

# Victorian Certificate of Education 2000

## MATHEMATICAL METHODS

### Written examination 1 (Facts, skills and applications)

**Friday 3 November 2000: 9.00 am to 10.45 am**

**Reading time: 9.00 am to 9.15 am**

**Writing time: 9.15 am to 10.45 am**

**Total writing time: 1 hour 30 minutes**

### PART I

### MULTIPLE-CHOICE QUESTION BOOK

#### Directions to students

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions). Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.

Part II consists of a separate question and answer book.

You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

A detachable formula sheet for use in both parts is in the centrefold of this book.

#### **At the end of the task**

Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).

You may retain this question book.

## Structure of book

| <i>Number of questions</i> | <i>Number of questions to be answered</i> | <i>Number of marks</i> |
|----------------------------|-------------------------------------------|------------------------|
| 27                         | 27                                        | 27                     |

## Directions to students

### Materials

Question book of 14 pages, including one blank page for rough working.

Answer sheet for multiple-choice questions.

You may bring to the examination up to four pages (two A4 sheets) of pre-written notes.

You may use an approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve-sketching.

You should have at least one pencil and an eraser.

### The task

Detach the formula sheet from the centre of this book during reading time.

Please ensure that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.

Answer **all** questions.

There is a total of 27 marks available for Part I.

All questions should be answered on the answer sheet provided for multiple-choice questions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

### At the end of the task

Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).

You may retain this question book.

### Specific instructions to students

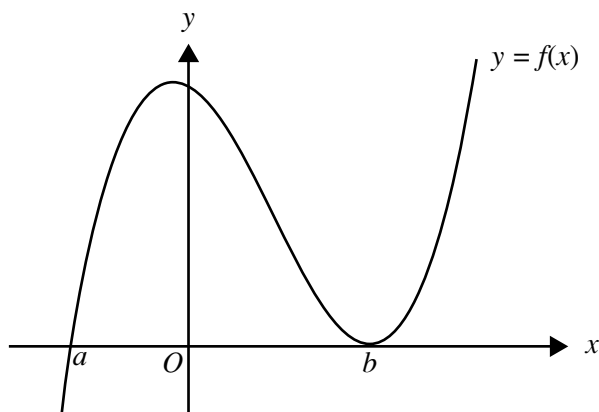
This part consists of 27 questions.

Answer **all** questions in this part on the answer sheet provided for multiple-choice questions. A correct answer scores 1, an incorrect answer scores 0.

Marks will not be deducted for incorrect answers. You should attempt every question.

No mark will be given if more than one answer is completed for any question.

#### Question 1



The graph shown could be that of a function  $f$  whose rule is

- A.  $f(x) = (x - a)(x - b)^2$
- B.  $f(x) = (x + a)(x - b)^2$
- C.  $f(x) = (x - a)(x + b)^2$
- D.  $f(x) = (x + a)(x + b)^2$
- E.  $f(x) = (x - a)^2(x - b)$

#### Question 2

The graph whose equation is  $y = \sqrt{x}$  is reflected in the  $x$ -axis and then translated 2 units to the right and 1 unit down. The equation of the new graph is

- A.  $y = \sqrt{(x - 2)} + 1$
- B.  $y = -\sqrt{(x - 2)} - 1$
- C.  $y = -\sqrt{(x + 2)} - 1$
- D.  $y = -\sqrt{(x - 2)} + 1$
- E.  $y = \sqrt{(x - 1)} + 2$

**Question 3**

The equations of the vertical and horizontal asymptotes of the graph whose equation is  $y = \frac{2}{x-4} + 3$  are, respectively,

- A.  $x = -4$ ,  $y = 3$
- B.  $x = 2$ ,  $y = 3$
- C.  $x = 3$ ,  $y = 4$
- D.  $x = 4$ ,  $y = -3$
- E.  $x = 4$ ,  $y = 3$

**Question 4**

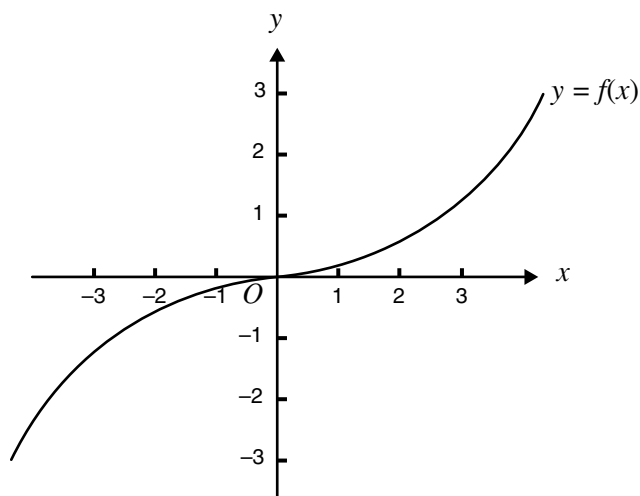
| $x$ | $y$ |
|-----|-----|
| 1   | 1.7 |
| 2   | 3.2 |
| 3   | 1.5 |
| 4   | 0.5 |
| 5   | 1.2 |
| 6   | 2.6 |
| 7   | 3.4 |
| 8   | 2.3 |

