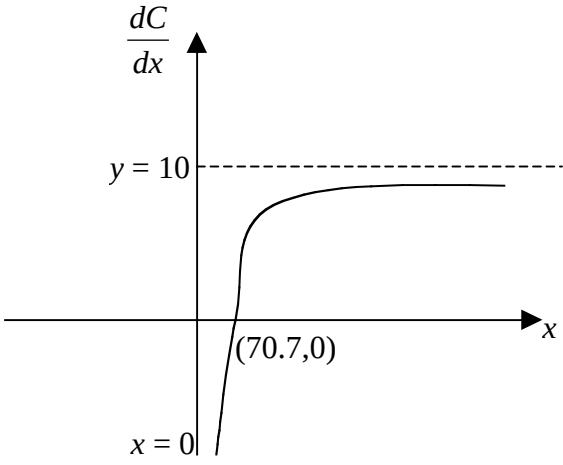


Question 1

i. Total Cost = $50,000 + 25x + 10x^2$ (1 mark)	ii. $C = \frac{50,000 + 25x + 10x^2}{x}$ (1 mark)
iii. $C = 50,000x^{-1} + 25 + 10x$ $\frac{dC}{dx} = -50,000x^{-2} + 10 = 0$ for turning point $\frac{50,000}{x^2} = 10$ (1 mark) $10x^2 = 50,000$ $x^2 = 5,000$ $x = 70.7$ (1 mark) If $x < 70.7$, $\frac{dC}{dx} < 0$ If $x > 70.7$, $\frac{dC}{dx} > 0$ (1 mark) $\therefore$ minimum cost when $x = 70.7$ $x = 71$ to nearest whole number.	iv. $x > 0$ (1 mark) v.  • 1 mark for shape • 1 mark for x intercept • 1 mark for labeling vertical asymptote • 1 mark for labeling horizontal asymptote

Suggested Solutions

Question 2

a.

$$y = 20x + 20$$

$$\text{Gradient of tangent} = 20$$

(1 mark)

b.

$$y = Ax^2$$

$$y = 20x + 20$$

At point of tangency

$$Ax^2 = 20x + 20 \quad (1 \text{ mark})$$

$$Ax^2 - 20x - 20 = 0$$

At point of tangency, only one solution

$$b^2 - 4ac = 0 \quad (1 \text{ mark})$$

$$400 + 80A = 0$$

$$80A = -400$$

$$A = -5 \quad (1 \text{ mark})$$

c.

$$y = -5x^2$$

$$y = 20x + 20$$

$$-5x^2 = 20x + 20$$

$$5x^2 + 20x + 20 = 0$$

$$x^2 + 4x + 4 = 0$$

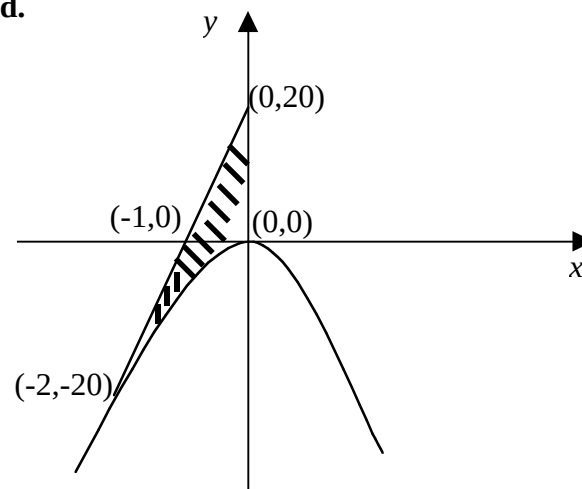
$$(x + 2)^2 = 0$$

$$x = -2 \quad (1 \text{ mark})$$

$$\text{When } x = -2, y = -5 \times -2^2$$

$$y = -20 \quad (1 \text{ mark})$$

$$(-2, -20)$$

d.

$$y = 20x + 20$$

X intercept when $y = 0$

$$20x + 20 = 0$$

$$20x = -20$$

$$x = -1$$

- 1 mark for parabola positioned correctly
- 1 mark for line
- 1 mark for point (0,20)
- 1 mark for point (-1,0)
- 1 mark for point (-2,-20)

Question 2

e.(i)

1 mark for shading correct region in graph on page 2.

e.(ii)

$$A = \int_{-2}^0 (20x + 20 + 5x^2) dx \quad (1 \text{ mark})$$

$$A = 10x^2 + 20x + \frac{5x^3}{3} \Big|_{-2}^0 \quad (1 \text{ mark})$$

$$A = 0 - [40 - 40 - \frac{40}{3}]$$

$$A = 13\frac{1}{3} \text{ sq units. } (1 \text{ mark})$$

f.

$$y = 2x + c$$

$$y = 20x + 20$$

$$\text{When } x = -3, y = -40$$

$$-40 = 2 \times -3 + c$$

$$-40 = -6 + c$$

$$c = -34$$

$$\therefore y = 2x - 34$$

(1 mark)

Question 3

a.

