

Question 1 D $x-2 \neq 0$ $\therefore$ Vertical asymptote at $x = 2$ $x-2 \overline{) \frac{1}{x-1}}$ $\frac{x-2}{x-2}$ $\frac{1}{x-2}$ $f(x) = 1 + \frac{1}{x-2}$ $\therefore$ Horizontal asymptote at $y = 1$	Question 2 D Interchange x and y $x = \frac{1}{2}e^{y+1}$ $2x = e^{y+1}$ $y+1 = \log_e(2x)$ $y = \log_e(2x) - 1$
Question 3 A $f(x)$ represents a semi circle with centre $(0,0)$ and radius 2. This semi circle is an increasing function for $-2 \leq x \leq 0$ $\therefore [-2,0]$	Question 4 B A maximum value occurs when $\cos \frac{\pi(t-1)}{4} = -1$ So $\frac{\pi(t-1)}{4} = \pi, 3\pi, 5\pi, \dots$ $\pi(t-1) = 4\pi, 12\pi, 20\pi, \dots$ $(t-1) = 4, 12, 20, \dots$ $t = 5, 13, 21, \dots$ A maximum value occurs when $t = 5$
Question 5 D $\sin 2\theta = -\cos 2\theta$ $\tan 2\theta = -1$ $2\theta = \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}, 3\pi - \frac{\pi}{4}, 4\pi - \frac{\pi}{4}$ $0 \leq \theta \leq 2\pi \quad 0 \leq 2\theta \leq 4\pi$ $2\theta = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}$ $\theta = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$ Difference between largest and smallest solution is $\frac{15\pi}{8} - \frac{3\pi}{8} = \frac{12\pi}{8} = \frac{3\pi}{2}$	Question 6 E This graph has an amplitude of 4 and is not translated up or down.
Question 7 A $\log_a \left(\frac{b}{c} \right)^2 = 2 \log_a \left(\frac{b}{c} \right)$ $= 2[\log_a b - \log_a c]$ $= 2[1.6 - 2.2]$ $= 2[-0.6]$ $= -1.2$	Question 8 E $P(2) = 8 + 4a + 2b - 24 \quad (1)$ $P(-3) = -27 + 9a - 3b - 24 \quad (2)$ $4a + 2b = 16 \quad (1a)$ $9a - 3b = 51 \quad (2a)$ $(1a) \times 3 \quad 12a + 6b = 48$ $(2a) \times 2 \quad 18a - 6b = 102$ Add $\frac{30a}{a} = 150$ $a = 5$ Substituting $a = 5$ in $(1a)$ $20 + 2b = 16$ $2b = -4 \Rightarrow b = -2$

Question 9 A $y = b + 2a^{3x}$ $y - b = 2a^{3x}$ $\frac{y - b}{2} = a^{3x}$ $3x = \log_a \left(\frac{y - b}{2} \right)$ $x = \frac{1}{3} \log_a \left(\frac{y - b}{2} \right)$	Question 10 C Since the vertical asymptote has the equation $x = 1$, then the graph of $\log_e x$ has been translated one unit to the right, so $A = -1$. When $x = 2$, $y = 3$, so $\log_e 1 + B = B = y$ $\Rightarrow B = 3$
Question 11 B Solutions are $x = 0$ (double root) and $x = 3$ $\therefore$ graph has equation $y = Kx^2(x - 3)$ When $x < 3$, $y < 0$ When $x > 3$, $y > 0$ $\therefore K$ is positive, so graph could be $y = x^2(x - 3)$ $\therefore y = x^3 - 3x^2$	Question 12 A This graph has amplitude = 3 It is translated 1 unit up. Period = $\pi = \frac{2\pi}{n}$ $\therefore n = 2$ It is a sin graph When $\theta = 0$, $3\sin 2\left(\theta + \frac{\pi}{4}\right) + 1 = 4$ so not B, C, D or E
Question 13 B Point of intersection is the point where $x^2 + 1 = \frac{1}{x}$ Entering the two equations $y = x^2 + 1$ and $y = \frac{1}{x}$ in the graphics calculator, it can be seen that 0.682 is closest to the x coordinate for the point of intersection.	Question 14 E $y = \sin \log_e(x)$ Let $u = \log_e(x)$ $\frac{du}{dx} = \frac{1}{x}$ $y = \sin u$ $\frac{dy}{du} = \cos u$ $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$ $\frac{dy}{dx} = \cos u \times \frac{1}{x}$ $\frac{dy}{dx} = \cos \log_e x \times \frac{1}{x}$ $\frac{dy}{dx} = \frac{\cos \log_e x}{x}$

