MAV Mathematical Methods Examination 1 Answers & Solutions

Part I (Multiple-choice) Answers

1. **E**	2. **D**	3. **D**	4. **C**	5. **D**
6. **D**	7. **B**	8. **C**	9. **B**	10. **D**
11. **C**	12. **E**	13. **A**	14. **A**	15. **E**
16. **B**	17. **A**	18. **D**	19. **A**	20. **A**
21. **E**	22. **C**	23. **A**	24. **E**	25. **A**
26. **D**	27. **B**			

Question 1 **[E]**

$f(x) = 7 - 6\sin(4x)$

amplitude = 6

period = $\frac{2\pi}{4} = \frac{\pi}{2}$

Question 2 **[D]**

$\sin^2(2\theta) = \frac{3}{4}$

$\sin(2\theta) = \pm\frac{\sqrt{3}}{2}$

$2\theta = \frac{\pi}{3}, \frac{2\pi}{3} + 2n\pi (n = -2, -1, 0, 1)$

$\theta = -\frac{5\pi}{6}, -\frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$

Question 3 **[D]**

$y = a\cos nx$

period = 2π , amplitude = 1

Double: period = 4π $\therefore n = \frac{1}{2}$

Amplitude = 2

$\Rightarrow y = 2\cos(0.5x) - 1$

Question 4 **[C]**

Reflect $y = \tan x$ in the x-axis. A or C

There has been a horizontal and vertical translation, hence C

Question 5 **[D]**

Turning point: $x = -\frac{b}{2a}$

$= \frac{-6}{-6} = 1$

Substitute $x = 1$ into $y = -3x^2 + 6x - 3a$

$y = -3 + 6 - 3a$

$y = 3 - 3a$

Question 6 **[D]**

$\left(2x^2 - \frac{3}{x}\right)^6$

General term: ${}^6C_r\left(2x^2\right)^{6-r}\left(-\frac{3}{x}\right)^r$

$\therefore 12 - 2r - r = 0$

$\therefore r = 4$

$= {}^6C_4\left(2x^2\right)^2\left(-\frac{3}{x}\right)^4$

$= {}^6C_4(2)^2(-3)^4$

$= 15 \times 4 \times 81$

$= 4860$

Question 7 **[B]**

$2\log_2 x - \log_2(x+4) = 1$

$\Rightarrow \log_2\left(x^2\right) - \log_2(x+4) = 1$

$\Rightarrow \log_2\left(\frac{x^2}{x+4}\right) = 1$

$\Rightarrow \frac{x^2}{x+4} = 2$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow (x-4)(x+2) = 0$

$\Rightarrow x = 4, -2$ Note: –2 is not possible

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Question 8 **[C]**

$3^{2x} - 3^{x+1} = 54$

$\Rightarrow 3^{2x} - 3 \times 3^x - 54 = 0$

Let $a = 3^x$

$a^2 - 3a - 54 = 0$

$(a - 9)(a + 6) = 0$

$a = 9$ or $a = -6$

$3^x = 9$ or $3^x = -6$

$x = 2$

Note: $3^x = -6$ is not possible.

Question 9 **[B]**

$3x - 5 \neq 0$

Hence $x = \frac{5}{3}$ is an asymptote.

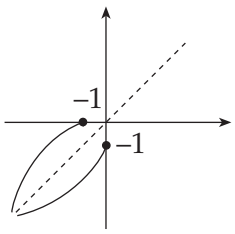
The largest domain is $\left(\frac{5}{3}, 0\right)$

$b = \frac{5}{3} = 1\frac{2}{3}$

Question 10 **[D]**

$y = -4(x + 1)^2$

Inverse: $x = -4(y + 1)^2$



$y = -\sqrt{-\frac{x}{4}} - 1$

$= \frac{-\sqrt{-x}}{2} - 1$

dom: $(-\infty, 0]$

Question 11 **[C]**

Note a and b are negative.

$y = \frac{1}{x}$ has been reflected in the x-axis

$y = -\frac{1}{x}$ and then translated $-a$ units to the left and $-b$ units down.

$y = -\frac{1}{x - a} + b$

Question 12 **[E]**

The curve has a stationary point of inflection at (B, C).