The data in the above table would be best modelled using

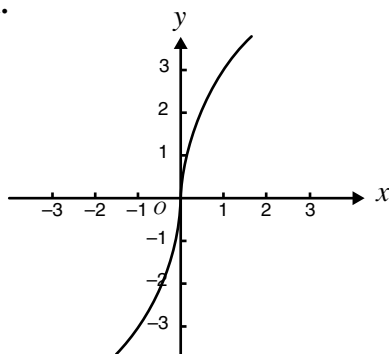
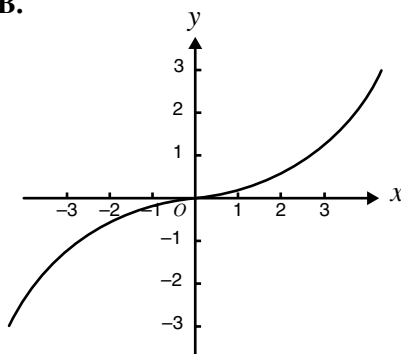
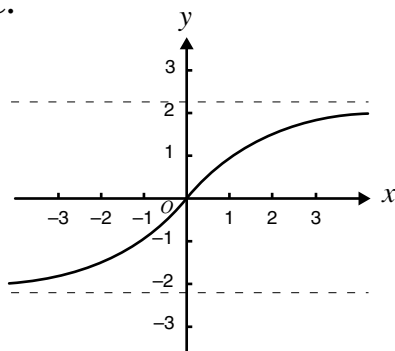
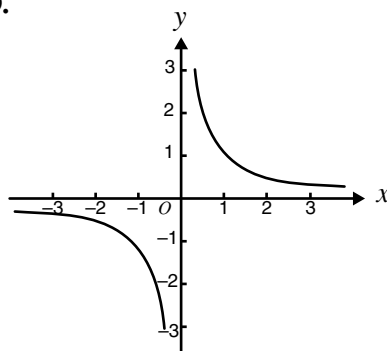
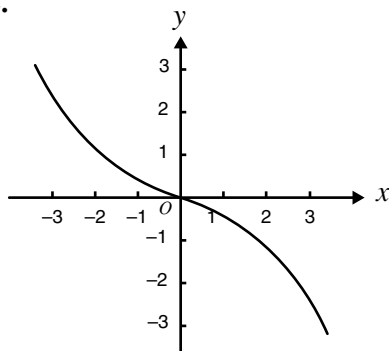
- A. a linear function.
- B. a power function.
- C. an exponential function.
- D. a circular function.
- E. a logarithmic function.

**Question 5**

The graph of the function with equation  $y = f(x)$  is shown below.



Which one of the following is most likely to be the graph of the inverse function?

**A.****B.****C.****D.****E.****TURN OVER**

**Question 6**

Let  $f: D_1 \rightarrow \mathbb{R}, f(x) = \frac{1}{x+2}$  where  $D_1$  is the largest domain for which  $f$  is defined.

Let  $g: D_2 \rightarrow \mathbb{R}, g(x) = e^{2x}$  where  $D_2$  is the largest domain for which  $g$  is defined.

Let  $h: D_3 \rightarrow \mathbb{R}, h(x) = \frac{1}{x+2} - e^{2x}$  where  $D_3$  is the largest domain for which  $h$  is defined.

Which one of the following is true?

- A.  $D_1 = D_3$  and  $\text{Range}(f) = \text{Range}(h)$
- B.  $D_1 \neq D_3$  and  $\text{Range}(f) = \text{Range}(h)$
- C.  $D_2 = D_3$  and  $\text{Range}(g) = \text{Range}(h)$
- D.  $D_1 = D_3$  and  $\text{Range}(g) \neq \text{Range}(h)$
- E.  $D_1 \neq D_3$  and  $\text{Range}(g) = \text{Range}(h)$

**Question 7**

Which one of the following functions does **not** have an inverse function?

- A.  $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2x - 5$
- B.  $g: [0, \infty) \rightarrow \mathbb{R}, g(x) = x^2$
- C.  $h: \mathbb{R} \rightarrow \mathbb{R}, h(x) = x^3$
- D.  $k: [-2, 2] \rightarrow \mathbb{R}, k(x) = \sqrt{(4 - x^2)}$
- E.  $m: \mathbb{R}^+ \rightarrow \mathbb{R}, m(x) = 2 - \frac{3}{x}$

**Question 8**

The first six rows of Pascal's triangle are shown below.

|   |   |    |    |    |    |   |   |   |
|---|---|----|----|----|----|---|---|---|
|   |   |    |    | 1  |    |   |   |   |
|   |   |    | 1  |    | 1  |   |   |   |
|   |   | 1  |    | 2  |    | 1 |   |   |
|   | 1 |    | 3  |    | 3  |   | 1 |   |
|   | 1 | 4  |    | 6  |    | 4 |   | 1 |
|   | 1 | 5  | 10 |    | 10 | 5 |   | 1 |
| 1 | 6 | 15 | 20 | 15 | 6  |   | 1 |   |

When  $(x + a)^7$  is expanded into a polynomial in decreasing powers of  $x$ , from left to right, the fifth term is

- A.  $21x^2a^5$
- B.  $15x^3a^4$
- C.  $35x^3a^4$
- D.  $35$
- E.  $35x^4a^3$

**Question 9**

If  $x = 4$  is a solution of the equation  $\log_e(ax + 2) = 3$ , then the exact value of  $a$  is

