



## Victorian Certificate of Education 2002

# MATHEMATICAL METHODS (CAS) PILOT STUDY

## Written examination 1 (Facts, skills and applications)

Friday 8 November 2002

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.45 am (1 hour 30 minutes)

### PART I MULTIPLE-CHOICE QUESTION BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions). Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.

Part II consists of a separate question and answer book.

You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

#### Structure of book

| <i>Number of questions</i> | <i>Number of questions to be answered</i> | <i>Number of marks</i> |
|----------------------------|-------------------------------------------|------------------------|
| 27                         | 27                                        | 27                     |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or computer algebra (CAS) calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

#### Materials supplied

- Question book of 15 pages, with a detachable sheet of miscellaneous formulas in the centrefold and one blank page for rough working.
- Answer sheet for multiple-choice questions.

#### Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

#### At the end of the examination

- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
- You may retain this question book.

**Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.**

Working space

### Instructions for Part I

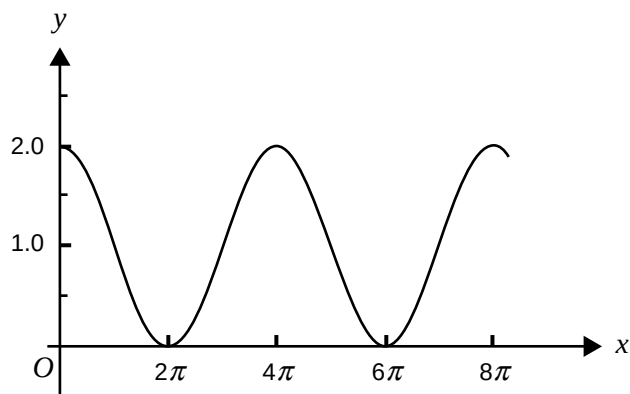
Answer all questions in pencil on the answer sheet for multiple-choice questions.

A correct answer scores 1, an incorrect answer scores 0. Marks will **not** be deducted for incorrect answers.

No mark will be given if more than one answer is completed for any question.

#### Question 1

The diagram below shows part of the graph of a circular function.



A possible equation for the function whose graph is shown is

- A.  $y = 1 + \sin(x)$
- B.  $y = 1 + \sin\left(\frac{x}{2}\right)$
- C.  $y = 1 + \cos(x)$
- D.  $y = 1 - \cos\left(\frac{x}{2}\right)$
- E.  $y = 1 + \cos\left(\frac{x}{2}\right)$

#### Question 2

For the equation  $\sin(2x) = 1$ , the sum of the solutions in the interval  $[0, 4\pi]$  is

- A.  $\frac{\pi}{4}$
- B. 1.09975
- C.  $\frac{(4n+1)\pi}{4}$
- D.  $\frac{3\pi}{2}$
- E.  $7\pi$

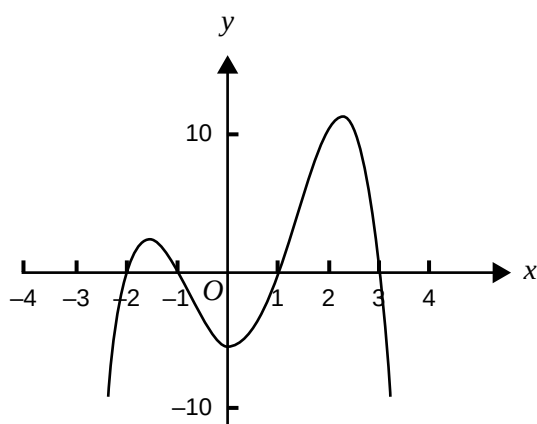
**Question 3**

On a particular day, the temperature  $y$ , in degrees Celsius, can be modelled by the function whose rule is  $y = 9 - 5\sin\left(\frac{\pi t}{12}\right)$ , where  $t$  is the time in hours after midnight. The maximum temperature for this particular day occurs at

- A. 3.00 am
- B. 6.00 am
- C. 12.00 noon
- D. 6.00 pm
- E. 12.00 midnight

**Question 4**

The diagram below shows part of the graph of a polynomial function.

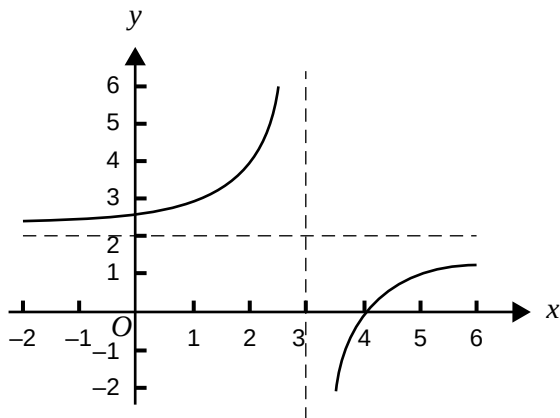


A possible rule for the polynomial function is

- A.  $y = -(x - 2)(x + 1)(x - 1)(x + 3)$
- B.  $y = (x - 2)(x - 1)(x + 1)(x + 3)$
- C.  $y = (x + 2)(x + 1)(x - 1)(x - 3)$
- D.  $y = -(x + 2)(x + 1)(x - 1)(3 - x)$
- E.  $y = (x + 2)(x + 1)(x - 1)(3 - x)$

**Question 5**

Part of the graph of the function with rule  $y = \frac{a}{x+b} + c$  is shown below.

