

Mathematical Methods Examination 2

Solutions

Question 1.

a.

Initial population = 500
 $\Rightarrow A = 500$

1A

b.

$$P = 500e^{kt}$$

When $t = 5$, $P = 746$

$$\Rightarrow 746 = 500e^{5k}$$

1M

$$\Rightarrow e^{5k} = \frac{746}{500}$$

$$\Rightarrow 5k = \log_e \frac{746}{500}$$

1M

$$\Rightarrow k = \frac{1}{5} \log_e \frac{746}{500}$$

$$\approx 0.08$$

c.

$$P(t) = 500 e^{0.08t}$$

$$t \in [0, 8]$$

1A

1A

d.

$$P(8) = 500 e^{0.64}$$

$$\approx 948 \text{ kangaroos}$$

1A

e.

$$K(t) = 948 (0.9)^{(t-8)}$$

$$t \in [8, \infty)$$

1A

1A

f. i.

$$\text{When } t = 11, K(t) = 948 (0.9)^3$$

$$\approx 691.09$$

$\therefore$ 692 kangaroos (round up)

1A

ii.

$$500 = 948(.9)^{(t-8)}$$

$$\log_e \frac{500}{948} = (t-8) \log_e (.9)$$

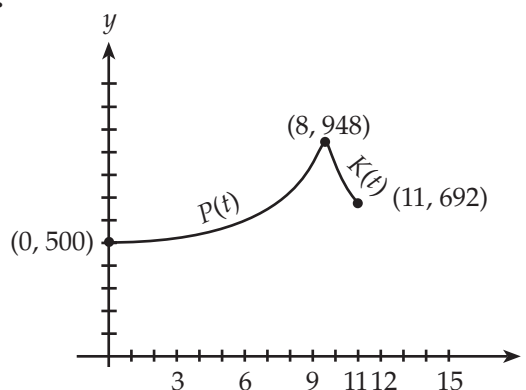
$$t-8 \approx 6.07$$

$$t \approx 14.07$$

$\therefore$ during 2004

1A

g.



Shape 1
 Labels 1
 End pts 1
(14 marks)

Question 2. a.

$$\Pr(X \leq k) = 0.9, \mu = 30, \sigma = 7$$

inv Norm (0.9, 30, 7)

$$k \approx 38.97$$

$$\approx 39$$

1A

b.

$$\Pr(X = 2)$$

$$= {}^{15}C_2 (0.1)^2 (0.9)^{13} \text{ where } n = 2, p = 0.1$$

$$\approx 0.2669$$

$$\approx 0.27$$

1A

c. i. $\Pr(X > 35)$

$$\approx 0.2375$$

norm cdf (35, 1E99, 30, 7)

1A

ii. $\Pr((X \geq 39) | (X > 35))$

$$\approx \frac{0.1}{0.2375}$$

$$\approx 0.42105$$

$$\approx 0.42$$

1M

1A

d. $E(X) = \frac{nD}{N}$

$$= \frac{6 \times 13}{52}$$

$$= 1.5$$

1A

e. i.

Probability of a win

$$= \frac{\binom{13}{5}\binom{39}{1}}{\binom{52}{6}} + \frac{\binom{13}{6}\binom{39}{0}}{\binom{52}{6}} \quad \mathbf{1M}$$

$$\approx 0.0025497$$

$$\approx 0.0025 \quad \mathbf{1A}$$

ii.

$$\Pr(X \geq 1)$$

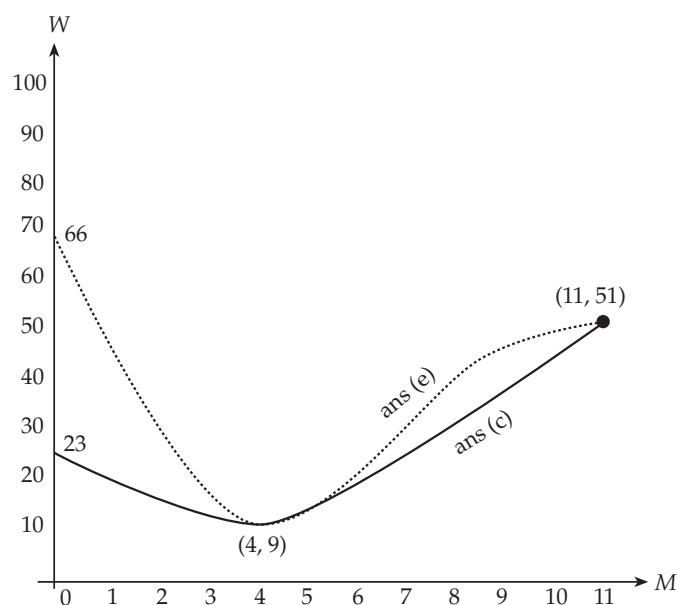
$$= 1 - \Pr(X = 0)$$

$$= 1 - \binom{n}{0} (0.002549)^0 (0.997451)^n = 0.0127 \quad \mathbf{1M}$$

$$0.997451^n = 0.9873$$

$$n = \frac{\log 0.9873}{\log 0.997451} \quad \mathbf{1M}$$

$$= 5 \quad \mathbf{1A}$$

(12 marks)**Question 3****a.**Mark and label (4, 9) & (11, 51) **1 A****b.**

Turning point at (4, 9)

$$\Rightarrow W = K(M - 4)^2 + 9$$

Find K using point (11, 51)

$$\Rightarrow 51 = K(7)^2 + 9$$

$$\Rightarrow 42 = 49K$$

$$\Rightarrow K = \frac{6}{7} \quad \mathbf{1A}$$

$$W = \frac{6}{7}(M - 4)^2 + 9 \quad \text{(or expanded version)} \quad \mathbf{1A}$$

$$M \in [0, 11] \quad \mathbf{1A}$$

c.

