

MAV Mathematical Methods Examination 2 Solutions

Question 1

$$V = \pi r^2 l + \frac{4}{3} \pi r^3 \text{ cm}^3$$

a. S. A. = $2\pi r l + 4\pi r^2 \text{ cm}^2$ [A]

b. If S. A. = 400 cm^2
 $400 = 2\pi r l + 4\pi r^2$ [M]

$$\frac{400 - 4\pi r^2}{2\pi r} = l$$

$$\Rightarrow l = \frac{200 - 2\pi r^2}{\pi r} \text{ cm} \quad [A]$$

c. $V = \pi r^2 \left(\frac{200 - 2\pi r^2}{\pi r} \right) + \frac{4}{3} \pi r^3$ [M]

$$= r(200 - 2\pi r^2) + \frac{4}{3} \pi r^3$$

$$= 200r - \frac{2}{3} \pi r^3 \text{ cm}^3 \quad [A]$$

d. $V > 0 \Rightarrow 200r - \frac{2}{3} \pi r^3 > 0$

$$r > 0 \Rightarrow 200 - \frac{2}{3} \pi r^2 > 0$$

$$\Rightarrow \frac{2}{3} \pi r^2 < 200$$

$$\Rightarrow r^2 < \frac{300}{\pi}$$

$$\Rightarrow r < \sqrt{\frac{300}{\pi}} \quad (r > 0)$$

Domain of V : $0 < r < \sqrt{\frac{300}{\pi}} \text{ cm}$ [A][A]

e. $350 = 200r - \frac{2}{3} \pi r^3$

Solve for r : $\frac{2}{3} \pi r^3 - 200r + 350 = 0$

From TABLE, $r = 8.7$ or 1.8 .

Ans : $r = 8.7 \text{ cm}$

[A]

f. $\frac{dV}{dr} = 200 - 2\pi r^2$; max where $\frac{dV}{dr} = 0$ [M]

$$\Rightarrow 100 - \pi r^2 = 0 \Rightarrow r = \frac{10}{\sqrt{\pi}} \quad [A]$$

$$l = 0 \quad [A]$$

i.e. a sphere [A]

g. Max volume = $\frac{4}{3} \pi r^3$
 $= \frac{4}{3} \times \pi \times \frac{1000}{\pi \sqrt{\pi}}$ [M]

$$= \frac{4000}{3\sqrt{\pi}} \text{ cm}^3 \quad [A]$$

Question 2

a. mark (5.1, 1.49), (12.3, 1.01) [A][A]

b. amplitude = $\frac{1}{2}(1.49 - 1.01)$; $a = 0.24$ [A]

$c \rightarrow$ vertical translation = $1.01 + 0.24$

$$c = 1.25 \quad [A]$$

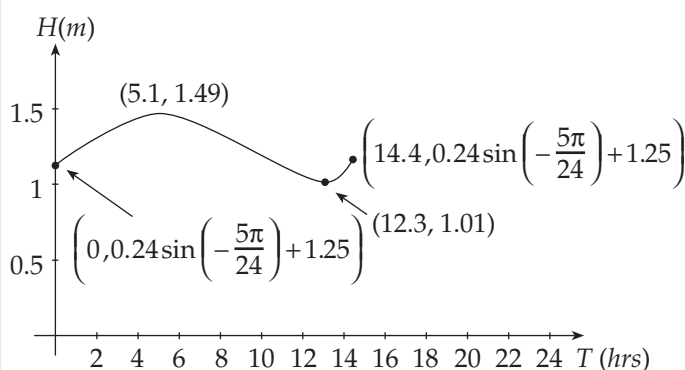
c. $n = \frac{2\pi}{\text{period}};$

high tide to low tide = $\frac{1}{2}$ period [M]

$$\therefore n = \frac{2\pi}{2(12.3 - 5.1)}$$

$$= \frac{\pi}{7.2} = \frac{5\pi}{36} \quad [M]$$

d.



[A][A]

- e. Use calculator to find $H = 1.2$, or Solve

$$1.2 = 0.24 \sin \left[\frac{5\pi}{36} (T - 1.5) \right] + 1.25$$

$$T = 1:01 \text{ am}$$

[A]

$$\frac{dH}{dT} = 0.1024 \text{ m/hr}$$

[A]

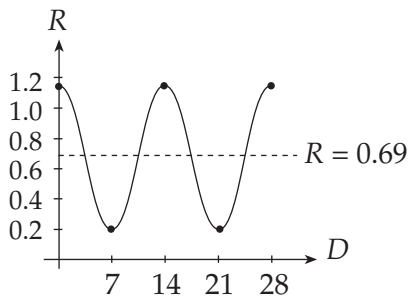
f. $R_{\max} = 1.18$ metres

[A]

g. $R_{\min} = 0.2$ metres

[A]

h. $R = 0.49 \cos \left(\frac{\pi}{7} D \right) + 0.69$



[A][A]

Question 3

a. $f(x) = 3e^{2-x} - 1$; $g(x) = x$

Point E; $f(x) = 4 \Rightarrow 3e^{2-x} - 1 = 4$

$$\Rightarrow e^{2-x} = \frac{5}{3}$$

$$\Rightarrow 2 - x = \log_e \frac{5}{3}$$

$$x = 2 - \log_e \frac{5}{3}$$

$$(2 - \log_e \frac{5}{3}, 4)$$

[A]

Point G; $f(x) = 0 \Rightarrow 3e^{2-x} - 1 = 0$

$$\Rightarrow e^{2-x} = \frac{1}{3}$$

$$\Rightarrow 2 - x = -\log_e 3$$

$$\Rightarrow x = 2 + \log_e 3$$

$$(2 + \log_e 3, 0)$$

[A]

