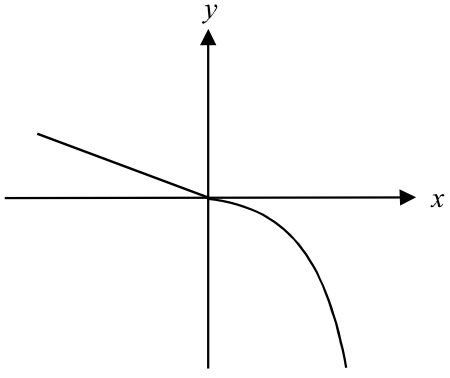
B.



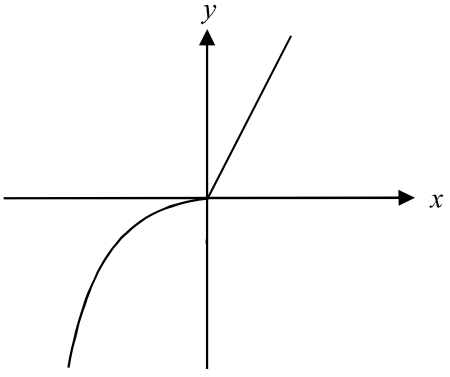
C.



D.



E.



Question 9

The tenth and eleventh rows of Pascal's triangle are shown below.

1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

The coefficient of the x^4 term in the expansion of $(x+2)^{10}$ is

A. 84×2^6
B. 120×2^6
C. 210×2^6
D. 252×2^6
E. 330×2^6

Question 10

x	1	2	3	4	5	6	7	8
y	1.6	2.5	3.1	3.6	3.8	3.9	4.0	4.1

The data in the table above could be modelled best using
A. a logarithmic function
B. an exponential function
C. a circular function
D. a cubic function
E. a linear function

Question 11

The exact solution to the equation $5 \times 2^{3x} = 1$ is given by

A. $-\frac{1}{3}$
B. $\frac{1}{3} \times 2^{\frac{1}{5}}$
C. $\frac{1}{5}\log_2\left(\frac{1}{3}\right)$
D. $3\log_2\left(\frac{1}{5}\right)$
E. $\log_2\left(\frac{1}{\sqrt[3]{5}}\right)$

Question 12

The expression $e^{\left(\log_e(2x)+2\log_e(x^3)\right)}$ is equal to

A. $2x^6$
B. $2x^7$
C. e^{x^3}
D. $2x + x^5$
E. $2x + x^6$

Question 13

If $y = \dfrac{1}{\sqrt{1-\cos(x)}}$ then $\dfrac{dy}{dx}$ is given by

A. $\dfrac{-\sin(x)}{2(1-\cos(x))^{\frac{3}{2}}}$

B. $\dfrac{\sin(x)}{2(1-\cos(x))^{\frac{3}{2}}}$

C. $\dfrac{-1}{(1-\cos(x))^{\frac{1}{2}}}$

D. $\dfrac{-1}{2(1-\cos((x))^{\frac{3}{2}}}$

E. $\dfrac{1}{2(1-\cos(x))^{\frac{3}{2}}}$

Question 14

The derivative of $e^{\tan(2x)}$ is

A. $2e^{2\tan(2x)}$

B. $\sec(2x)e^{\tan(2x)}$

C. $\sec^2(2x)e^{\tan(2x)}$

D. $2e^{\tan(2x)}\sec^2(2x)$

E. $e^{\tan(2x)} + 2\sec^2(2x)e^{\tan(2x)}$

Question 15

Given that $y = \log_e\left(2x^2 + 5x\right)$, then the rate of change of y with respect to x when $x = 1$ is

A. $\dfrac{4}{7}$

B. 1

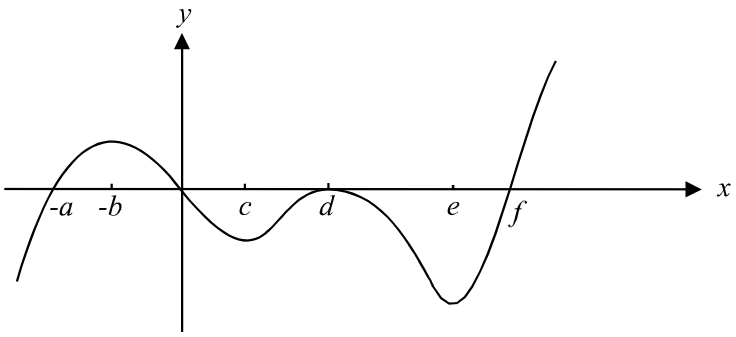
C. $\dfrac{9}{7}$

D. $\log_e 7$

E. $\log_e 9$

Question 16

The graph of the function $y = f(x)$ is shown on the graph below.



