

Part 1: Multiple-choice questions**Question 1**

For the function $f(x) = 7 - 6\sin(4x)$, the amplitude and period respectively are

- A. $7, \frac{\pi}{3}$
- B. $-6, 8\pi$
- C. $-6, \frac{\pi}{2}$
- D. $7, \frac{\pi}{2}$
- E. $6, \frac{\pi}{2}$

Question 2

Exact solutions to the equation $4\sin^2(2\theta) - 3 = 0$, where $-\pi \leq \theta \leq \pi$ are

- A. $\theta = -0.5 \sin^{-1}\left(\frac{3}{4}\right), 0.5 \sin^{-1}\left(\frac{3}{4}\right)$ only
- B. $\theta = -\frac{\pi}{3}, \frac{\pi}{3}$ only
- C. $\theta = -\frac{\pi}{6}, \frac{\pi}{6}$ only
- D. $\theta = -\frac{5\pi}{6}, -\frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$
- E. $\theta = -\frac{\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{3}$ only

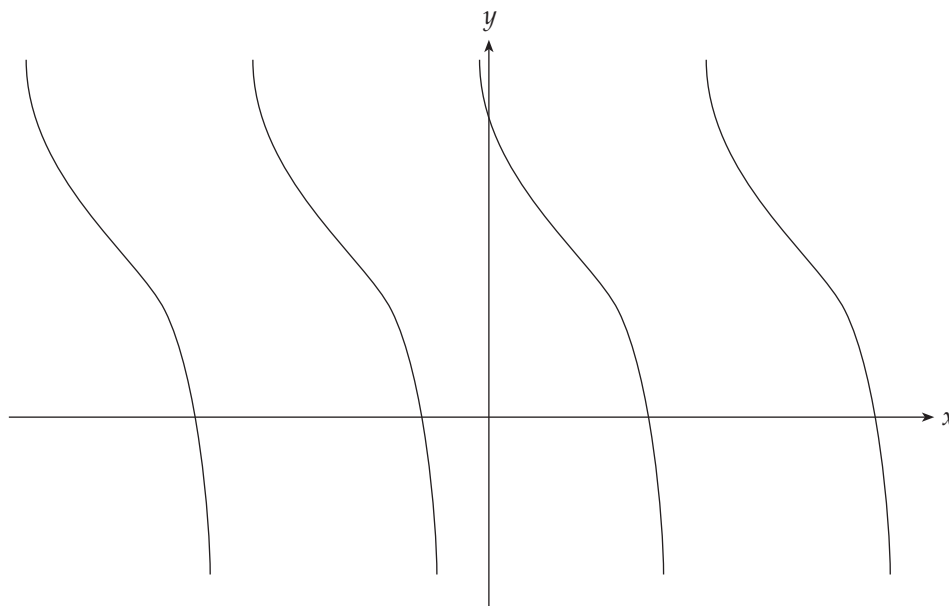
Question 3

The graph of $y = \cos x$ is transformed by doubling the amplitude, doubling the period and then translating 1 unit vertically down. The new function would be

- A. $y = 2\cos(x) - 1$
- B. $y = 2\cos(2x) - 1$
- C. $y = 0.5\cos(2x) - 1$
- D. $y = 2\cos(0.5x) - 1$
- E. $y = 2(\cos(0.5x) - 1)$

Question 4

If a , b and c are positive constants, a possible equation for the function shown could be



- A. $f(x) = a - b \tan(x)$
- B. $f(x) = a + b \tan(x + c)$
- C. $f(x) = b - a \tan(x + c)$
- D. $f(x) = a + b \tan(x)$
- E. $f(x) = a \tan(x - b) + c$

Question 5

If $y = -3x^2 + 6x - 3a$, where a is a constant, then the y coordinate of the turning point is

- A. $-1 + a$
- B. $1 - a$
- C. $-3 + 3a$
- D. $3 - 3a$
- E. a

Question 6

The term independent of x in the expansion $(2x^2 - \frac{3}{x})^6$ is

- A. 90
- B. -90
- C. 324
- D. 4860
- E. -4860

Question 7

If $2\log_2(x) - \log_2(x + 4) = 1$ then x equals

- A. -2
- B. 4
- C. -2 or 4
- D. $\frac{1 \pm \sqrt{17}}{2}$
- E. $\frac{1 + \sqrt{17}}{2}$