- A.  $\frac{\log_e 3 - 2}{4}$
- B.  $\frac{e}{4}$
- C.  $\frac{e^3}{4} - 2$
- D. 4.5210
- E.  $\frac{e^3 - 2}{4}$

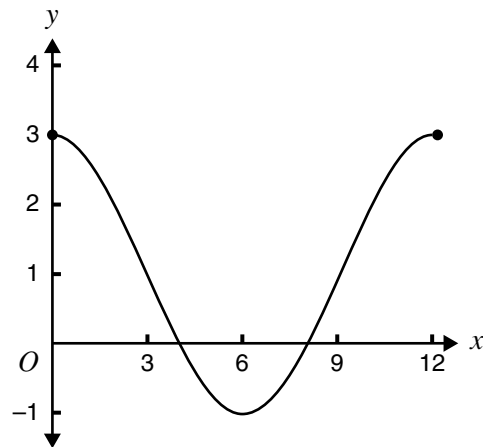
**Question 10**

$e^{(2 \log_e x - \log_e 2x)}$  is equal to

- A.  $2 \log_e \left( \frac{1}{2} \right)$
- B. 0
- C. 1
- D.  $x^2 - 2x$
- E.  $\frac{x}{2}$

**Question 11**

The diagram below shows one cycle of the graph of a circular function.



The amplitude, period and range of the function are, respectively,

|    | amplitude | period          | range     |
|----|-----------|-----------------|-----------|
| A. | 2         | $\frac{\pi}{6}$ | $[0, 12]$ |
| B. | 2         | 12              | $[-1, 3]$ |
| C. | 3         | 12              | $[0, 12]$ |
| D. | 4         | $\frac{\pi}{6}$ | $[0, 12]$ |
| E. | 4         | 12              | $[-1, 3]$ |

**Question 12**

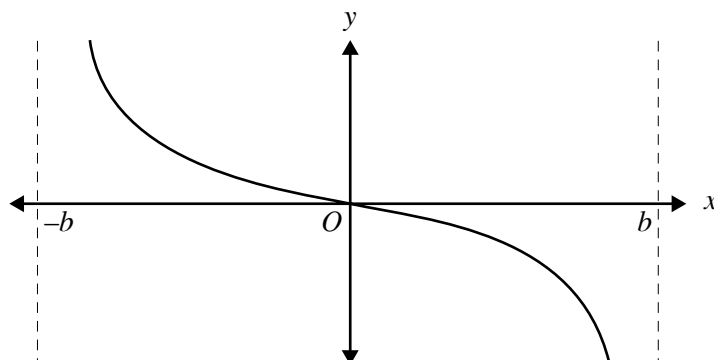
For the equation  $2\sin 3x = 1$ , the **sum** of the solutions in the interval  $[0, \pi]$  is equal to

- A.  $\frac{\pi}{3}$
- B.  $2\pi$
- C.  $5\pi$
- D.  $6\pi$
- E.  $15\pi$

**Question 13**

The diagram shows one cycle of the graph with equation  $y = \tan ax$ .

Vertical asymptotes have equations  $x = b$  and  $x = -b$ .



Possible values of  $a$  and  $b$  are

- |    | $a$            | $b$              |
|----|----------------|------------------|
| A. | $-3$           | $\frac{\pi}{6}$  |
| B. | $-3$           | $\frac{2\pi}{3}$ |
| C. | $-\frac{1}{3}$ | $\frac{\pi}{6}$  |
| D. | $-\frac{1}{3}$ | $\frac{2\pi}{3}$ |
| E. | $3$            | $\frac{\pi}{6}$  |

**Question 14**

Using the approximation formula,  $f(x+h) \approx f(x) + hf'(x)$  where  $f(x) = \sqrt{x}$  with  $x = 16$ , an approximate value of  $\sqrt{15.96}$  is given by

- A.  $f(4) + 0.04f'(4)$
- B.  $f(16) + 0.04f'(16)$
- C.  $f(16)$
- D.  $f(4) - 0.04f'(4)$
- E.  $f(16) - 0.04f'(16)$

**Question 15**

For the curve with equation  $y = -x^3 - x^2 + 2x + 2$ , the subset of  $R$  for which the gradient of the curve is positive is closest to

- A.  $(-\infty, -1.215)$
- B.  $(-1.215, 0.548)$
- C.  $(0.548, \infty)$
- D.  $(-1.000, 1.414)$
- E.  $(2.000, \infty)$

**TURN OVER**

**Question 16**

Rainwater is being collected in a water tank. The volume,  $V \text{ m}^3$ , of water in the tank after time,  $t$  hours, is given by  $V = 2t^2 - 3t + 2$ . The average rate of change of volume over the first ten hours in  $\text{m}^3$  per hour is

- A. 10
- B. 17
- C. 19
- D. 37
- E. 172

**Question 17**

If  $f(x) = e^{-x}(x^3 - 4)$  then  $f'(x)$  is

- A.  $e^{-x}(3x^2 - 4)$
- B.  $e^{-x}(x^3 + 3x^2 - 4)$
- C.  $e^{-x}(-x^3 + 3x^2 - 4)$
- D.  $e^{-x}(-x^3 + 3x^2 + 4)$
- E.  $-3x^2 e^{-x}$