Point F; $3e^{2-x} - 1 = x$

from calculator: $x = 2$

$$(2, 2)$$

[A]

b. Area AFG = area of $\triangle + \int_{x_1}^{x_2} (3e^{2-x} - 1) dx$ [M]

$$= 2 + \int_2^{2+\log_e 3} (3e^{2-x} - 1) dx$$
 [M]

$$= 2 + [-3e^{2-x} - x]_2^{2+\log_e 3}$$
 [M]

$$\text{Area} = (4 - \log_e 3) \text{ sq. metres}$$

[A]

c. Inverse: Let $x = 3e^{2-y} - 1$

[M]

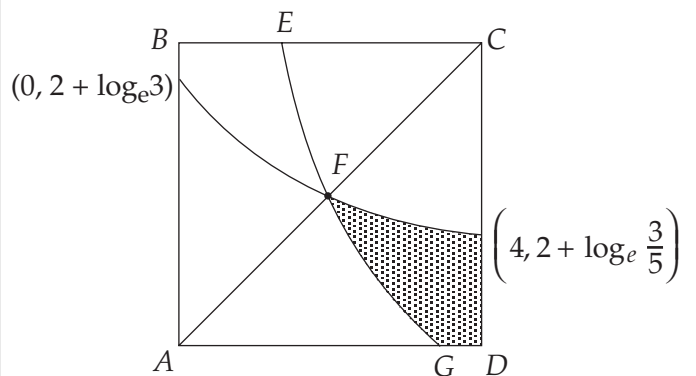
$$\frac{x+1}{3} = e^{2-y}$$

$$2 - y = \log_e \left(\frac{x+1}{3} \right)$$

$$y = 2 - \log_e \left(\frac{x+1}{3} \right)$$

$$f^{-1}(x) = 2 - \log_e \left(\frac{x+1}{3} \right)$$

[A]



sketch [A]

end points [A]

d. New panel area

$$= \int_2^{2+\log_e 3} ((f^{-1}(x)) - f(x)) dx + \int_{2+\log_e 3}^4 f^{-1}(x) dx$$

OR

$$= \int_2^4 f^{-1}(x) dx - \int_2^{2+\log_e 3} f(x) dx$$
 [M]

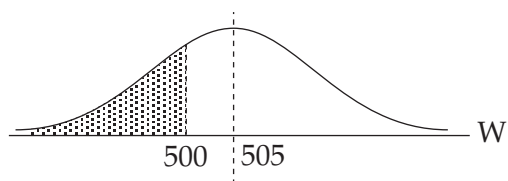
$$= 3.4459 - 0.9014$$
 [M]

$$= 2.5 \text{ square metres}$$

[A]

Question 4

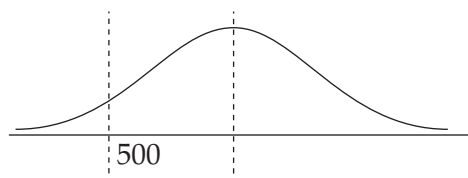
a.



$$\Pr(W < 500) = \text{normalcdf}(-1 \text{ E } 99, 500, 505, 10) \quad [\text{M}]$$

$$\approx 0.3085 \quad [\text{A}]$$

b.



$$\Pr(W \geq 500) = 0.95$$

OR

$$\Pr(W < 500) = .05$$

$$\text{inv normal}(.05, 0, 1) \rightarrow Z = -1.64485 \quad [\text{M}]$$

$$z = \frac{x - \mu}{\sigma}$$

$$\begin{aligned} \text{If } \sigma = 10 \text{ then } \frac{500 - \mu}{10} &= -1.64485 \\ \Rightarrow 500 - \mu &= -16.4485 \\ \Rightarrow \mu &= 516.4485 \text{ gm} \quad [\text{A}] \end{aligned}$$

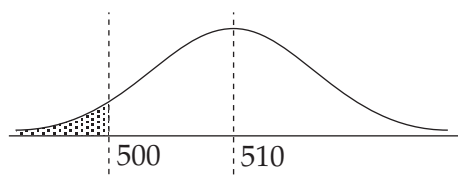
$$\begin{aligned} \text{c. If } \mu = 505 \text{ then } \frac{500 - 505}{\sigma} &= -1.64485 \quad [\text{M}] \\ \Rightarrow \frac{5}{\sigma} &= 1.64485 \\ \Rightarrow \sigma &= 3.0398 \quad [\text{A}] \end{aligned}$$

d. In case (b) the mean is higher and would cost the company too much.

In case (c) the standard deviation of 3 gms may be difficult to achieve, but it would be preferable from a cost point of view.

[A][A]

e.



$$\mu = 510; \quad \sigma = \sqrt{40}$$

$$\Pr(\text{bag} < 500 \text{ gm})$$

$$= \text{normalcdf}(-1 \text{ E } 99, 500, 510, \sqrt{40})$$

$$= 0.056923 \quad [\text{M}]$$

Let X be the number of bags which weigh less than 500 gm.

$$\Pr(X \geq 3) = 1 - \Pr(X \leq 2)$$

$$= 1 - \text{bimomcdf}(5, 0.056923, 2) \quad [\text{M}]$$

$$\approx 0.0017 \quad [\text{A}]$$

f.

Hypergeometric

$$N = 25$$

$$D = 5$$

$$n = 5$$

$$x \geq 3$$

$$\Pr(\text{box rejected}) =$$

$$\frac{\binom{20}{C_2} \binom{5}{C_3} + \binom{20}{C_1} \binom{5}{C_4} + \binom{20}{C_0} \binom{5}{C_5}}{\binom{25}{C_5}}$$

[M]

$$\approx 0.0377$$

[A]