Question 13 **[A]**

Translation 3 to right $\therefore$ B = – 3

Substitute $(7, -1) \Rightarrow -1 = A \log_e 4$

$$A = \frac{-1}{\log_e 4}$$

Question 14 **[A]**

$f(0) = -1$

$f(2) = e^2$

Gradient $= \frac{e^2 - -1}{2 - 0}$

$= \frac{e^2 + 1}{2}$

Question 15 **[E]**

When $x = \pi$, $y = \frac{\pi}{-1} = -\pi$

From calculator: tangent is $y = -x$

$\Rightarrow$ normal: $y = x - 2\pi$

Question 16 **[B]**

Chain rule: $\frac{dy}{dx} = \frac{1}{\tan x} \times \sec^2 x$

$= \frac{\cos x}{\sin x} \times \frac{\sec x}{\cos x}$

$= \frac{\sec x}{\sin x}$

Question 17 **[A]**

$y = \left(\sqrt{\left(x^2 + 1\right)}\right)^3 = \left(x^2 + 1\right)^{\frac{3}{2}}$

$\frac{dy}{dx} = \frac{3}{2}\left(x^2 + 1\right)^{\frac{1}{2}} \times 2x$

$= 3x\left(x^2 + 1\right)^{\frac{1}{2}}$

$= 3xy^{\frac{1}{3}}$

Question 18 **[D]**

From calculator: note: point of inflection at $x = 1$

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Question 19 **[A]**

Use TABLE on calculator (or graphics calculator program).

$$A = \frac{1}{2}\left[f(0) + f(0.5) + f(1) + f(1.5) + f(2) + f(2.5)\right]$$
$$= 231.01 \text{ square units}$$

Question 20 **[A]**

$f(x)$ is gradient function of $h(x)$

$h'(x) = 0$ when $x \approx -2.3$

$h'(x) < 0$ when $x < -2.3$

$h'(x) > 0$ when $x > -2.3$

Either A or D

$$f(x) = (x+1)^3 + 2$$
$$h(x) = \int \left((x+1)^3 + 2\right) dx$$
$$= \frac{(x+1)^4}{4} + 2x + c$$

Hence A

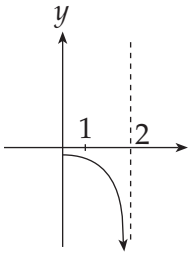
Question 21 **[E]**

$$\int \frac{e^{3x} + 1}{e^x} dx$$
$$= \int \left(e^{2x} + e^{-x}\right) dx$$
$$= \frac{e^{2x}}{2} - e^{-x} + c$$

Question 22 **[C]**

$$\int_1^a \frac{1}{(x-2)^3} dx = -\frac{1}{2}$$
$$\left[\frac{1}{-2(x-2)^2}\right]_1^a = -\frac{1}{2}$$
$$\frac{1}{-2(a-2)^2} + \frac{1}{2} = -\frac{1}{2}$$
$$\frac{1}{-2(a-2)^2} = -1$$
$$2(a-2)^2 = 1$$
$$(a-2)^2 = \frac{1}{2}$$
$$a - 2 = \pm\sqrt{\frac{1}{2}}$$
$$a = 2 \pm \sqrt{\frac{1}{2}}$$
$$= 2 \pm \frac{\sqrt{2}}{2}$$

cannot be $2 + \frac{\sqrt{2}}{2}$



Question 23 **[A]**

From calculator:

$$\text{Area} = \left|\int_0^1 \left(2^x (x-1)^3\right) dx\right|$$
$$= 0.289$$

Question 24 **[E]**

$10a = 1$

$a = 0.1$

$\text{E(X)} = 1 \times 0.2 + 2 \times 0.4 + 3 \times 0.3$

$= 1.9$

$\therefore \text{E(2X} - 1) = 2 \times 1.9 - 1$

$= 2.8$

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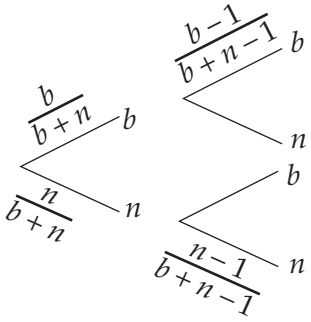
Question 25 **[A]**

Pr(correct) = 0.2

$$\Pr(20 \text{ correct}) = {}^{33}C_{20}(0.2)^{20}(0.8)^{13}$$
$$= {}^{33}C_{13}(0.2)^{20}(0.8)^{13}$$