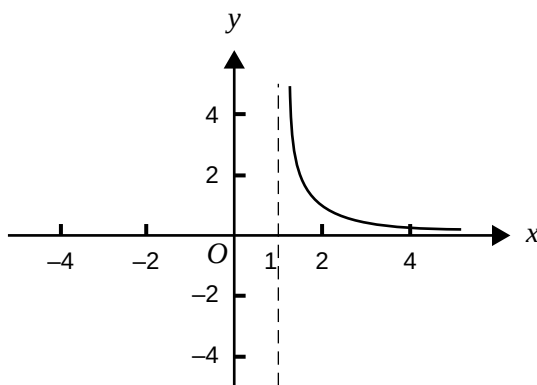


The values of  $a$ ,  $b$  and  $c$  respectively are

- |    | $a$ | $b$ | $c$ |
|----|-----|-----|-----|
| A. | 2   | 3   | 2   |
| B. | 2   | -3  | 2   |
| C. | -2  | -3  | 2   |
| D. | -2  | -2  | 3   |
| E. | 2   | -2  | 3   |

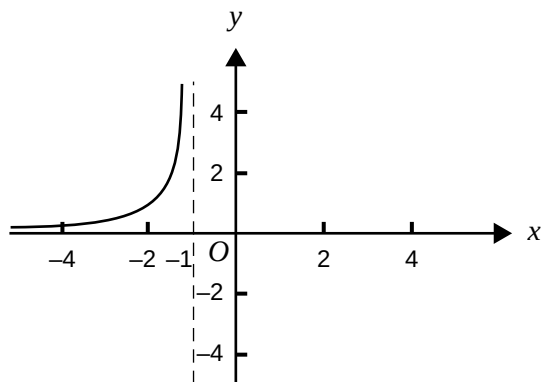
**Question 6**

Part of the graph of the function with equation  $y = f(x)$  is shown below.

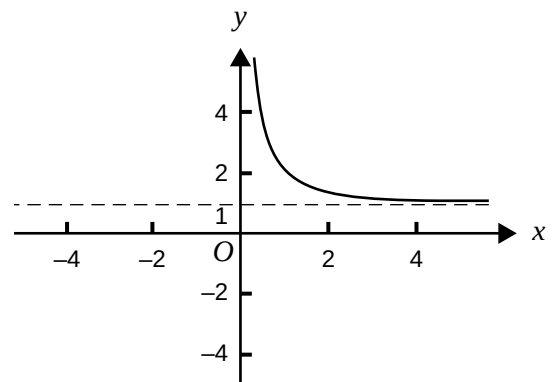


Which one of the following is most likely to be the graph of the function with equation  $y = f(-x)$ ?

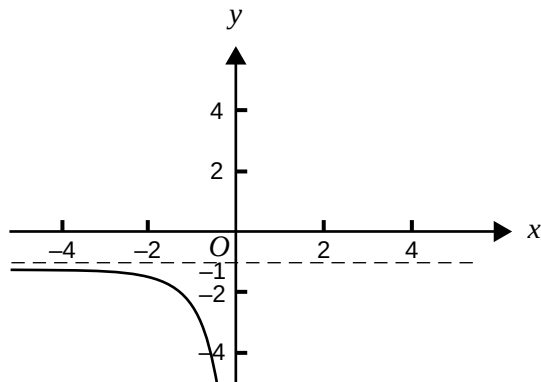
**A.**



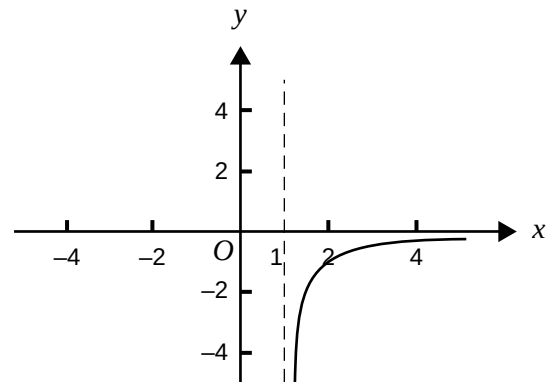
**B.**



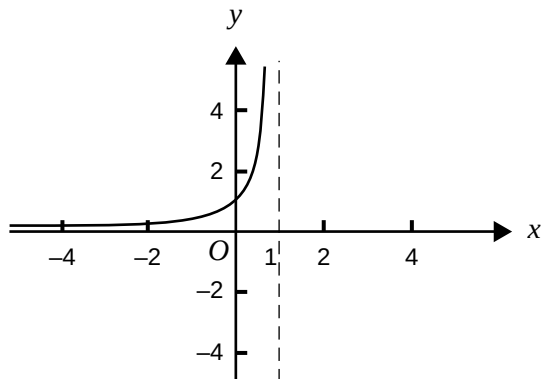
**C.**



**D.**

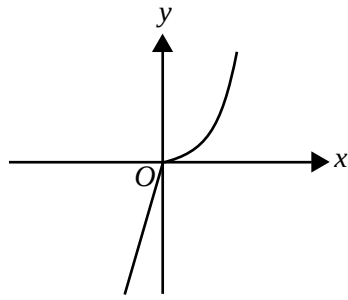


**E.**

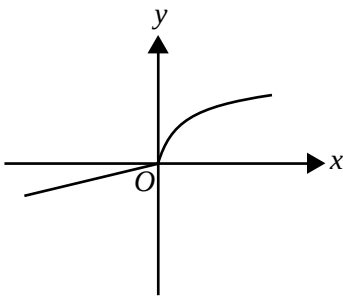
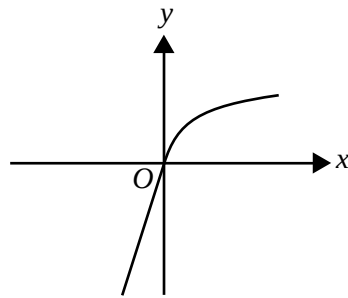
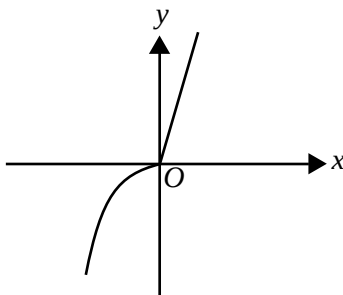
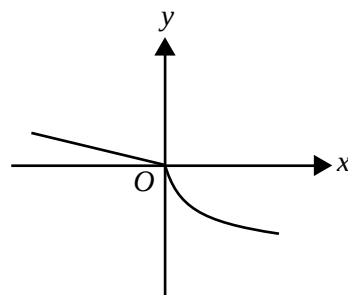
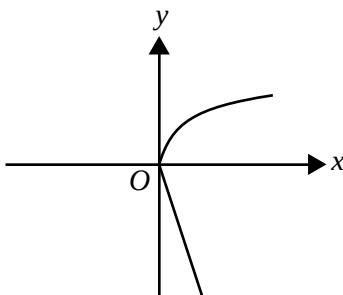


**Question 7**

Part of the graph of the function  $f$  is shown below. (The same scale has been used on both axes.)