$f'(x) < 0$ for

A. $x \in (-b, c) \cup (d, e)$
B. $x \in [-b, c] \cup [d, e]$
C. $x \in (-\infty, -a) \cup (0, d) \cup (d, f)$
D. $x \in (-\infty, -a) \cup (c, d) \cup (e, \infty)$
E. $x \in (-\infty, -b) \cup (c, d) \cup (e, \infty)$

Question 17

The approximation formula $f(x+h) \approx f(x) + hf'(x)$, where $f(x) = x^3$ with $x = 5$, when used to find an approximate value to $4 \cdot 95^3$, gives

A. $f(0 \cdot 05) + f(5)$
B. $f(0 \cdot 05) - 5f'(0 \cdot 05)$
C. $f(0 \cdot 05) + 5f'(0 \cdot 05)$
D. $f(5) - 0 \cdot 05f'(5)$
E. $f(5) + 0 \cdot 05f'(5)$

Question 18

The left rectangle approximation with rectangles of width 1 unit is used to find the approximate area of the region bounded by the *x*-axis, the line $x = 3$ and the curve with equation $y = \sqrt{9 - (x-3)^2}$. That approximate area is

A. $\sqrt{5}$
B. $\sqrt{5} + 3$
C. $\sqrt{8} + 3$
D. $\sqrt{5} + \sqrt{8}$
E. $\sqrt{5} + \sqrt{8} + 3$

Question 19

An antiderivative of $-2\sin\left(\frac{x}{2}\right)$ is

A. $-\cos\left(\frac{x}{2}\right)$

B. $-2\cos\left(\frac{x}{2}\right)$

C. $-4\cos\left(\frac{x}{2}\right)$

D. $\cos\left(\frac{x}{2}\right)$

E. $4\cos\left(\frac{x}{2}\right)$

Question 20

$\int_0^1 (5x-1)^6\,dx$ is equal to

A. $\frac{16383}{35}$

B. $\frac{3277}{7}$

C. $\frac{16383}{7}$

D. $\frac{16384}{7}$

E. $\frac{16385}{7}$

Question 21

The definite integral $\int_a^b (f(x)-g(x))dx$, where $b > a$, can also be written as

A. $-\int_a^b (f(x)-g(x))dx$

B. $-\int_b^a (f(x)+g(x))dx$

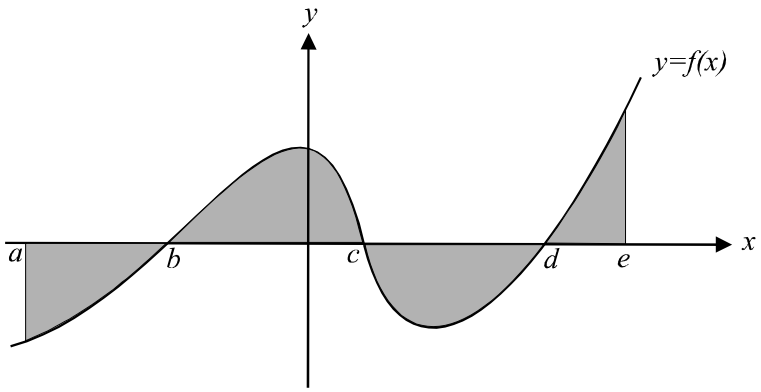
C. $-\int_b^a f(x)dx+\int_a^b g(x)dx$

D. $-\int_b^a f(x)dx-\int_a^b g(x)dx+\int_b^b f(x)dx$

E. $-\int_{b}^{a} f(x)dx + \int_{a}^{b} g(x)dx + \int_{a}^{b} g(x)dx$

Question 22

Part of the graph of $y = f(x)$ is shown below.