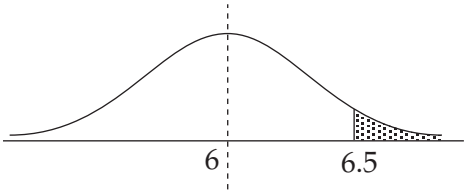
Question 26 **[D]**

$$\Pr(\text{pair}) = \frac{\binom{b}{2}\binom{n}{0} + \binom{b}{0}\binom{n}{2}}{\binom{b+n}{2}}$$



$$\Pr(\text{pair}) = \frac{b(b-1) + n(n-1)}{(b+n)(b+n-1)}$$

Question 27 **[B]**



invNorm (0.95) = 1.6449

$$z = \frac{x - \mu}{\sigma}$$

$$1.6449 = \frac{6.5 - 6}{\sigma}$$

$$\Rightarrow \sigma = 0.304$$

Part II Solutions

Question 1

$4\cos^2 x + 4\sin x = 1$

$\Rightarrow 4 - 4\sin^2 x + 4\sin x = 1$ **[M]**

$\Rightarrow 4\sin^2 x - 4\sin x - 3 = 0$

$\Rightarrow (2\sin x - 3)(2\sin x + 1) = 0$ **[M]**

$\Rightarrow \sin x = \frac{3}{2}$ (not possible) or $\sin x = \frac{-1}{2}$

$\Rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$ **[A]**

Question 2

a.
$$\frac{x^2 + 2x + 2}{(x+1)^2}$$
$$= \frac{x^2 + 2x + 1 - 1 + 2}{(x+1)^2}$$
$$= \frac{(x+1)^2 + 1}{(x+1)^2}$$
$$= 1 + \frac{1}{(x+1)^2}$$
$$= \frac{1}{(1+x)^2} + 1$$ **[A][A]**

b. $f_1(x) = \frac{2}{x^2} + 2$ **[A]**

c. $(2, \infty)$ **[A]**

Question 3

a. $d = 1$ **[A]**

$\frac{dy}{dx} = 3x^2 + 2bx + c$

$\Rightarrow 0 = 3 + 2b + c$

$2b + c = -3 \quad \ldots\ldots ①$

$y = x^3 + bx^2 + cx + 1$

$2 = 1 + b + c + 1$

$\therefore b + c = 0 \quad \ldots\ldots ②$

Solve ① and ② simultaneously

$b = -3, c = 3$

$\Rightarrow y = x^3 - 3x^2 + 3x + 1$ **[A][A]**

b. $y = (x-1)^3 + 2$ **[A]**

c. If $y = 0$

$\Rightarrow (x-1)^3 + 2 = 0$

$\Rightarrow (x-1)^3 = -2$

$\Rightarrow x - 1 = \sqrt[3]{-2}$

$\Rightarrow x = 1 + \sqrt[3]{-2}$ **[M]**

Question 4

a. $m = \frac{8}{4} = 2$ **[A]**

$y = 2x^4 - 8$ **[A]**

b. $y = 2(x^4 - 4)$

$= 2(x^2 - 2)(x^2 + 2)$

$= 2\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right)\left(x^2 + 2\right)$

When $y = 0$, $x = \pm\sqrt{2}$ **[A][A]**

c. $\text{A} = \left| \int_{-\sqrt{2}}^{\sqrt{2}} (2x^4 - 8)dx \right|$ **[M]**

(= 18.10) for checking

$= \left| \left[\frac{2}{5}x^5 - 8x \right]_{-\sqrt{2}}^{\sqrt{2}} \right|$

$= \left| \left(\frac{2 \times 4\sqrt{2}}{5} - 8\sqrt{2} \right) - \left(\frac{2 \times -4\sqrt{2}}{5} + 8\sqrt{2} \right) \right|$

$= \frac{64}{5}\sqrt{2}$ **[A]**

Question 5

Y	0	1	2	3	4
$f(Y)$	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

[A]

$\text{E}(Y) = 3$; $\text{E}(Y^2) = 10$ **[A]**

$\text{Var}(Y) = \text{E}(Y^2) - [\text{E}(Y)]^2$

$= 1 \Rightarrow \sigma = 1$ **[A]**

Question 6

$\Pr(X \geq 3) = 1 - \Pr(X \leq 2)$

$= 1 - \text{binomcdf}(10, 0.4, 2)$ **[M]**

$= 0.8327$ **[A]**