Which one of the following is most likely to be the graph of the inverse function of  $f$ ?

**A.****B.****C.****D.****E.**

**Question 8**

|     |     |     |     |     |     |      |
|-----|-----|-----|-----|-----|-----|------|
| $x$ | 0   | 1   | 2   | 3   | 4   | 5    |
| $y$ | 1.0 | 1.6 | 2.7 | 4.5 | 7.4 | 12.2 |

The data in the above table would most closely be modelled by

A.  $y = \tan\left(\frac{x}{2}\right)$

B.  $y = \log_e\left(\frac{x}{2}\right)$

C.  $y = e^{\frac{x}{2}}$

D.  $y = \frac{x}{2} + 1$

E.  $y = \frac{x^2}{2}$

**Question 9**

The **linear** factors of  $x^4 + x^3 - 3x^2 - 3x$  over  $R$  are

A.  $x, x + 1, x^2 - 3$

B.  $x, x + 1, x + \sqrt{3}, x - \sqrt{3}$

C.  $x, x + 1$

D.  $x + 1, x + \sqrt{3}, x - \sqrt{3}$

E.  $x + 1, x^3 - 3x$

**Question 10**

Given that the functions with rules  $f(x) = e^x$  and  $g(x) = \log_e(x)$  are inverse functions, which one of the following statements is true?

A. The graph of  $f$  is the reflection of the graph of  $g$  in the  $x$ -axis.

B. If  $f$  is increasing over an interval, then  $g$  is decreasing over the same interval.

C.  $f(g(x)) = x$

D.  $f(x)g(x) = 1$

E. If  $f(x) = y$ , then  $g(y) = \frac{1}{x}$  for all  $x \neq 0$ .



**Question 11**

Which one of the following functions is **not** a one-to-one function?

- A.  $f: R \rightarrow R, f(x) = e^x$
- B.  $f: R^+ \rightarrow R, f(x) = \log_e(x)$
- C.  $f: R \setminus \{0\} \rightarrow R, f(x) = \frac{1}{x}$
- D.  $f: R \rightarrow R, f(x) = \sin(x)$
- E.  $f: R \rightarrow R, f(x) = x^3$

**Question 12**

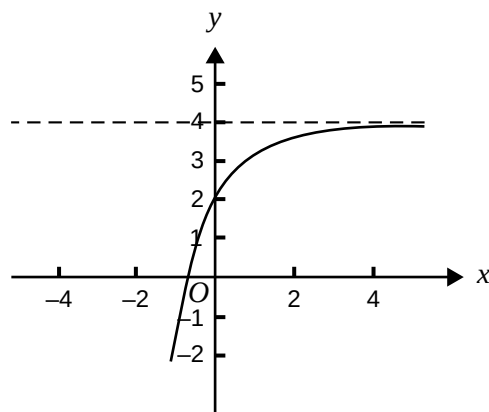
The function  $f$  defined by  $f: A \rightarrow R$ , where  $f(x) = (x - 3)^2 + 2$ , will have an inverse function if  $A$  is the set

- A.  $R$
- B.  $R^+ \cup \{0\}$
- C.  $\{x: x \geq 2\}$
- D.  $\{x: x \leq 3\}$
- E.  $\{x: x \leq 11\}$

|                            |
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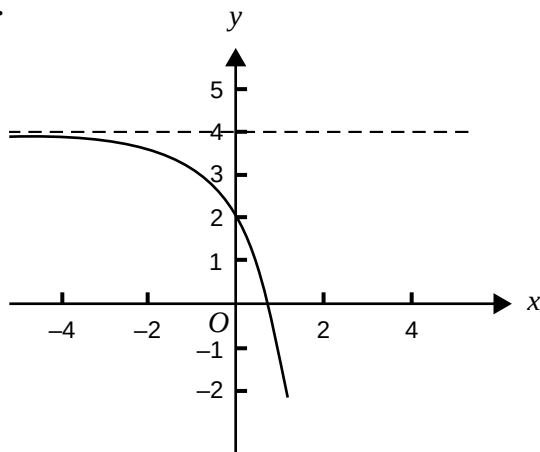
**Question 13**

Part of the graph of the function with equation  $y = f(x)$  is shown below.

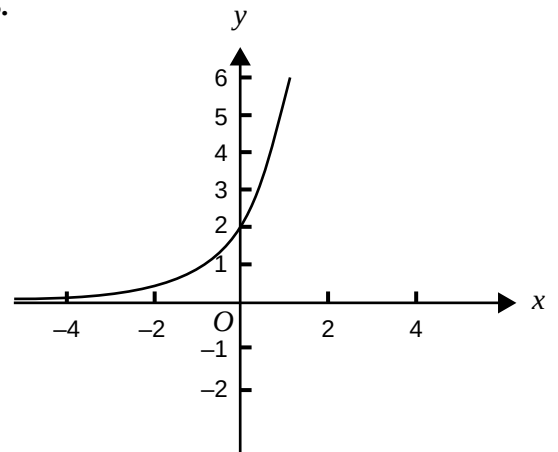


Which one of the following is most likely to be the corresponding part of the graph of the function with equation  $y = f'(x)$ ?