The total area of the shaded region shown on the graph above is given by

A. $\int_a^e f(x)dx$

B. $\int_b^e f(x)dx - \int_a^d f(x)dx$

C. $\int_d^e f(x)dx - \int_c^d f(x)dx + \int_b^c f(x)dx - \int_a^b f(x)dx$

D. $\int_d^e f(x)dx + \int_c^d f(x)dx + \int_b^c f(x)dx - \int_a^b f(x)dx$

E. $\int_d^e f(x)dx - \int_c^d f(x)dx + \int_0^c f(x)dx - \int_b^0 f(x)dx + \int_a^b f(x)dx$

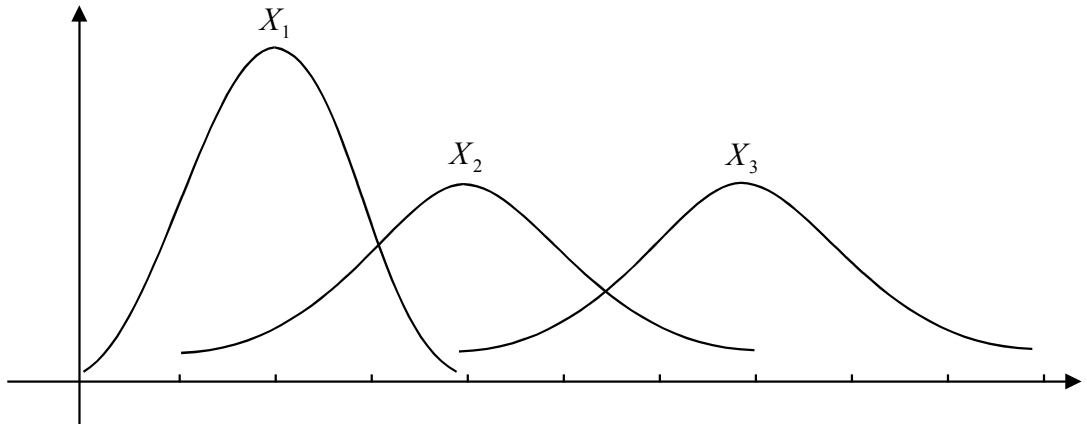
Question 23

Kate has 20% likelihood of forgetting to take her Maths text book to class. Whether or not she takes her text book to class one lesson is independent of whether she takes it the next time. Let X be a random variable representing the number of times Kate forgot to take her maths text book to five consecutive maths lessons. The type of probability distribution that X would follow would most likely be

A. discrete
B. binominal
C. hypergeometric
D. continuous
E. normal

Question 24

Three normal distributions X_1, X_2 and X_3 are shown in the diagram below.



In relation to these three normal distributions, which one of the following statements is true?

A. distribution X_1 has the highest mean
B. distribution X_2 has the lowest mean
C. distribution X_2 has the lowest variance
D. distribution X_3 has the highest variance
E. distribution X_1 has the lowest variance

Question 25

Jane has a packet of jelly beans. The packet contains 20 red, 30 white, 10 black, 15 green and 25 yellow jelly beans. Jane randomly selects a jelly bean and eats it. She repeats this process until she has eaten 5 jelly beans. The variance of the number of black jelly beans eaten by Jane is closest to

A. 0.048
B. 0.147
C. 0.432
D. 0.5
E. 0.788

Question 26

A random variable *X* follows a normal distribution and has a mean of 30 and a standard deviation of 5. If Z has the standard normal distribution, then the probability that Z is greater than – 2 is the same as

A. $\Pr(X > 25)$
B. $1 - \Pr(X > 20)$
C. $\Pr(X < 35)$
D. $\Pr(X < 40)$
E. $1 - \Pr(X < 40)$

Question 27

Mick rolls a fair die 6 times. The probability that he obtains an even number on at least half of those throws is given by

A. $\dfrac{1}{{}^6C_3}$

B. ${}^6C_3(0\cdot 5)^6$

C. $0\cdot 5^5\left({}^6C_3+{}^6C_4+{}^6C_5+{}^6C_6\right)$

D. $1-\left\{(0\cdot 3)^6+3(0\cdot 3)^5+3(0\cdot 3)^4\right\}$

E. $1-\left\{(0\cdot 5)^6+3(0\cdot 5)^5+{}^6C_2(0\cdot 5)^6\right\}$

PART II

Question 1

There are 15 players in George's soccer squad. The coach only conducts training if 12 or more players are available to attend.
The probability distribution of the number of players who attend a training session is shown in the table below.

Number of players attending a training session X	12	13	14	15
Probability $\Pr(X = x)$	a	$2a$	$3a$	$4a$

a. Find the value of a.

b. What is the expected number of players to attend a training session?

c. What is the probability that at least 13 players will be at a training session?