**Question 18**

If  $y = \frac{\tan x}{x}$  then  $\frac{dy}{dx}$  is

- A.  $\frac{\frac{x}{\cos^2 x} - \tan x}{x^2}$
- B.  $\frac{1}{\cos^2 x}$
- C.  $\frac{\tan x - \frac{x}{\cos^2 x}}{x^2}$
- D.  $\frac{x}{\cos^2 x} + \tan x$
- E.  $\frac{\frac{x}{\cos^2 x} - \tan x}{x}$

**Question 19**

$\int_1^4 (2f(x) + 1)dx$  can be written as

- A.  $2\int_1^4 (f(x) + 1)dx$
- B.  $2\int_1^4 f(x)dx + 1$
- C.  $\int_1^4 2f(x)dx$
- D.  $2\int_1^4 f(x)dx + 3$
- E.  $2\int_1^4 f(x)dx + x$

**Question 20**

Using the left rectangle approximation with rectangles of width 1, the area of the region bounded by the  $x$ -axis,  $y$ -axis, the line  $x = 3$  and by the curve whose equation is  $y = e^x$  is approximated by

- A.  $1 + e + e^2$
- B.  $1 + e + e^2 + e^3$
- C.  $e + e^2 + e^3$
- D.  $e^3 - e$
- E.  $\frac{\frac{1}{2} + e + e^2 + e^3}{2}$

**Question 21**

An antiderivative of  $\frac{1}{3x} + \sin 2x$ ,  $x > 0$ , is

- A.  $\frac{-1}{3x^2} + 2 \cos 2x$
- B.  $\frac{1}{3} \log_e 3x - 2 \cos 2x$
- C.  $\frac{1}{3} \log_e x - \frac{1}{2} \cos 2x$
- D.  $\log_e 3x + 2 \cos 2x$
- E.  $\frac{-1}{3x^2} - \frac{1}{2} \cos 2x$

**Question 22**

If  $\frac{dy}{dx} = \frac{2}{(4x+1)^{\frac{3}{2}}}$  and  $c$  is a real constant, then  $y$  is

- A.  $\frac{1}{(4x+1)^{\frac{1}{2}}} + c$
- B.  $\frac{1}{5(4x+1)^{\frac{5}{2}}} + c$
- C.  $\frac{-1}{2(4x+1)^{\frac{1}{2}}} + c$
- D.  $\frac{-1}{10(4x+1)^{\frac{5}{2}}} + c$
- E.  $\frac{-1}{(4x+1)^{\frac{1}{2}}} + c$

**Question 23**

The probability distribution for the discrete random variable  $X$  is given by

|              |     |      |      |      |
|--------------|-----|------|------|------|
| $x$          | 0   | 1    | 2    | 3    |
| $\Pr(X = x)$ | $k$ | $2k$ | $4k$ | $8k$ |

The value of  $k$  is

- A.  $\frac{1}{35}$
- B.  $\frac{1}{34}$
- C.  $\frac{1}{15}$
- D.  $\frac{1}{4}$
- E. 15

**Question 24**

The number,  $X$ , of cars waiting in the right-hand turn lane at a set of traffic lights as the lights change has the following probability distribution.

|              |     |     |     |     |     |
|--------------|-----|-----|-----|-----|-----|
| $x$          | 0   | 1   | 2   | 3   | 4   |
| $\Pr(X = x)$ | 0.2 | 0.2 | 0.3 | 0.2 | 0.1 |

The variance of  $X$ , correct to two decimal places, is

- A. 1.25
- B. 1.56
- C. 1.80
- D. 2.19
- E. 4.80

**Question 25**

Vandals enter a factory where computer chips are manufactured and mix 24 normal chips with 12 faulty chips in a box. The proprietor discovers the mixed box and selects a sample of  $r$  chips for testing, where  $r > 3$ .

The probability that she selects exactly 3 faulty chips is

- A.  ${}^r C_3 \left(\frac{1}{3}\right)^{r-3} \left(\frac{2}{3}\right)^3$
- B.  ${}^r C_3 \left(\frac{2}{3}\right)^{r-3} \left(\frac{1}{3}\right)^3$
- C.  $\left(\frac{1}{3}\right)^3$
- D.  $\frac{{}^{24} C_3 \times {}^{12} C_{r-3}}{{}^{36} C_r}$
- E.  $\frac{{}^{12} C_3 \times {}^{24} C_{r-3}}{{}^{36} C_r}$

**Question 26**

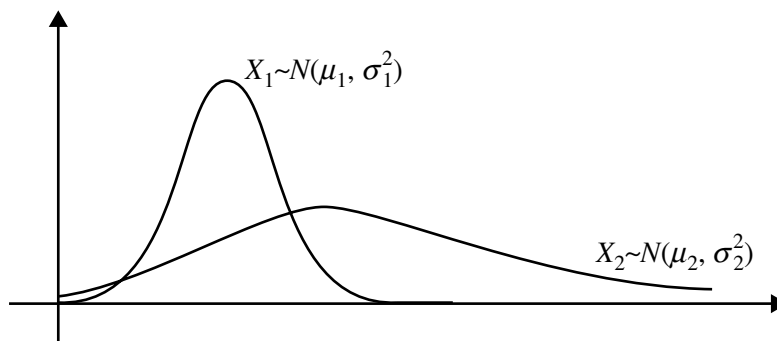
Andrea throws a netball towards a goal ring. If the ball passes through the ring, she scores a goal. Andrea knows that on average she scores a goal 17 times out of every 20 throws. The result of each throw is independent of the previous throw.