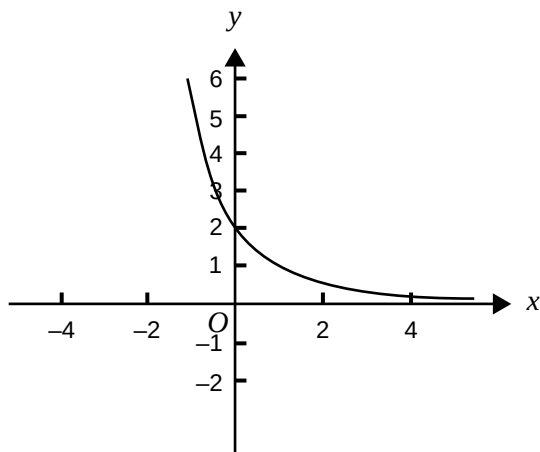
**A.**



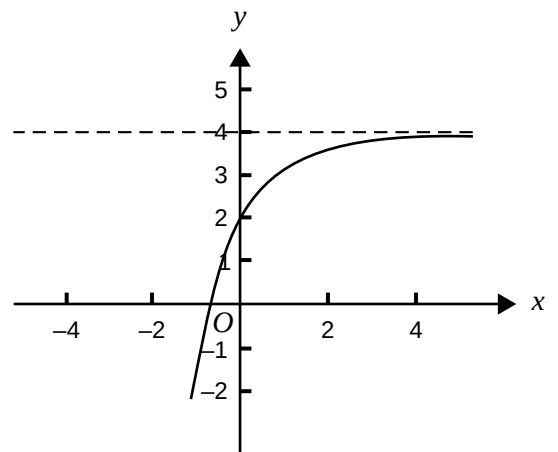
**B.**



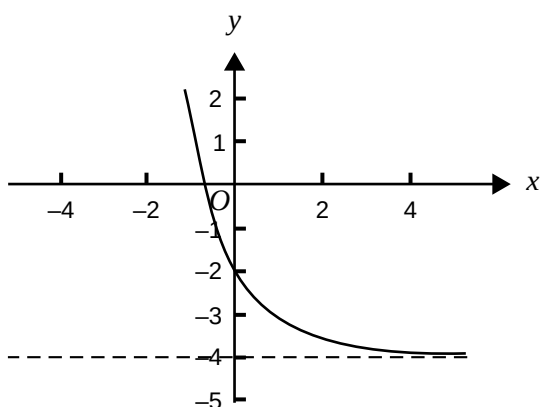
**C.**



**D.**



**E.**



**Question 14**

If  $y = \log_e(\cos(2x))$ , then  $\frac{dy}{dx}$  is equal to

- A.  $2 \tan(2x)$
- B.  $-2 \tan(2x)$
- C.  $\frac{1}{\cos(2x)}$
- D.  $\frac{-1}{2 \sin(2x)}$
- E.  $-2 \sin(2x)$

**Question 15**

The equation of the **normal** to the curve with equation  $y = x \sin(x)$  at the point on the curve with  $x$  coordinate  $\pi$ , is

- A.  $y = -(x - \pi)\pi$
- B.  $y = (x - \pi)\pi$
- C.  $y = -\frac{1}{\pi}(x - \pi)$
- D.  $y = \frac{1}{\pi}(x - \pi)$
- E.  $y = \frac{-1}{x \sin(x)}$

**Question 16**

If  $y = e^{-x} - 1$ , then the rate of change of  $y$  with respect to  $x$ , when  $x = 0$  is

- A.  $-e$
- B.  $-2$
- C.  $-1$
- D.  $0$
- E.  $e - 1$

**Question 17**

Using the approximation formula,  $f(x + h) \approx f(x) + h f'(x)$ , where  $f(x) = x^2$  with  $x = 3$ , an approximate value for  $3.02^2$  is given by

- A.  $f(3) + 0.02 f'(3)$
- B.  $f(9) + 0.04 f'(9)$
- C.  $f(3)$
- D.  $f(3) - 0.02 f'(3)$
- E.  $f(9) - 0.04 f'(9)$

**Question 18**

A continuous function  $f$  has the following properties.

$$f(0) = 0$$

$$f'(0) = 0$$

$$f(-3) = 0$$

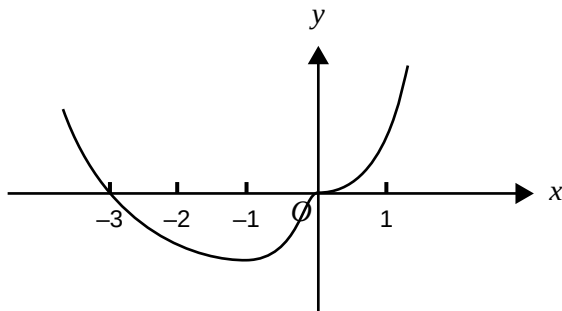
$$f'(-1) = 0$$

$$f'(x) > 0 \quad \text{for} \quad \{x: x < -1\}$$

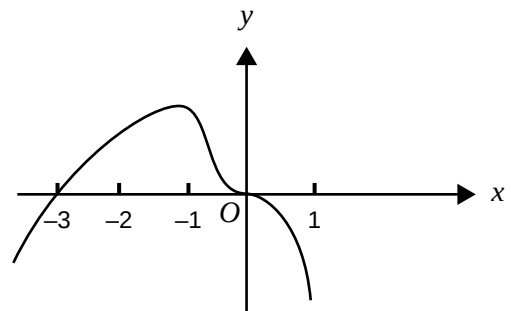
$$f'(x) < 0 \quad \text{for} \quad \{x: x > -1\} \setminus \{0\}$$

Which one of the following is most likely to represent the graph of  $f$ ?