Question 15 A $f'(x) = x^2 \times \frac{d}{dx} \cos 2x + \cos 2x \times \frac{d}{dx} x^2$ $= x^2 \times (-2 \sin 2x) + \cos 2x \times 2x$ $= -2x^2 \sin 2x + 2x \cos 2x$ $= 2x(\cos 2x - x \sin 2x)$	Question 16 E For turning points $f'(x) = 0$ $\frac{(5 + e^x)e^x - e^x(e^x)}{(5 + e^x)^2} = 0$ $\therefore e^x(5 + e^x - e^x) = 0$ $\therefore 5e^x = 0$ $\therefore e^x = 0$ Which is not possible $x \in \mathbb{R}$, so there are no stationary points.
Question 17 E $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + 1} dx$ Let $u = \sin x + 1$ $du = \cos x dx$ When $x = 0, u = 1$ When $x = \frac{\pi}{2}, u = 2$ $\int_1^2 \frac{du}{u} = [\log_e u]_1^2$ $= \log_e 2 - \log_e 1$ $= \log_e 2$ $= 0.693$	Question 18 C $f(x) = \int -7e^{-2x} dx$ $f(x) = \frac{7}{2} e^{-2x} + c$ When $x = 0$ $14 = \frac{7}{2} + c$ $c = 10.5$ Hence, $f(x) = 3.5e^{-2x} + 10.5$ $f(x) = Ae^{-2x} + B$ $A = 3.5, B = 10.5$
Question 19 B Reading the graph from left to right, $f(x)$ has a negative gradient, then a positive gradient, then a negative gradient. Hence A or B. When $x = 0$, the gradient is positive so not A.	Question 20 B $y = \frac{2}{x}$ $\int_1^3 y dx = 2 \int_1^3 \frac{1}{x} dx = 2[\log_e x]_1^3$ $= 2[\log_e 3 - \log_e 1]$ $= 2 \log_e 3$ $= 2.2$ which is closest to 2

Question 21 D

$$\mu = np$$

$$6 = 10 \times p$$

$$0.6 = p$$

$$\Pr(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

$$\begin{aligned} \Pr(X = 8) &= \binom{10}{8} (0.6)^8 (0.4)^2 \\ &= 0.121 \end{aligned}$$

Question 22 E

$$\sum \Pr(X = x) = 1$$

$$\frac{6 - 4a}{8} + \frac{3 + 2a}{8} + \frac{3 - 2a}{8} = 1$$

$$\frac{12 - 4a}{8} = 1$$

$$12 - 4a = 8$$

$$4 = 4a$$

$$1 = a$$

Question 23 D

Binomial with $n = 4$, $p = 0.7$, $x = 3$ or 4

$$\Pr(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

$$\Pr(X = 3) + \Pr(X = 4)$$

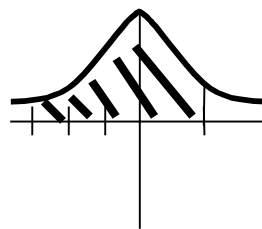
$$= \binom{4}{3} (0.7)^3 (0.3)^1 + \binom{4}{4} (0.7)^4 (0.3)^0$$

$$= 0.4116 + 0.2401$$

$$= 0.6517$$

which is closest to 0.7

Question 24 C



$$\Pr X < Z = 0.7$$

$$Z = 0.524$$

$$Z = \frac{a - \mu}{\sigma}$$

$$0.524 = \frac{a - 2}{2.5}$$

$$a - 2 = 1.31$$

$$a = 3.31$$

Question 25 E

$$\text{Total Outcomes} = \binom{52}{13}$$

$$\text{Favourable Outcomes} = \binom{4}{1} \times \binom{48}{12}$$

$$\begin{aligned} \text{Pr.} &= \frac{\binom{4}{1} \times \binom{48}{12}}{\binom{52}{13}} \\ &= 0.44 \end{aligned}$$

Question 26 A

$$f'(x) = \frac{(x^2 - 1) \times 1 - x(2x)}{(x^2 - 1)^2}$$

$$f'(x) = \frac{x^2 - 1 - 2x^2}{(x^2 - 1)^2}$$

$$f'(x) = \frac{-(x^2 + 1)}{(x^2 - 1)^2}$$

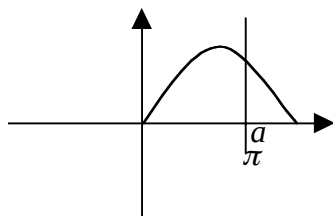
For $x \in \mathbb{R} / \{-1, 1\}$ the denominator is always positive and the numerator is always negative

$$\therefore f'(x) < 0$$

$$f'(x) \neq 0 \text{ because } x^2 \neq -1$$

$$f(x) \text{ can equal } 0 \text{ when } x = 0$$

Question 27 C



$$\int_0^a 2 \sin x dx = \frac{1}{3} \int_a^\pi 2 \sin x dx$$

$$-2 \cos x \Big|_0^a = \frac{1}{3} [-2 \cos x]_a^\pi$$

$$-6[\cos a - \cos 0] = -2[\cos \pi - \cos a]$$

$$3[\cos a - 1] = [-1 - \cos a]$$

$$3 \cos a - 3 = -1 - \cos a$$

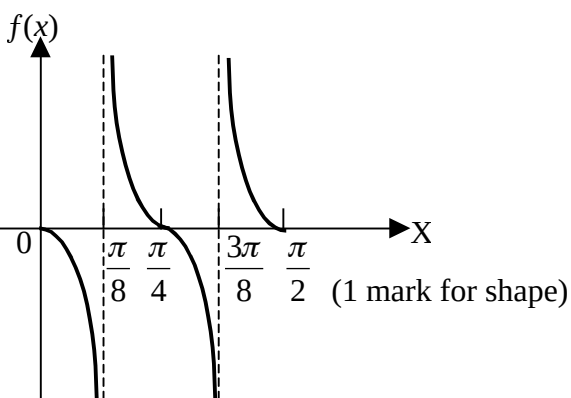
$$4 \cos a = 2$$

$$\cos a = \frac{1}{2}$$

$$a = \frac{\pi}{3}$$

Question 1

a.