If Andrea were to throw the netball 10 times towards a goal ring, the probability of obtaining more than 8 goals is

- A.  ${}^{10}C_9(0.15)^1(0.85)^9$
- B.  ${}^{10}C_9(0.15)^1(0.85)^9 + (0.85)^{10}$
- C.  ${}^{10}C_8(0.15)^2(0.85)^8 + {}^{10}C_9(0.15)^1(0.85)^9 + (0.85)^{10}$
- D.  $(0.85)^{10}$
- E.  $\frac{{}^{17}C_8 \times {}^3C_2}{{}^{20}C_{10}}$

**Question 27**

The diagram below shows two normal distribution curves with means  $\mu_1$  and  $\mu_2$  and variances  $\sigma_1^2$  and  $\sigma_2^2$  respectively.



Which one of the following sets of statements is true?

- A.  $\mu_1 < \mu_2$  and  $\sigma_1 < \sigma_2$
- B.  $\mu_1 > \mu_2$  and  $\sigma_1 < \sigma_2$
- C.  $\mu_1 < \mu_2$  and  $\sigma_1 > \sigma_2$
- D.  $\mu_1 > \mu_2$  and  $\sigma_1 > \sigma_2$
- E.  $\mu_1 > \mu_2$  and  $\sigma_1 = \sigma_2$

Working space



SUPERVISOR TO ATTACH PROCESSING LABEL HERE

**STUDENT NUMBER**

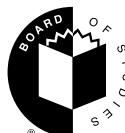
**Letter**

**Figures**

**Words**

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## **Victorian Certificate of Education 2000**

# **MATHEMATICAL METHODS**

## **Written examination 1 (Facts, skills and applications)**

**Friday 3 November 2000: 9.00 am to 10.45 am**

**Reading time: 9.00 am to 9.15 am**

**Writing time: 9.15 am to 10.45 am**

**Total writing time: 1 hour 30 minutes**

### **PART II**

## **QUESTION AND ANSWER BOOK**

### **Directions to students**

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions). Part I consists of a separate question book and must be answered on the answer sheet provided for multiple-choice questions.

Part II consists of this question and answer book.

You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

A detachable formula sheet for use in both parts is in the centrefold of the Part I question book.

#### **At the end of the task**

Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II).

## Structure of book

| <i>Number of questions</i> | <i>Number of questions to be answered</i> | <i>Number of marks</i> |
|----------------------------|-------------------------------------------|------------------------|
| 8                          | 8                                         | 23                     |

## Directions to students

### Materials

Question and answer book of 8 pages, including two blank pages for rough working.

You may bring to the examination up to four pages (two A4 sheets) of pre-written notes.

You may use an approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve-sketching.

### The task

Detach the formula sheet from the centre of the Part I book during reading time.

Ensure that you write your **student number** in the space provided on the cover of this book.

The marks allotted to each question are indicated at the end of the question.

There is a total of 23 marks available for Part II.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example,  $\pi$ ,  $e$ , surds or fractions. A decimal approximation will not be accepted if an exact answer is required to a question.

Where an exact answer is required to a question, appropriate working must be shown and calculus must be used to evaluate derivatives and definite integrals.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

All written responses should be in English.

### At the end of the task

Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II).

Working space

**TURN OVER**

**Specific instructions to students**

Answer **all** questions in this part in the spaces provided.

**Question 1**

The resistance, in ohm, of an electrical component for an appliance is required to lie within the range  $(50 \pm 5)$  ohm. If this condition is not satisfied, the component is rejected as unsatisfactory.

The components are mass-produced with resistance being approximately normally distributed with a mean of 50 ohm and a standard deviation of 3 ohm.

Find the probability, correct to three decimal places, that a randomly selected component is unsatisfactory.

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2 marks

**Question 2**

A jar contains fifteen jellybeans, of which twelve are green. Four jellybeans are taken from the jar at random and eaten. If  $X$  is the number of green jellybeans taken from the jar and eaten, calculate  $\Pr(X \geq 3)$  correct to three decimal places.

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3 marks

**Question 3**

Jack drew the graph of the function

$$f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = (x-1)(x-3)(x+2) + 4$$

by adding the ordinates of the graphs of two functions.

- a. Write down the rules of two functions that Jack could have used.

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- b. Find the exact values of all the roots of the equation  $f(x) = 0$ .