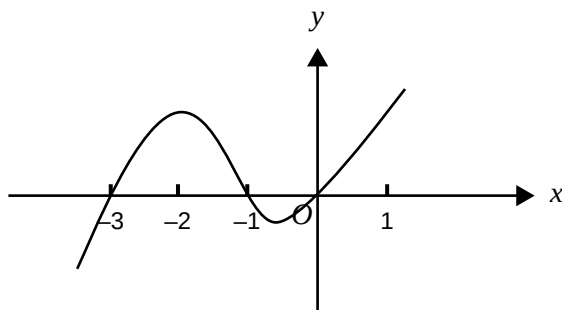
**A.**



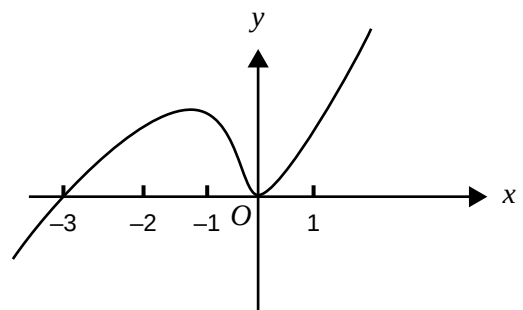
**B.**



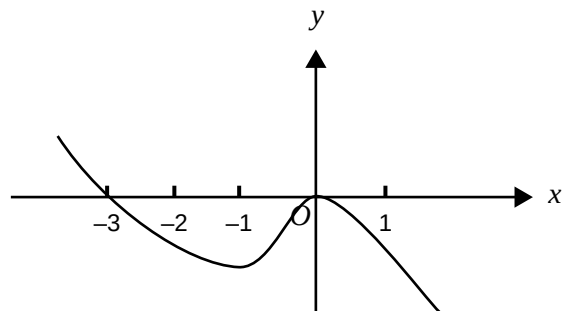
**C.**



**D.**



**E.**



**Question 19**

Let  $g(x) = e^{f(x)}$ . If  $g'(x) = -2xe^{-x^2}$ , then a rule for  $f$  is

- A.  $f(x) = -x^2$
- B.  $f(x) = -\frac{x^3}{3}$
- C.  $f(x) = x^2$
- D.  $f(x) = e^{-x^2}$
- E.  $f(x) = -4x^2$

**Question 20**

If  $f'(x) = 2\cos(5x)$  and  $c$  is a real constant, then  $f(x)$  is equal to

- A.  $-\frac{2}{5}\sin(5x) + c$
- B.  $\frac{2}{5}\sin(5x) + c$
- C.  $-10\sin(5x) + c$
- D.  $10\sin(5x) + c$
- E.  $-2\sin\left(\frac{5x^2}{2}\right) + c$

**Question 21**

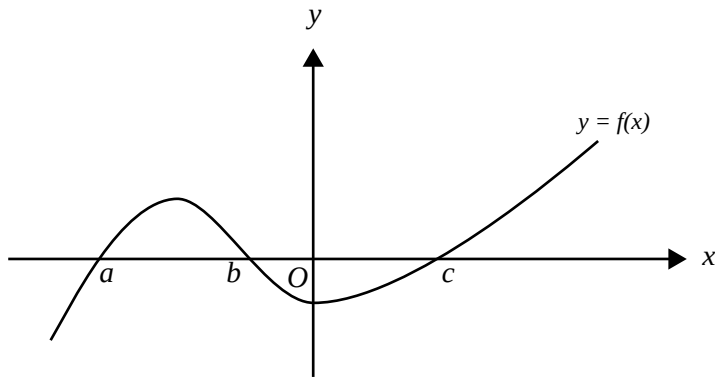
A truck leaves town A and takes 30 minutes to travel in a straight line to town B. The velocity,  $v$ , in kms per minute, of the truck at time  $t$  minutes on this journey is given by  $v(t) = \sin^2\left(\frac{\pi t}{30}\right)$ .

The distance, in kms, between town A and town B is

- A.  $\frac{\pi}{30}$
- B.  $\frac{\pi}{15}$
- C.  $\frac{30}{\pi}$
- D. 15
- E. 30

**Question 22**

Part of the graph of the function  $f$  is shown below.



The total area bounded by the graph of the function with rule  $y = f(x)$  and the  $x$ -axis on the interval  $[a, c]$  is given by

- A.  $\int_a^c f(x)dx$
- B.  $\int_a^b f(x)dx + \int_b^c f(x)dx$
- C.  $-\int_a^0 f(x)dx + \int_0^c f(x)dx$
- D.  $\int_a^b f(x)dx + \int_c^b f(x)dx$
- E.  $\int_a^b f(x)dx - \int_b^0 f(x)dx + \int_0^c f(x)dx$

**Question 23**

A continuous random variable  $X$  is the distance, measured in hundreds of kilometres, that a particular car will travel on a full tank of petrol. It has the probability density function

$$f(x) = \begin{cases} -2x + 8 & 3 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

The probability that the car will travel more than 350 kilometres on a full tank of petrol is

- A. 0.08
- B. 0.25
- C. 0.5
- D. 0.75
- E. 0.92

**Question 24**

A bag contains twenty casino chips which are used to represent cash. Five of the chips have value \$10 each and the other fifteen chips have value \$5 each. If four casino chips are drawn from the bag at random without replacement, what is the probability that there will be at least one chip of value \$10?