Question 8

If $9^x - 3^{(x+1)} = 54$ then x equals

- A. -6
- B. -2
- C. 2
- D. 9
- E. -6 or 9

Question 9

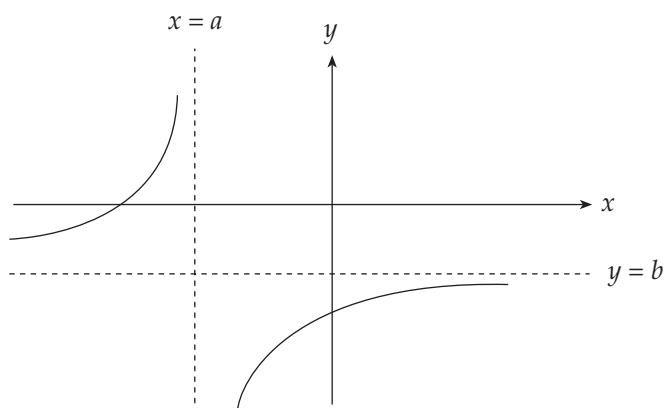
Let $h: D \rightarrow \mathbb{R}, h(x) = \frac{2}{(3x-5)^2} + 1$ where D is the maximal domain of h . The smallest value of b such that $g: (b, \infty) \rightarrow \mathbb{R}$ with $g(x) = h(x)$ is a one to one function is

- A. 2
- B. $1\frac{2}{3}$
- C. 1
- D. $-1\frac{2}{3}$
- E. 5

Question 10

The inverse function, $f^{-1}(x)$ of $f: (-\infty, -1] \rightarrow \mathbb{R}, f(x) = -4(x+1)^2$ is

- A. $f^{-1}: (-\infty, 0] \rightarrow \mathbb{R}, f^{-1}(x) = \frac{\sqrt{-x}}{2} - 1$
- B. $f^{-1}: (-\infty, 0] \rightarrow \mathbb{R}, f^{-1}(x) = \frac{\sqrt{x}}{2} - 1$
- C. $f^{-1}: (-\infty, 0] \rightarrow \mathbb{R}, f^{-1}(x) = \frac{\pm\sqrt{-x}}{2} - 1$
- D. $f^{-1}: (-\infty, 0] \rightarrow \mathbb{R}, f^{-1}(x) = \frac{-\sqrt{-x}}{2} - 1$
- E. $f^{-1}: (-\infty, -1] \rightarrow \mathbb{R}, f^{-1}(x) = \frac{-\sqrt{-x}}{2} - 1$

Question 11

The rule for the above graph could be

- A. $y = \frac{-1}{x + a} + b$
- B. $y = \frac{1}{x + a} + b$
- C. $y = \frac{-1}{x - a} + b$
- D. $y = \frac{-1}{x - a} - b$
- E. $y = \frac{-1}{x + a} - b$

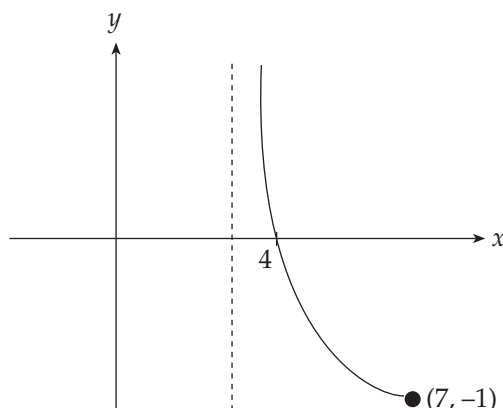
Question 12

The curve of a cubic function, $f(x)$, in the form $f: R \rightarrow R$, where $f(x) = A(x - B)^3 + C$ and A, B and C are **positive** real constants will have a

- A. positive gradient for all $x \in R$.
- B. local minimum followed by a local maximum.
- C. a local maximum followed by a local minimum.
- D. a stationary point of inflection or a local maximum and local minimum.
- E. a stationary point of inflection.

Question 13

The rule for the function of the graph shown below is of the form $y = A \log_e(x + B)$, where A and B are real constants.