$$\text{Pr} = \frac{\binom{7}{1} \binom{3}{1}}{\binom{10}{2}} \quad (1 \text{ mark})$$

$$\text{Pr} = 0.467 \quad (1 \text{ mark})$$

b.

$$\text{Pr} = \frac{\binom{5}{2} \binom{5}{0}}{\binom{10}{2}} \quad (1 \text{ mark})$$

$$\text{Pr} = 0.222 \quad (1 \text{ mark})$$

c.

$$\text{Mean} = n \times \frac{D}{N}$$

$$n = 2, N = 7, D = 3 \quad (1 \text{ mark})$$

$$\text{Mean} = 2 \times \frac{3}{7} = 0.857 \quad (1 \text{ mark})$$

d.

$$\text{Pr}(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

$$n = 3, x = 2, p = \frac{2}{3} \quad (1 \text{ mark})$$

$$\text{Pr}(X = 2) = \binom{3}{2} \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^1$$

$$\text{Pr}(X = 2) = 0.444 \quad (1 \text{ mark})$$

Question 3 (continued)

e. (i)

$$\text{Pr. Laura wins a set} = \frac{1}{5}$$

$$\text{Pr. Laura wins at least one set} = 1 - \text{Pr Laura wins 0 sets} \quad (1 \text{ mark})$$

$$\text{Pr. Laura wins at least one set} = 1 - \binom{3}{0} \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^3 \quad (1 \text{ mark})$$

$$\text{Pr. Laura wins at least one set} = 1 - \frac{64}{125}$$

$$\text{Pr. Laura wins at least one set} = \frac{61}{125} = 0.49 \quad (1 \text{ mark})$$

e.(ii)

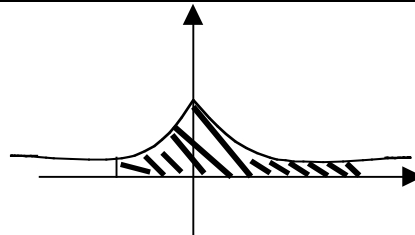
Binomial

$$\text{Mean} = np$$

$$\text{Mean} = 10 \times \frac{4}{5}$$

$$\text{Mean} = 8 \quad (1 \text{ mark})$$

f.(i)



$$\begin{aligned} \text{Pr}(X > 27) &= 1 - \text{Pr}(X < 27) \\ &= 1 - \text{Pr}\left(Z < \frac{27 - 30}{2}\right) \quad (1 \text{ mark}) \\ &= 1 - \text{Pr}(Z < -1.5) \\ &= 1 - \text{Pr}(Z > 1.5) \\ &= 1 - [1 - \text{Pr}(Z < 1.5)] \\ &= \text{Pr}(Z < 1.5) \\ &= 0.9332 \quad (1 \text{ mark}) \end{aligned}$$

f.(ii)

$$1 - \text{Pr}(X=0) = 0.9844$$

$$\text{Pr}(X=0) = 0.0156$$

Let there be n balls in the box

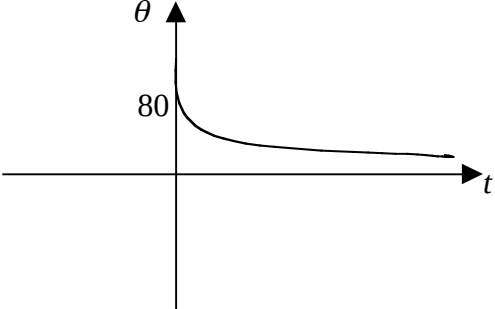
$$(0.5)^n = 0.0156 \quad (1 \text{ mark})$$

$$n \log_{10}(0.5) = \log_{10} 0.0156$$

$$n = \frac{\log_{10} 0.0156}{\log_{10} 0.5}$$

$$n = 6 \quad (1 \text{ mark})$$

Question 4

a.(i) 100 °C (1 mark)	a.(ii) 100 = 20 + A × e⁰ (1 mark) 100 = 20 + A A = 80 (1 mark)
a.(iii) From the information given, T = 70 °C when t = 10 (1 mark)	a.(iv) $T = T_0 + Ae^{kt}$ 70 = 20 + 80e^{10k} 50 = 80e^{10k} 0.625 = e^{10k} (1 mark) 10k = log_e 0.625 (1 mark) 10k = -0.47 k = -0.047 k = -0.05 to two decimal places (1 mark)
b.(i) $T - T_0 = Ae^{kt}$ $\theta = 80e^{-0.05t}$ (1 mark) $\frac{d\theta}{dt} = -4e^{-0.05t}$ (1 mark)	b.(ii) $\frac{d\theta}{dt} = -4e^{-0.05t}$ For all values of t, e^{-0.5t} > 0 ∴ -4 × e^{-0.5t} < 0 for all values of t (1 mark)
b.(iii) When t = 10 $\frac{d\theta}{dt} = -4e^{-0.5}$ = -2.426 °C/ min (1 mark)	b.(iv) $\theta = 80e^{-0.05t}$  • 1 mark for shape • 1 mark for point (0,80)

END OF SUGGESTED SOLUTIONS
2002 Mathematical Methods Trial Examination 2

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