- A.  $1 - \frac{{}^{15}C_4}{{}^{20}C_4}$
- B.  $\frac{{}^5C_1 \times {}^{15}C_3}{{}^{20}C_4}$
- C.  $\frac{1}{4}$
- D.  $1 - \left(\frac{3}{4}\right)^4$
- E.  ${}^4C_1 \times \left(\frac{1}{4}\right) \times \left(\frac{3}{4}\right)^3$

**Question 25**

The number of defective computer parts, in a box of computer parts ready for sale, is a random variable with a binomial distribution, with mean 10 and standard deviation 3. If a computer part is drawn at random from the box, the probability that it is defective is

- A. 0.1
- B. 0.3
- C. 0.5
- D. 0.7
- E. 0.9

**Question 26**

The random variable  $X$  is normally distributed with mean 4.7 and standard deviation 1.2. If  $Z$  is a random variable which has the standard normal distribution, then the probability that  $X$  is less than 3.5 is equal to

- A.  $\Pr(Z < 1)$
- B.  $\Pr(Z > 1)$
- C.  $1 - \Pr(Z < -1)$
- D.  $\Pr(Z > -1)$
- E.  $\Pr(-1 < Z < 1)$

**Question 27**

Black Mountain coffee is sold in packets labelled 250 grams. The packing process produces packets whose weight is a normally distributed random variable with a standard deviation of 3 grams. In order to guarantee that only 1% of packages are under the labelled weight, the actual mean weight (in grams) would be required to be closest to

- A. 243
- B. 247
- C. 250
- D. 254
- E. 257



# Victorian Certificate of Education 2002

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

**STUDENT NUMBER**

Letter

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| Figures |  |  |  |  |  |  |  |  |  |
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## MATHEMATICAL METHODS (CAS) PILOT STUDY

### Written examination 1 (Facts, skills and applications)

Friday 8 November 2002

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.45 am (1 hour 30 minutes)

### PART II QUESTION AND ANSWER BOOK

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**Structure of book**

| <i>Number of questions</i> | <i>Number of questions to be answered</i> | <i>Number of marks</i> |
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| 6                          | 6                                         | 23                     |

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**Materials supplied**

- Question and answer book of 10 pages, including two blank pages for rough working.

**Instructions**

- Detach the formula sheet from the centre of the Part I book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

**At the end of the examination**

- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II).

**Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.**



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### Instructions for Part II

Answer all questions in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

Appropriate working should be shown if more than one mark is available.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

#### Question 1

The life of a light globe, in hours, can be modelled by the random variable  $X$  with probability density function

$$f(x) = \begin{cases} \frac{c}{x^2} & \text{if } x > 100 \\ 0 & \text{if } x \leq 100 \end{cases}$$

- a. Find the value of  $c$ .

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- b. Find the median life of a light globe according to this model.

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2 + 2 = 4 marks

Working space

**Question 2**

A roulette wheel has thirty-seven slots, numbered 0 to 36. Slot 0 is green. Eighteen of the remaining slots are black, and the other eighteen are red. In a single game, a person spins the wheel and at the same time rolls a ball around the wheel in the opposite direction. As the wheel slows, the ball falls into one of the slots. The wheel is carefully balanced so that the ball is equally likely to fall into any of the slots.

- a. In a single game, what is the probability, correct to three decimal places, that the ball falls into a red slot?

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Several games are played one after the other. Assume that the result of each game is independent of the result of any other game.

- b. What is the probability, correct to three decimal places, that the first time that the ball falls into a red slot is in the fifth game?

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- c. What is the probability, correct to three decimal places, that the ball falls into a red slot before the fifth game?

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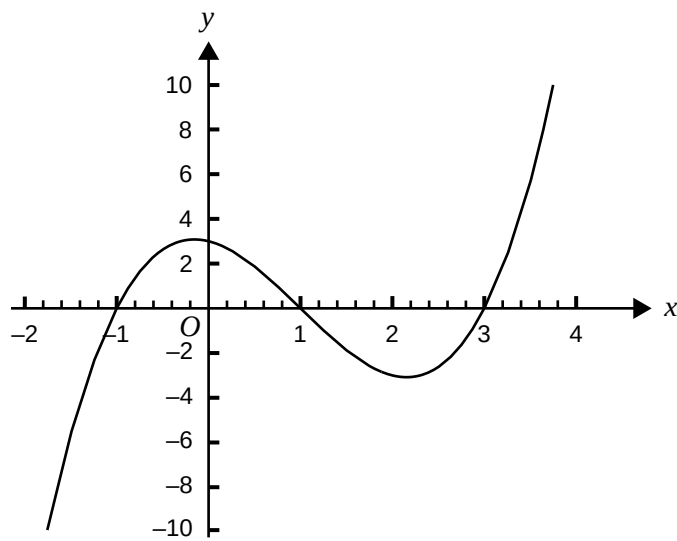
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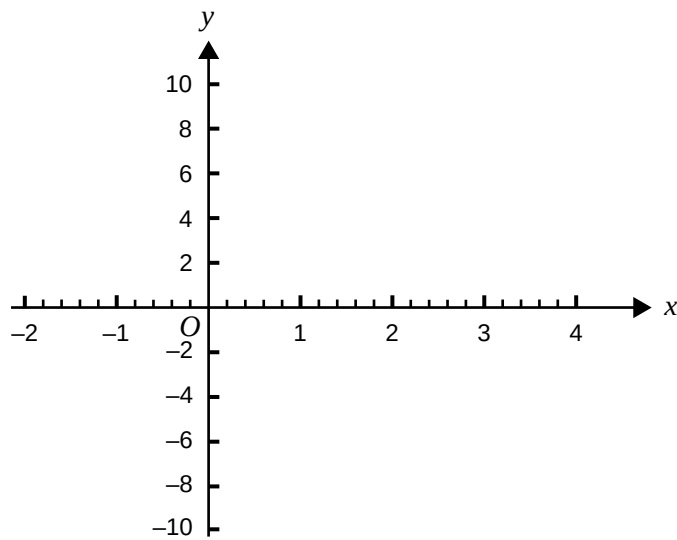
1 + 1 + 2 = 4 marks

**Question 3**

The following diagram shows part of the graph of a function  $f$ :



On the set of axes below, sketch the corresponding part of the graph of  $|f|$ .



2 marks

**Question 4**

A cubic polynomial function with rule  $f(x) = ax^3 + bx^2 + cx + d$ , is uniquely defined by the following conditions.

$$f(-1) = 1 \quad f'(1) = -2$$

$$f(1) = \frac{5}{3} \quad f'(0) = 0$$

- a. Each of the above conditions can be written as a linear equation in terms of  $a$ ,  $b$ ,  $c$  and  $d$ .