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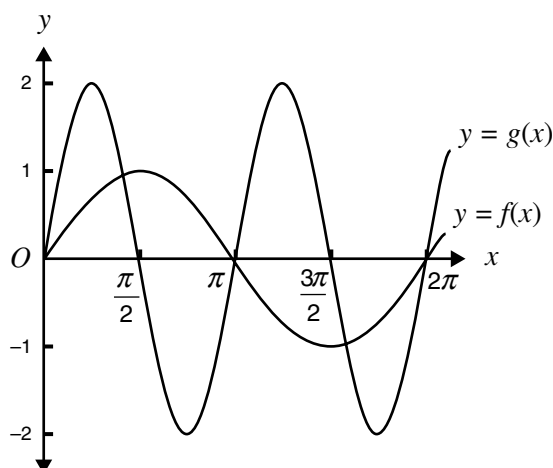


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1 + 3 = 4 marks

**Question 4**

The diagram below shows the graphs of two circular functions,  $f$  and  $g$ .



State the type of transformation, together with any relevant scale factors, distances or directions, required to transform the graph whose equation is  $y = f(x)$  into the graph whose equation is  $y = g(x)$ .

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2 marks

**TURN OVER**

**Question 5**

Find the exact solutions of the equation  $\sin 2x = \sqrt{3} \cos 2x$ ,  $-\pi \leq x \leq \pi$ .

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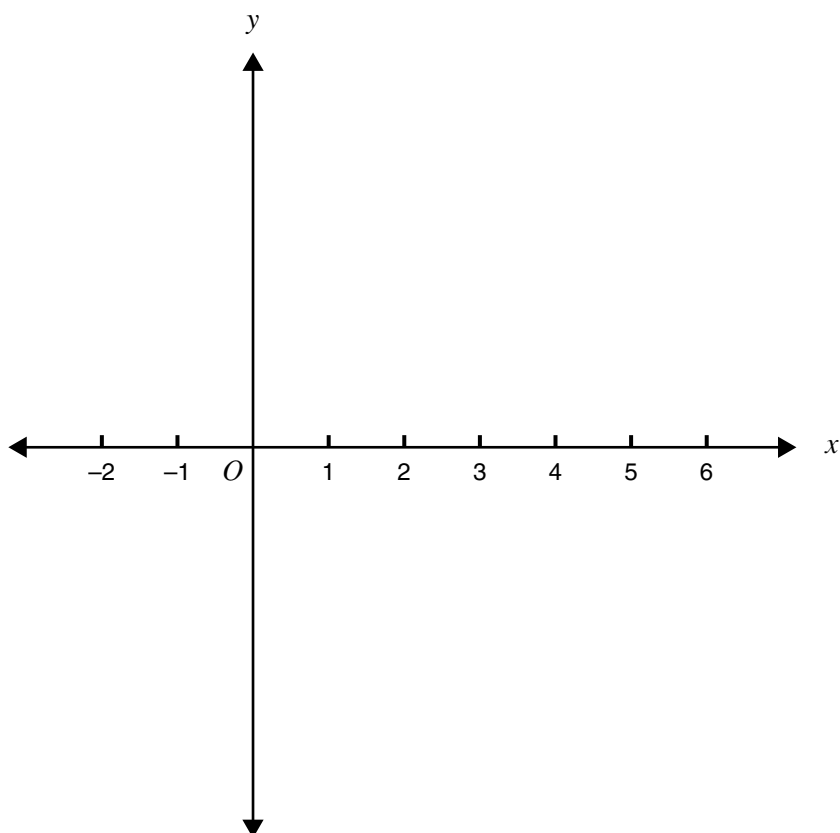
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2 marks

**Question 6**

On the set of axes provided below, sketch a continuous curve with equation  $y = f(x)$  having the following properties

$$\begin{aligned} f(0) &= 0 & f'(0) &= 0 \\ f(4) &= 0 & f'(3) &= 0 \\ f'(x) &< 0 \text{ for } \{x : x > 3\} \\ f'(x) &> 0 \text{ for } \{x : x < 3\} \setminus \{0\} \end{aligned}$$



3 marks

**Question 7**

Given  $f: (0, 200] \rightarrow \mathbb{R}$ ,  $f(x) = (100 - x)\log_{10} x$  find

- i. the maximum value of  $f(x)$ , correct to three decimal places

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- ii. the values of  $x$  for which  $f'(x) = 0$

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- iii. the value of  $f'(x)$  when  $f(x) = 0$  and  $x > 1$ .

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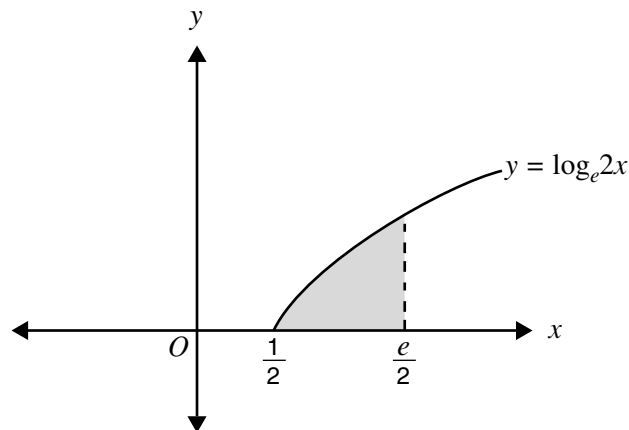


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1 + 1 + 1 = 3 marks

**Question 8**

The graph of the function  $f: [\frac{1}{2}, \infty) \rightarrow \mathbb{R}$ , where  $f(x) = \log_e 2x$  is shown below.



- a. If  $y = x \log_e 2x - x$ , find  $\frac{dy}{dx}$ .