The values of A and B are

- A. $A = -\frac{1}{\log_e 4}$ $B = -3$
- B. $A = -\frac{1}{\log_e 10}$ $B = 3$
- C. $A = -1$ $B = -3$
- D. $A = -1.789$ $B = 2.477$
- E. $A = -0.721$ $B = 3$

Question 14

The average rate of change of the function $f(x) = (x - 1)e^x$ with respect to x over $[0, 2]$ is

- A. $\frac{e^2 + 1}{2}$
- B. $2e^2$
- C. xe^x
- D. e^2
- E. $\frac{e^2}{2}$

Question 15

The equation of the normal to the curve of the function with equation $y = \frac{x}{\cos(x)}$ at the point where $x = \pi$ is

- A. $y = x$
- B. $y = x + 2\pi$
- C. $y = -x$
- D. $y = -x + 2\pi$
- E. $y = x - 2\pi$

Question 16

The derivative of $\log_e(\tan(x))$ is

- A. $\frac{\sec(x)}{\tan(x)}$
- B. $\frac{\sec(x)}{\sin(x)}$
- C. $\sec^2(x)\tan(x)$
- D. $\sec^2(x)$
- E. $\frac{\tan(x)}{\sec^2(x)}$

Question 17

If $y = \left(\sqrt{x^2 + 1}\right)^3$ then $\frac{dy}{dx}$ equals

- A. $3xy^{\frac{1}{3}}$
- B. $3xy^3$
- C. $\frac{3}{2}\sqrt{x^2 + 1}$
- D. $2x\sqrt{x^2 + 1}$
- E. $6x\sqrt{x^2 + 1}$

Question 18

For the curve of the function with equation $y = (x - 1)^3(x + 2)$, the largest subset of R for which the gradient of the graph is positive is

- A. $(-\infty, -2)$
- B. $(-\infty, -1.25)$
- C. $(-1.25, \infty)$
- D. $(-1.25, 1) \cup (1, \infty)$
- E. $(1, \infty)$

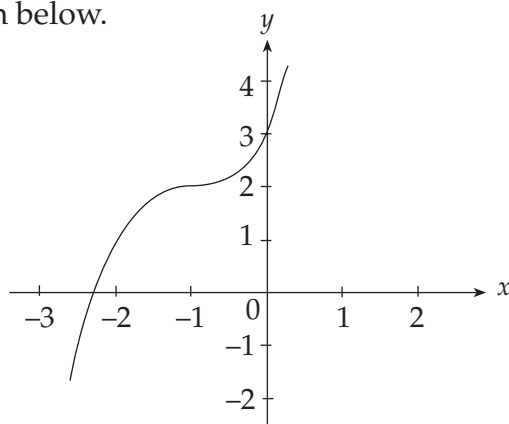
Question 19

The approximate area, in square units correct to two decimal places, bounded by the graph of $y = 10^x$ and the x -axis, using the left end point between $x = 0$ and $x = 3$ and using rectangular strips of width 0.5 is