Write down the corresponding linear equation for each condition.

$$f(-1) = 1 \quad \underline{\hspace{15cm}}$$

$$f(1) = \frac{5}{3} \quad \underline{\hspace{15cm}}$$

$$f'(1) = -2 \quad \underline{\hspace{15cm}}$$

$$f'(0) = 0 \quad \underline{\hspace{15cm}}$$

- b. Express this as a system of simultaneous linear equations in matrix form.

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- c. Solve this system of simultaneous linear equations exactly, and state the rule of the cubic polynomial function.

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2 + 2 + 2 = 6 marks

Working space

**Question 5**

The graph of the function  $f$  is obtained from the graph of  $y = \frac{1}{x}$  by the following transformations applied in the order given.

- a dilation by a factor of  $\frac{1}{2}$  from the  $y$ -axis
- a reflection in the  $y$ -axis
- a translation of 3 units parallel to the  $x$ -axis
- a translation of 1 unit parallel to the  $y$ -axis

**a.** Write down the rule for  $f$ .

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**b.** Hence state the domain and range of  $f$ .

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1 + 2 = 3 marks



**Question 6**

Given  $f:[0, \frac{\pi}{2}] \rightarrow \mathbb{R}$ ,  $f(x) = 2 \sin(3x) - \frac{3}{2}$ , find

- a. the exact values of  $x$  for which  $f(x) = -\frac{1}{2}$

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- b. the value of  $f'(1)$ , correct to three decimal places

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- c. the exact interval over which the rate of change of  $f$  with respect to  $x$  is positive.

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2 + 1 + 1 = 4 marks

# **MATHEMATICAL METHODS (CAS)**

## **PILOT STUDY**

### **Written examinations 1 and 2**

#### **FORMULA SHEET**

##### **Directions to students**

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

## Mathematical Methods CAS Formulas

### Mensuration

area of a trapezium:  $\frac{1}{2}(a+b)h$

curved surface area of a cylinder:  $2\pi rh$

volume of a cylinder:  $\pi r^2 h$

volume of a cone:  $\frac{1}{3}\pi r^2 h$

volume of a pyramid:  $\frac{1}{3}Ah$

volume of a sphere:  $\frac{4}{3}\pi r^3$

area of a triangle:  $\frac{1}{2}bc \sin A$

### Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$$

approximation:  $f(x+h) \approx f(x) + hf'(x)$

average value:  $\frac{1}{b-a} \int_a^b f(x)dx$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

product rule:  $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

quotient rule:  $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

### Statistics

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

mean:  $\mu = E(X)$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

transition matrices:  $S_n = T^n \times S_0$

variance:  $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

| Discrete distributions   |                                                                                                                                                                    |                                          |                                                                                                           |
|--------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------|-----------------------------------------------------------------------------------------------------------|
|                          | $\Pr(X = x)$                                                                                                                                                       | mean                                     | variance                                                                                                  |
| general                  | $p(x)$                                                                                                                                                             | $\mu = \sum x p(x)$                      | $\sigma^2 = \sum (x - \mu)^2 p(x)$<br>$= \sum x^2 p(x) - \mu^2$                                           |
| binomial                 | ${}^n C_x p^x (1-p)^{n-x}$                                                                                                                                         | $np$                                     | $np(1-p)$                                                                                                 |
| hypergeometric           | $\frac{{}^D C_x {}^{N-D} C_{n-x}}{{}^N C_n}$                                                                                                                       | $n \frac{D}{N}$                          | $n \frac{D}{N} \left(1 - \frac{D}{N}\right) \left(\frac{N-n}{N-1}\right)$                                 |
| Continuous distributions |                                                                                                                                                                    |                                          |                                                                                                           |
|                          | $\Pr(a < X < b)$                                                                                                                                                   | mean                                     | variance                                                                                                  |
| general                  | $\int_a^b f(x)dx$                                                                                                                                                  | $\mu = \int_{-\infty}^{\infty} x f(x)dx$ | $\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)dx$<br>$= \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$ |
| normal                   | If $X$ is distributed $N(\mu, \sigma^2)$ and $Z = \frac{X - \mu}{\sigma}$ , then $Z$ is distributed $N(0, 1)$ , $f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$ |                                          |                                                                                                           |