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- b. Hence, find the exact area of the shaded region.

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1 + 3 = 4 marks  
**TURN OVER**

Working space



# **MATHEMATICAL METHODS**

## **Written examinations 1 and 2**

### **FORMULA SHEET**

#### **Directions to students**

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

## Mathematical Methods Formulas

### Mensuration

area of a trapezium:  $\frac{1}{2} (a + b)h$

curved surface area of a cylinder:  $2\pi rh$

volume of a cylinder:  $\pi r^2 h$

volume of a cone:  $\frac{1}{3} \pi r^2 h$

volume of a pyramid:  $\frac{1}{3} Ah$

volume of a sphere:  $\frac{4}{3} \pi r^3$

area of a triangle:  $\frac{1}{2} bc \sin A$

### Calculus

$$\frac{d}{dx} (x^n) = nx^{n-1}$$

$$\frac{d}{dx} (e^{ax}) = ae^{ax}$$

$$\frac{d}{dx} (\log_e x) = \frac{1}{x}$$

$$\frac{d}{dx} (\sin ax) = a \cos ax$$

$$\frac{d}{dx} (\cos ax) = -a \sin ax$$

$$\frac{d}{dx} (\tan ax) = \frac{a}{\cos^2 ax} = a \sec^2 ax$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e x + c, \text{ for } x > 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + c$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + c$$

product rule:  $\frac{d}{dx} (uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

quotient rule:  $\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

approximation:  $f(x+h) \approx f(x) + hf'(x)$

### Statistics and Probability

$$\Pr(A) = 1 - \Pr(A^c)$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

mean:  $\mu = E(X)$

variance:  $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

| Discrete distributions   |                                                                                                                 |                     |                                                                               |
|--------------------------|-----------------------------------------------------------------------------------------------------------------|---------------------|-------------------------------------------------------------------------------|
|                          | $\Pr(X = x)$                                                                                                    | mean                | variance                                                                      |
| general                  | $p(x)$                                                                                                          | $\mu = \sum x p(x)$ | $\sigma^2 = \sum (x - \mu)^2 p(x)$<br>$= \sum x^2 p(x) - \mu^2$               |
| binomial                 | ${}^n C_x p^x (1-p)^{n-x}$                                                                                      | $np$                | $np(1-p)$                                                                     |
| hypergeometric           | $\frac{{}^D C_x {}^{N-D} C_{n-x}}{{}^N C_n}$                                                                    | $n \frac{D}{N}$     | $n \frac{D}{N} \left( \frac{N-D}{N} \right) \left( \frac{N-D-1}{N-1} \right)$ |
| Continuous distributions |                                                                                                                 |                     |                                                                               |
| normal                   | If $X$ is distributed $N(\mu, \sigma^2)$ and $Z = \frac{X - \mu}{\sigma}$ , then $Z$ is distributed $N(0, 1)$ . |                     |                                                                               |