$$\text{Intercept } (0, 22\frac{5}{7}) \rightarrow (0, 23) \quad \mathbf{1A}$$

Shape and points **1A****d. i.**S.P. when $M = 4, 11$

$$\Rightarrow W'(M) = K(M - 4)(M - 11) \quad \mathbf{1A}$$

ii.

$$W'(M) = K(M^2 - 15M + 44)$$

$$W(M) = K\left(\frac{M^3}{3} - \frac{15M^2}{2} + 44M\right) + C \quad \mathbf{1M}$$

Substitute (4, 9) and (11, 51)

$$\Rightarrow 9 = \frac{232}{3}K + C \quad \dots(1)$$

$$51 = \frac{121}{6}K + C \quad \dots(2) \quad \mathbf{1M}$$

$$\text{Subtract } -42 = \frac{343}{6}K$$

$$\Rightarrow K = \frac{-36}{49}; \quad C = \frac{3225}{49} \quad (\approx 66) \quad \mathbf{1A}$$

$$W(M) = \frac{-36}{49}\left(\frac{M^3}{3} - \frac{15M^2}{2} + 44M\right) + \frac{3225}{49}$$

$$M \in [0, 11] \quad \mathbf{1A}$$

e.

$$\text{Intercept } \approx (0, 66) \quad \mathbf{1A}$$

Shape **1A****f. i.**Do not take cyclic nature into account. **1A****ii.**A sin/cos model $\therefore$ have cyclic nature. **1A****(15 marks)**

Question 4.

a.

$$\text{Period} = \frac{2\pi}{n} = 12 \text{ hours}$$

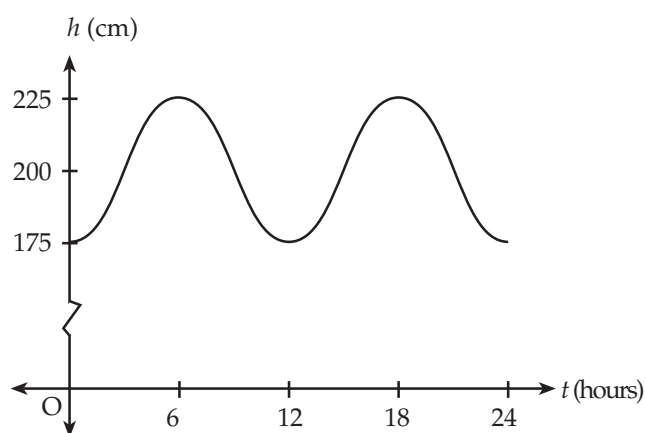
1M

$$n = \frac{2\pi}{12} = \frac{\pi}{6}$$

b.

$$b = \frac{400}{2} = 200 \text{ cm}$$

1A



$$h = 25 \sin \frac{\pi}{6}(t-3) + 200$$

$$= 25 \sin \left(\frac{\pi}{6}t - \frac{\pi}{2} \right) + 200$$

$$e = -\frac{\pi}{2}$$

1A

c.

$$A = -25$$

1A

$$n = \frac{\pi}{6}$$

1A

d.

$$h = -25 \cos \left(\frac{\pi}{6}t \right) + 200$$

$$\frac{dh}{dt} = \frac{25\pi}{6} \sin \left(\frac{\pi}{6}t \right)$$

1A

Maximum value of $\frac{dh}{dt}$ occurs when

$$\sin \left(\frac{\pi}{6}t \right) = 1, \text{ hence maximum rate of change}$$

1M

is $\frac{25\pi}{6}$ cm per hour.

1A

e.

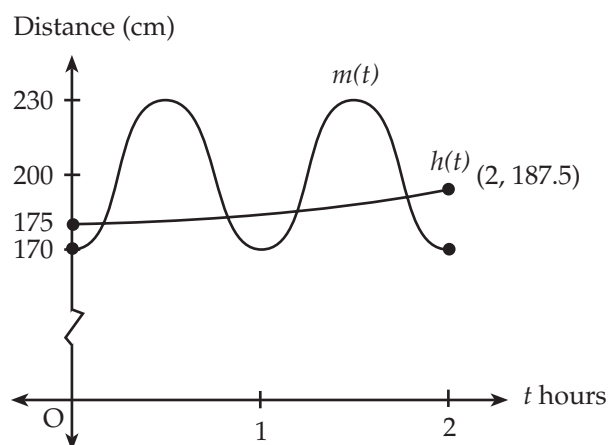
$$\text{Period} = \frac{2\pi}{n} = 1 \text{ hour}$$

$$n = 2\pi$$

$$m = 30 \sin \left(2\pi t - \frac{\pi}{2} \right) + 200$$

2A

f.



shape 1
end points 1

2A

g.

2 times per hour

$$= 2 \times 24 \text{ times per day}$$

$$= 48 \text{ times per day}$$

1A

h.

Using graphics calculator

$$t \approx 0.09350184 \text{ hours}$$

$$= 0.09350184 \times 60 \text{ minutes}$$

$$\approx 5.61 \text{ minutes}$$

$$\approx 6 \text{ minutes after noon or 12:06 p.m.}$$

1A

(14 marks)