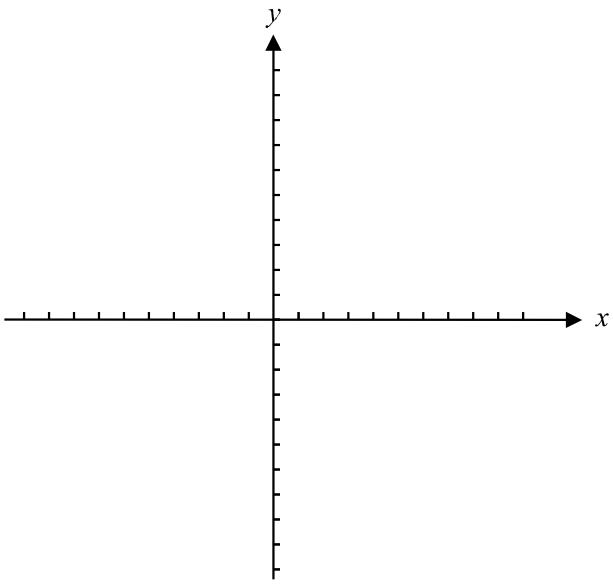
1 + 1 + 1 = 3 marks

Question 2

Consider the function

$$f:[a, \infty) \to R,\ f(x) = 2(x+1)^2 - 8$$

a. Sketch the graph of $y = f(x)$ if $a = -3$



b. Write down the least value of a if the inverse function $f^{-1}(x)$ is to exist.

_____

_____

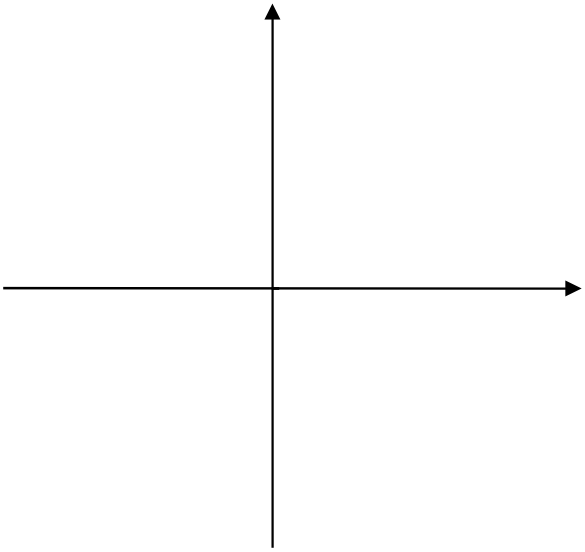
1 + 1 = 2 marks

Question 3

a. Sketch the graph of

$$f:[0°,\ 90°) \to R, \text{ where } f(x) = 2\tan(x°)$$

on the set of axes below.



b. Find the point of intersection between the graph of $y = f(x)$ and the graph of $y = 1$. (Express any degrees correct to 2 decimal places.)

1 + 2 = 3 marks

Question 4

The graph of the function $y = e^x$ underwent four transformations, which are listed below:

- a dilation by a factor 2 from the *y*-axis
- a dilation by a factor of 3 from the *x*-axis
- a translation of 4 units parallel to the *x*-axis in the positive direction
- a reflection in the *x*-axis

a. Write down the equation of the resulting function.

b. Write down the range of the resulting function.

2 + 1 = 3 marks

Question 5

Find the equation of the normal to the graph of $y = \dfrac{-1}{x+3} - 2$ at the point where $x = -2$.

2 marks

Question 6

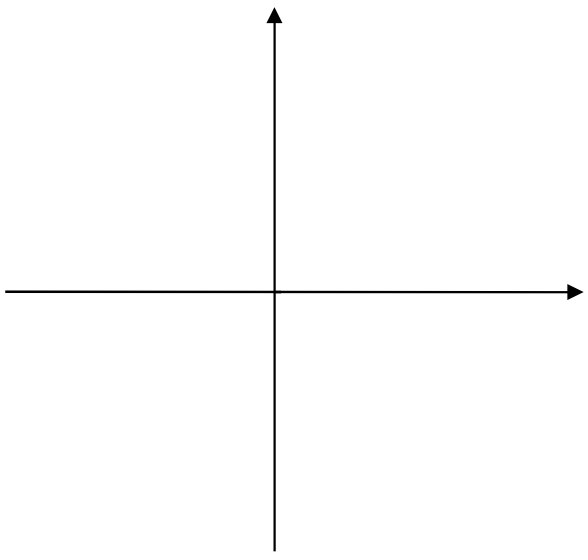
On the set of axes below, sketch a graph of the function *f*, showing all intercepts, where *f* is a continuous function and has the following properties.