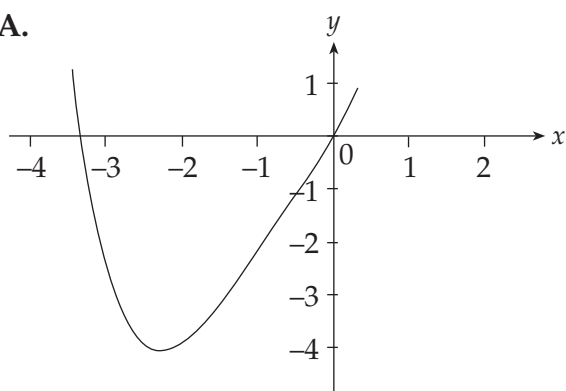
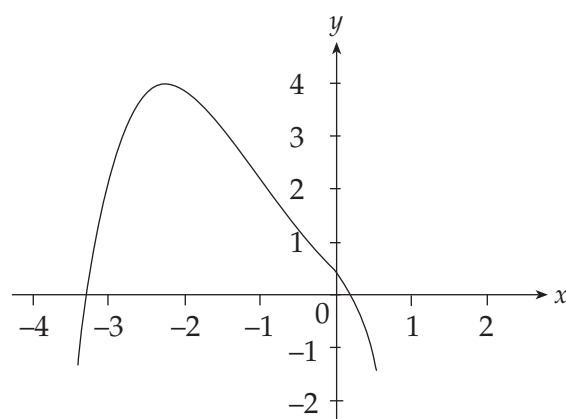
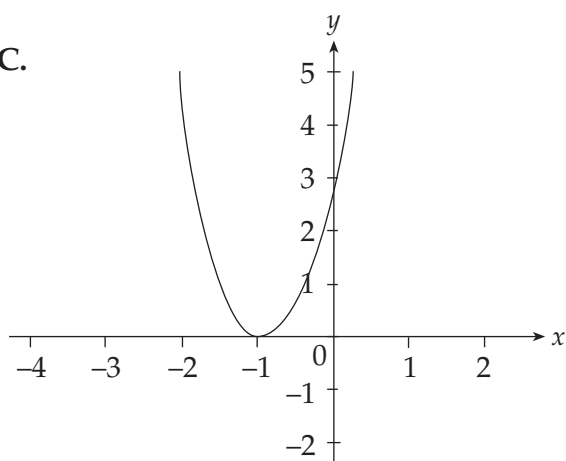
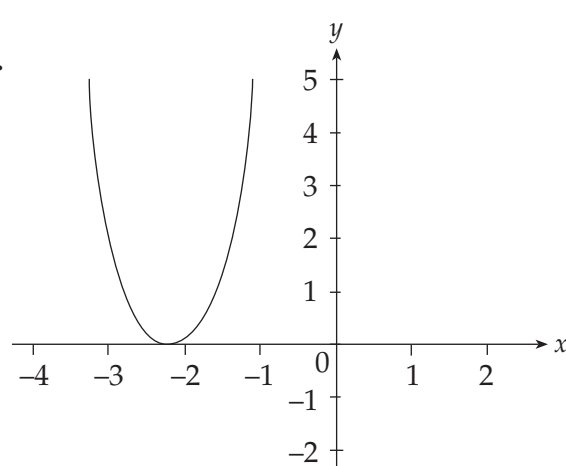
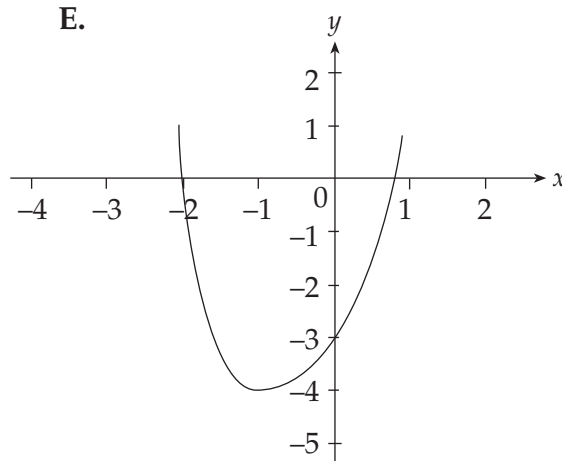
- A. 231.01
- B. 433.86
- C. 480.76
- D. 610.50
- E. 730.51

Question 20

The graph of $y = f(x)$ is shown below.



If h is a function such that $h'(x) = f(x)$, then the graph of h could be

A.**B.****C.****D.****E.**

Question 21

Note: c is a real constant.

$$\int \frac{e^{3x} + 1}{e^x} dx \text{ equals}$$

- A. $\frac{e^{2x}}{2} - e^x + c$
- B. $\frac{e^{2x}}{2} + e^{-x} + c$
- C. $2e^{2x} + e^{-x} + c$
- D. $2e^{2x} - e^{-x} + c$
- E. $\frac{e^{2x}}{2} - \frac{1}{e^x} + c$

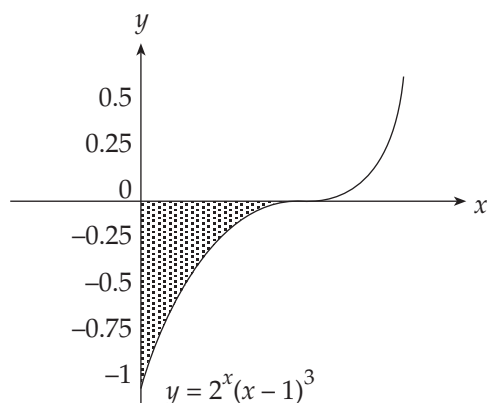
Question 22

$$\text{If } \int_1^a \frac{1}{(x-2)^3} dx = -\frac{1}{2} \text{ then } a \text{ equals}$$

- A. $2 + \frac{\sqrt{2}}{2}$
- B. $2 \pm \frac{\sqrt{2}}{2}$
- C. $2 - \frac{\sqrt{2}}{2}$
- D. $2 + \sqrt{2}$
- E. $2 - \sqrt{2}$