(1 mark for asymptote)

b. R (1 mark)

c. Reflection in the X axis (1 mark)

Question 3

a.

$$y = 2x^2 - 2x + 3$$

$$\frac{dy}{dx} = 4x - 2 \quad (1 \text{ mark})$$

When $x = 1$

$$\frac{dy}{dx} = 4 - 2 = 2 \quad (1 \text{ mark})$$

b.

$$\tan \theta = 2$$

$$\theta = \tan^{-1} 2$$

$$\theta = 63.4^\circ$$

$$\theta = 63^\circ \text{ to the nearest degree} \quad (1 \text{ mark})$$

This is the acute angle

$$\text{Obtuse angle} = 180 - 63 = 117^\circ \quad (1 \text{ mark})$$

Question 2

a.

$$\begin{aligned} & (2\sqrt{x})^3 + 3 \times (2\sqrt{x})^2 + 3 \times (2\sqrt{x}) + 1 \quad (1 \text{ mark}) \\ & = 8x^{\frac{3}{2}} + 12x + 6x^{\frac{1}{2}} + 1 \quad (1 \text{ mark}) \end{aligned}$$

b.

$$\begin{aligned} & \frac{8x^{\frac{3}{2}} + 12x + 6x^{\frac{1}{2}} + 1}{2\sqrt{x}} \\ & = 4x + 6\sqrt{x} + 3 + \frac{1}{2\sqrt{x}} \quad (1 \text{ mark}) \end{aligned}$$

Question 4

a.

$$2x - 2 \geq 0$$

$$2x \geq 2$$

$$x \geq 1$$

$$\text{Domain: } x \geq 1$$

$$\text{Range: } f(x) \geq 0 \quad (1 \text{ mark})$$

b.

Interchange x and y

$$x = \sqrt{2y - 2}$$

$$x^2 = 2y - 2 \quad (1 \text{ mark})$$

$$x^2 + 2 = 2y$$

$$\frac{1}{2}x^2 + 1 = y$$

$$f^{-1}(x) = \frac{1}{2}x^2 + 1 \quad (1 \text{ mark})$$

c.

$$\text{Domain: } x \geq 0$$

$$\text{Range: } f^{-1}(x) \geq 1 \quad (1 \text{ mark})$$

Question 5 Probability $= \frac{\binom{5}{1}\binom{6}{1}\binom{9}{1}}{\binom{20}{3}} \quad (1 \text{ mark})$ $= \frac{5 \times 6 \times 9}{1140}$ $= 0.237 \quad (1 \text{ mark})$	Question 6 $g'(x) = \cos \frac{5x}{2} - 2 \sin 3x$ $g(x) = \int (\cos \frac{5x}{2} - 2 \sin 3x) dx \quad (1 \text{ mark})$ $g(x) = \frac{2}{5} \sin \frac{5x}{2} + \frac{2}{3} \cos 3x + c$ $g(0) = \frac{2}{5} \times 0 + \frac{2}{3} \times 1 + c = 1$ $g(0) = 0 + \frac{2}{3} + c = 1$ $\therefore c = \frac{1}{3} \quad (1 \text{ mark})$ $g(x) = \frac{2}{5} \sin \frac{5x}{2} + \frac{2}{3} \cos 3x + \frac{1}{3} \quad (1 \text{ mark})$
Question 7 a. $3e^{-\frac{x}{2}} \cos(2x) = 0$ $3e^{-\frac{x}{2}} \neq 0$ $\therefore \cos(2x) = 0 \quad 0 \leq 2x \leq 4\pi \quad (1 \text{ mark})$ $2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \quad (1 \text{ mark})$	b. $y = e^{\frac{1}{x}}$ Let $u = \frac{1}{x}$ $y = e^u$ $\frac{dy}{du} = e^u$ $\frac{du}{dx} = -\frac{1}{x^2}$ $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$ $\frac{dy}{dx} = e^u \times \left(-\frac{1}{x^2}\right) \quad (1 \text{ mark})$ $\frac{dy}{dx} = -\frac{1}{x^2} e^{\frac{1}{x}}$ But $e^{\frac{1}{x}}$ is always positive, and $\frac{1}{x^2}$ is always positive So $\frac{dy}{dx}$ is always negative and therefore decreasing. However $x \neq 0$ since x is in the denominator. (1 mark)

END OF SUGGESTED SOLUTIONS
2002 Mathematical Methods Trial Examination 1

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