Table 1 Normal distribution – cdf

| $x$ | 0      | 1      | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|---|---|----|----|----|----|----|----|----|
| 0.0 | .5000  | .5040  | .5080  | .5120  | .5160  | .5199  | .5239  | .5279  | .5319  | .5359  | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.1 | .5398  | .5438  | .5478  | .5517  | .5557  | .5596  | .5636  | .5675  | .5714  | .5753  | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 35 |
| 0.2 | .5793  | .5832  | .5871  | .5910  | .5948  | .5987  | .6026  | .6064  | .6103  | .6141  | 4 | 8 | 12 | 15 | 19 | 23 | 27 | 31 | 35 |
| 0.3 | .6179  | .6217  | .6255  | .6293  | .6331  | .6368  | .6406  | .6443  | .6480  | .6517  | 4 | 8 | 11 | 15 | 19 | 23 | 26 | 30 | 34 |
| 0.4 | .6554  | .6591  | .6628  | .6664  | .6700  | .6736  | .6772  | .6808  | .6844  | .6879  | 4 | 7 | 11 | 14 | 18 | 22 | 25 | 29 | 32 |
| 0.5 | .6915  | .6950  | .6985  | .7019  | .7054  | .7088  | .7123  | .7157  | .7190  | .7224  | 3 | 7 | 10 | 14 | 17 | 21 | 24 | 27 | 31 |
| 0.6 | .7257  | .7291  | .7324  | .7357  | .7389  | .7422  | .7454  | .7486  | .7517  | .7549  | 3 | 6 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 0.7 | .7580  | .7611  | .7642  | .7673  | .7703  | .7734  | .7764  | .7793  | .7823  | .7852  | 3 | 6 | 9  | 12 | 15 | 18 | 21 | 24 | 27 |
| 0.8 | .7881  | .7910  | .7939  | .7967  | .7995  | .8023  | .8051  | .8078  | .8106  | .8133  | 3 | 6 | 8  | 11 | 14 | 17 | 19 | 22 | 25 |
| 0.9 | .8159  | .8186  | .8212  | .8238  | .8264  | .8289  | .8315  | .8340  | .8365  | .8389  | 3 | 5 | 8  | 10 | 13 | 15 | 18 | 20 | 23 |
| 1.0 | .8413  | .8438  | .8461  | .8485  | .8508  | .8531  | .8554  | .8577  | .8599  | .8621  | 2 | 5 | 7  | 9  | 12 | 14 | 16 | 18 | 21 |
| 1.1 | .8643  | .8665  | .8686  | .8708  | .8729  | .8749  | .8770  | .8790  | .8810  | .8830  | 2 | 4 | 6  | 8  | 10 | 12 | 14 | 16 | 19 |
| 1.2 | .8849  | .8869  | .8888  | .8907  | .8925  | .8944  | .8962  | .8980  | .8997  | .9015  | 2 | 4 | 6  | 7  | 9  | 11 | 13 | 15 | 16 |
| 1.3 | .9032  | .9049  | .9066  | .9082  | .9099  | .9115  | .9131  | .9147  | .9162  | .9177  | 2 | 3 | 5  | 6  | 8  | 10 | 11 | 13 | 14 |
| 1.4 | .9192  | .9207  | .9222  | .9236  | .9251  | .9265  | .9279  | .9292  | .9306  | .9319  | 1 | 3 | 4  | 6  | 7  | 8  | 10 | 11 | 13 |
| 1.5 | .9332  | .9345  | .9357  | .9370  | .9382  | .9394  | .9406  | .9418  | .9429  | .9441  | 1 | 2 | 4  | 5  | 6  | 7  | 8  | 10 | 11 |
| 1.6 | .9452  | .9463  | .9474  | .9484  | .9495  | .9505  | .9515  | .9525  | .9535  | .9545  | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 1.7 | .9554  | .9564  | .9573  | .9582  | .9591  | .9599  | .9608  | .9616  | .9625  | .9633  | 1 | 2 | 3  | 3  | 4  | 5  | 6  | 7  | 8  |
| 1.8 | .9641  | .9649  | .9656  | .9664  | .9671  | .9678  | .9686  | .9693  | .9699  | .9706  | 1 | 1 | 2  | 3  | 4  | 4  | 5  | 6  | 6  |
| 1.9 | .9713  | .9719  | .9726  | .9732  | .9738  | .9744  | .9750  | .9756  | .9761  | .9767  | 1 | 1 | 2  | 2  | 3  | 4  | 4  | 5  | 5  |
| 2.0 | .9772  | .9778  | .9783  | .9788  | .9793  | .9798  | .9803  | .9808  | .9812  | .9817  | 0 | 1 | 1  | 2  | 2  | 3  | 3  | 4  | 4  |
| 2.1 | .9821  | .9826  | .9830  | .9834  | .9838  | .9842  | .9846  | .9850  | .9854  | .9857  | 0 | 1 | 1  | 2  | 2  | 2  | 3  | 3  | 4  |
| 2.2 | .9861  | .9864  | .9868  | .9871  | .9875  | .9878  | .9881  | .9884  | .9887  | .9890  | 0 | 1 | 1  | 1  | 2  | 2  | 2  | 3  | 3  |
| 2.3 | .9893  | .9896  | .9898  | .9901  | .9904  | .9906  | .9909  | .9911  | .9913  | .9916  | 0 | 1 | 1  | 1  | 1  | 2  | 2  | 2  | 2  |
| 2.4 | .9918  | .9920  | .9922  | .9925  | .9927  | .9929  | .9931  | .9932  | .9934  | .9936  | 0 | 0 | 1  | 1  | 1  | 1  | 1  | 2  | 2  |
| 2.5 | .9938  | .9940  | .9941  | .9943  | .9945  | .9946  | .9948  | .9949  | .9951  | .9952  | 0 | 0 | 0  | 1  | 1  | 1  | 1  | 1  | 1  |
| 2.6 | .9953  | .9955  | .9956  | .9957  | .9959  | .9960  | .9961  | .9962  | .9963  | .9964  | 0 | 0 | 0  | 0  | 1  | 1  | 1  | 1  | 1  |
| 2.7 | .9965  | .9966  | .9967  | .9968  | .9969  | .9970  | .9971  | .9972  | .9973  | .9974  | 0 | 0 | 0  | 0  | 0  | 1  | 1  | 1  | 1  |
| 2.8 | .9974  | .9975  | .9976  | .9977  | .9977  | .9978  | .9979  | .9979  | .9980  | .9981  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 1  | 1  |
| 2.9 | .9981  | .9982  | .9982  | .9983  | .9984  | .9984  | .9985  | .9985  | .9986  | .9986  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.0 | .9987  | .9987  | .9987  | .9988  | .9988  | .9989  | .9989  | .9989  | .9990  | .9990  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.1 | .9990  | .9991  | .9991  | .9991  | .9992  | .9992  | .9992  | .9992  | .9993  | .9993  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.2 | .9993  | .9993  | .9994  | .9994  | .9994  | .9994  | .9994  | .9995  | .9995  | .9995  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.3 | .9995  | .9995  | .9995  | .9996  | .9996  | .9996  | .9996  | .9996  | .9996  | .9997  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.4 | .9997  | .9997  | .9997  | .9997  | .9997  | .9997  | .9997  | .9997  | .9997  | .9998  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.5 | .9998  | .9998  | .9998  | .9998  | .9998  | .9998  | .9998  | .9998  | .9998  | .9998  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.6 | .9998  | .9998  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.7 | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.8 | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |

END OF FORMULA SHEET