Question 23

The area, in square units, of the shaded region (the region bounded by the curve and the axes) shown correct to three decimal places is



- A. 0.289
- B. 0.578
- C. 0.587
- D. 0.876
- E. -0.289

Question 24

A random variable X has the following probability distribution.

x	0	1	2	3
$\Pr(X = x)$	a	$2a$	$4a$	$3a$

The value of $E(2X - 1)$ is

- A. 0.1
- B. 1.5
- C. 1.9
- D. 2.5
- E. 2.8

Question 25

An examination paper consists of 33 multiple-choice questions, each question having 5 possible answers. A student randomly guesses the answer to every question. The probability of her getting 20 correct is

- A. ${}^{33}C_{20} (0.2)^{20} (0.8)^{13}$
- B. ${}^{33}C_5 (0.2)^{20}$
- C. $\frac{20}{33}$
- D. ${}^{33}C_{20} (0.2)^{13} (0.8)^{20}$
- E. ${}^{33}C_{20} (0.2)^5$

Question 26

There are b identical black socks and n identical navy socks in a drawer. Two socks are taken from the drawer at random in the dark. The probability of obtaining a pair is

- A. $\frac{b(b-1)}{(b+n)}$
- B. $\frac{(b-n)}{(b+n)}$
- C. $\frac{b}{(b+n)}$
- D. $\frac{b(b-1) + n(n-1)}{(b+n)(b+n-1)}$
- E. $\frac{(b^2 + n^2)}{(b+n)^2}$

Question 27

The height (H) of trees in a plantation is known to be normally distributed with a mean of 6 metres. If $\Pr(H > 6.5) = 0.05$ then the standard deviation of the distribution is closest to

- A. 0.092
- B. 0.304
- C. 0.526
- D. 0.962
- E. 3.290

PART II
SHORT ANSWER QUESTIONS (23 marks)

Question 1

Find exact solutions for $4 \cos^2(x) + 4 \sin(x) = 1$ given that $0 < x < 2\pi$.

3 marks

Question 2

Let $f: \mathbb{R} \setminus \{-1\} \rightarrow \mathbb{R}$, where $f(x) = \frac{x^2 + 2x + 2}{(x + 1)^2}$.

- a. Express $f(x)$ in the form $\frac{A}{(x + 1)^2} + B$, where A and B are positive integers.

- b. Hence, if $f(x)$ is dilated a factor of 2 from the x -axis and then translated 1 unit to the right, write down the equation for this new function $f_1(x)$.

- c. State the range of $f_1(x)$.

$2 + 1 + 1 = 4$ marks

Question 3

The curve with equation $y = x^3 + bx^2 + cx + d$ has a stationary point at $(1, 2)$ and a y -intercept of 1.

- a. Find b, c and d .

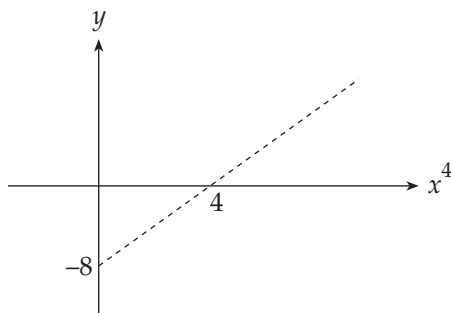
- b. Write y in the form $A(x - B)^3 + C$, where A, B and C are positive integers.

- c. Hence, show that the x -intercept is $\sqrt[3]{-2} + 1$.

3 + 1 + 1 = 5 marks

Question 4

An experiment was conducted to find the relationship between two variables x and y . The graph of y against x^4 is plotted below and was found to be linear.



- a. Find a rule for y in terms of x .

- b. Factorise or use another method to find exact solutions to $y = 0$.

- c. Find the exact area bounded by the quartic and the x -axis.

2 + 2 + 2 = 6 marks

Question 5

For a discrete random variable Y , the probability function is defined by $f(Y) = \frac{Y}{10}$. Complete the distribution table and find $E(Y)$ and hence find the standard deviation of Y .

Y	0	1	2	3	4
$f(Y)$					

3 marks

Question 6

The probability that a person dies from a certain disease is 0.4. What is the probability correct to 4 decimal places that out of 10 randomly selected patients, at least 3 will die as a result of the disease?

2 marks