Table 1 Normal distribution – cdf

| $x$ | 0      | 1      | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|---|---|----|----|----|----|----|----|----|
| 0.0 | .5000  | .5040  | .5080  | .5120  | .5160  | .5199  | .5239  | .5279  | .5319  | .5359  | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.1 | .5398  | .5438  | .5478  | .5517  | .5557  | .5596  | .5636  | .5675  | .5714  | .5753  | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 35 |
| 0.2 | .5793  | .5832  | .5871  | .5910  | .5948  | .5987  | .6026  | .6064  | .6103  | .6141  | 4 | 8 | 12 | 15 | 19 | 23 | 27 | 31 | 35 |
| 0.3 | .6179  | .6217  | .6255  | .6293  | .6331  | .6368  | .6406  | .6443  | .6480  | .6517  | 4 | 8 | 11 | 15 | 19 | 23 | 26 | 30 | 34 |
| 0.4 | .6554  | .6591  | .6628  | .6664  | .6700  | .6736  | .6772  | .6808  | .6844  | .6879  | 4 | 7 | 11 | 14 | 18 | 22 | 25 | 29 | 32 |
| 0.5 | .6915  | .6950  | .6985  | .7019  | .7054  | .7088  | .7123  | .7157  | .7190  | .7224  | 3 | 7 | 10 | 14 | 17 | 21 | 24 | 27 | 31 |
| 0.6 | .7257  | .7291  | .7324  | .7357  | .7389  | .7422  | .7454  | .7486  | .7517  | .7549  | 3 | 6 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 0.7 | .7580  | .7611  | .7642  | .7673  | .7703  | .7734  | .7764  | .7793  | .7823  | .7852  | 3 | 6 | 9  | 12 | 15 | 18 | 21 | 24 | 27 |
| 0.8 | .7881  | .7910  | .7939  | .7967  | .7995  | .8023  | .8051  | .8078  | .8106  | .8133  | 3 | 6 | 8  | 11 | 14 | 17 | 19 | 22 | 25 |
| 0.9 | .8159  | .8186  | .8212  | .8238  | .8264  | .8289  | .8315  | .8340  | .8365  | .8389  | 3 | 5 | 8  | 10 | 13 | 15 | 18 | 20 | 23 |
| 1.0 | .8413  | .8438  | .8461  | .8485  | .8508  | .8531  | .8554  | .8577  | .8599  | .8621  | 2 | 5 | 7  | 9  | 12 | 14 | 16 | 18 | 21 |
| 1.1 | .8643  | .8665  | .8686  | .8708  | .8729  | .8749  | .8770  | .8790  | .8810  | .8830  | 2 | 4 | 6  | 8  | 10 | 12 | 14 | 16 | 19 |
| 1.2 | .8849  | .8869  | .8888  | .8907  | .8925  | .8944  | .8962  | .8980  | .8997  | .9015  | 2 | 4 | 6  | 7  | 9  | 11 | 13 | 15 | 16 |
| 1.3 | .9032  | .9049  | .9066  | .9082  | .9099  | .9115  | .9131  | .9147  | .9162  | .9177  | 2 | 3 | 5  | 6  | 8  | 10 | 11 | 13 | 14 |
| 1.4 | .9192  | .9207  | .9222  | .9236  | .9251  | .9265  | .9279  | .9292  | .9306  | .9319  | 1 | 3 | 4  | 6  | 7  | 8  | 10 | 11 | 13 |
| 1.5 | .9332  | .9345  | .9357  | .9370  | .9382  | .9394  | .9406  | .9418  | .9429  | .9441  | 1 | 2 | 4  | 5  | 6  | 7  | 8  | 10 | 11 |
| 1.6 | .9452  | .9463  | .9474  | .9484  | .9495  | .9505  | .9515  | .9525  | .9535  | .9545  | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 1.7 | .9554  | .9564  | .9573  | .9582  | .9591  | .9599  | .9608  | .9616  | .9625  | .9633  | 1 | 2 | 3  | 3  | 4  | 5  | 6  | 7  | 8  |
| 1.8 | .9641  | .9649  | .9656  | .9664  | .9671  | .9678  | .9686  | .9693  | .9699  | .9706  | 1 | 1 | 2  | 3  | 4  | 4  | 5  | 6  | 6  |
| 1.9 | .9713  | .9719  | .9726  | .9732  | .9738  | .9744  | .9750  | .9756  | .9761  | .9767  | 1 | 1 | 2  | 2  | 3  | 4  | 4  | 5  | 5  |
| 2.0 | .9772  | .9778  | .9783  | .9788  | .9793  | .9798  | .9803  | .9808  | .9812  | .9817  | 0 | 1 | 1  | 2  | 2  | 3  | 3  | 4  | 4  |
| 2.1 | .9821  | .9826  | .9830  | .9834  | .9838  | .9842  | .9846  | .9850  | .9854  | .9857  | 0 | 1 | 1  | 2  | 2  | 2  | 3  | 3  | 4  |
| 2.2 | .9861  | .9864  | .9868  | .9871  | .9875  | .9878  | .9881  | .9884  | .9887  | .9890  | 0 | 1 | 1  | 1  | 2  | 2  | 2  | 3  | 3  |
| 2.3 | .9893  | .9896  | .9898  | .9901  | .9904  | .9906  | .9909  | .9911  | .9913  | .9916  | 0 | 1 | 1  | 1  | 1  | 2  | 2  | 2  | 2  |
| 2.4 | .9918  | .9920  | .9922  | .9925  | .9927  | .9929  | .9931  | .9932  | .9934  | .9936  | 0 | 0 | 1  | 1  | 1  | 1  | 1  | 2  | 2  |
| 2.5 | .9938  | .9940  | .9941  | .9943  | .9945  | .9946  | .9948  | .9949  | .9951  | .9952  | 0 | 0 | 0  | 1  | 1  | 1  | 1  | 1  | 1  |
| 2.6 | .9953  | .9955  | .9956  | .9957  | .9959  | .9960  | .9961  | .9962  | .9963  | .9964  | 0 | 0 | 0  | 0  | 1  | 1  | 1  | 1  | 1  |
| 2.7 | .9965  | .9966  | .9967  | .9968  | .9969  | .9970  | .9971  | .9972  | .9973  | .9974  | 0 | 0 | 0  | 0  | 0  | 1  | 1  | 1  | 1  |
| 2.8 | .9974  | .9975  | .9976  | .9977  | .9977  | .9978  | .9979  | .9979  | .9980  | .9981  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 1  | 1  |
| 2.9 | .9981  | .9982  | .9982  | .9983  | .9984  | .9984  | .9985  | .9985  | .9986  | .9986  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.0 | .9987  | .9987  | .9987  | .9988  | .9988  | .9989  | .9989  | .9989  | .9990  | .9990  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.1 | .9990  | .9991  | .9991  | .9991  | .9992  | .9992  | .9992  | .9992  | .9993  | .9993  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.2 | .9993  | .9993  | .9994  | .9994  | .9994  | .9994  | .9994  | .9995  | .9995  | .9995  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.3 | .9995  | .9995  | .9995  | .9996  | .9996  | .9996  | .9996  | .9996  | .9996  | .9997  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.4 | .9997  | .9997  | .9997  | .9997  | .9997  | .9997  | .9997  | .9997  | .9997  | .9998  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.5 | .9998  | .9998  | .9998  | .9998  | .9998  | .9998  | .9998  | .9998  | .9998  | .9998  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.6 | .9998  | .9998  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.7 | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.8 | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |

END OF FORMULA SHEET