$f(-2) = 3$ $\quad$ $f'(-2) = 0$

$f(0) = 4$ $\quad$ $f'(x) > 0$ for $x \in (-2, 3)$

$f'(x) < 0$ for $x \in (-\infty, -2) \cup (3, \infty)$

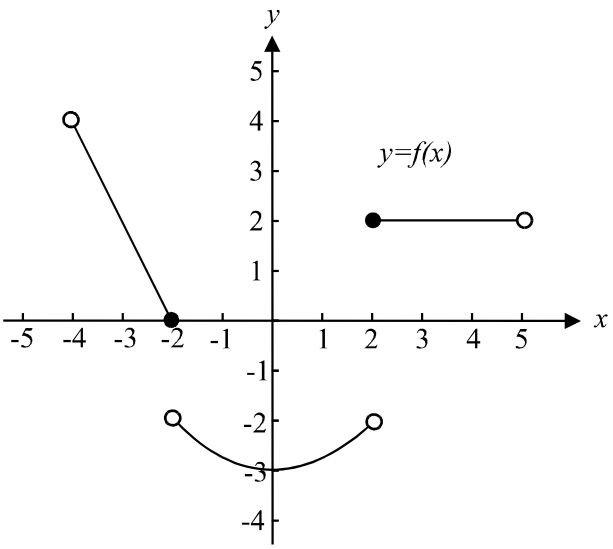
Also, for function *f*, the average rate of change between $x = 0$ and $x = 4$ is -1.



3 marks

Question 7

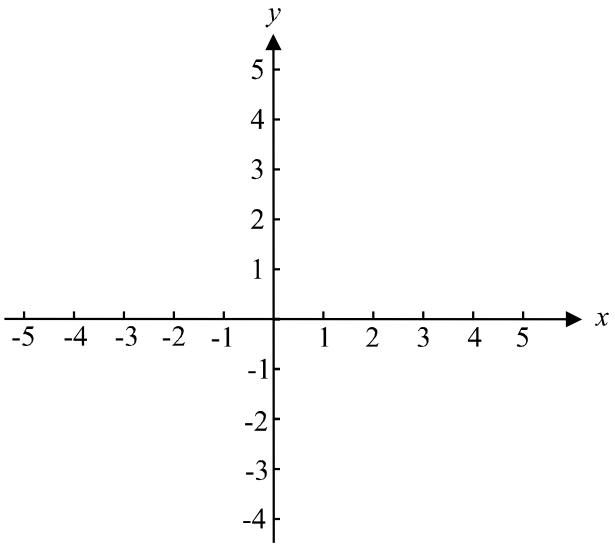
The graph of the function f is shown below:



a. Write down the range of f.

_____

b. Sketch the graph of the derived function f' on the set of axes below.



c. Write down the domain of f'.

_____

1 + 2 + 1 = 4 marks

Question 8

a. If $y = (x+2)e^{\frac{x}{2}}$, find $\frac{dy}{dx}$.

b. Hence find $\int (x+2)e^{\frac{x}{2}}\,dx$.

1 + 2 = 3 marks

MATHEMATICAL METHODS

TRIAL EXAMINATION 1

MULTIPLE- CHOICE ANSWER SHEET

STUDENT NAME:.....

INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example: (A) (■) (C) (D) (E)

The answer selected is B. Only one answer should be selected.

1. (A) (B) (C) (D) (E)	10. (A) (B) (C) (D) (E)	19. (A) (B) (C) (D) (E)
2. (A) (B) (C) (D) (E)	11. (A) (B) (C) (D) (E)	20. (A) (B) (C) (D) (E)
3. (A) (B) (C) (D) (E)	12. (A) (B) (C) (D) (E)	21. (A) (B) (C) (D) (E)
4. (A) (B) (C) (D) (E)	13. (A) (B) (C) (D) (E)	22. (A) (B) (C) (D) (E)
5. (A) (B) (C) (D) (E)	14. (A) (B) (C) (D) (E)	23. (A) (B) (C) (D) (E)
6. (A) (B) (C) (D) (E)	15. (A) (B) (C) (D) (E)	24. (A) (B) (C) (D) (E)
7. (A) (B) (C) (D) (E)	16. (A) (B) (C) (D) (E)	25. (A) (B) (C) (D) (E)
8. (A) (B) (C) (D) (E)	17. (A) (B) (C) (D) (E)	26. (A) (B) (C) (D) (E)
9. (A) (B) (C) (D) (E)	18. (A) (B) (C) (D) (E)	27. (A) (B) (C) (D) (E)