Trial Examination 2 Solutions

Question 1

a. i. At 10am $t = 1$, $\therefore r = \frac{1}{2}$ km **[A1]**

ii. Area $= \pi r^2$, Area $= \pi(\frac{1}{2})^2 = \frac{\pi}{4}$ km^2 **[A1]**

b. $r = \frac{t}{2}$, $\frac{dr}{dt} = \frac{1}{2}$ km/hr **[A1]**

c. $r = 10\ km$, $\therefore 10 = \frac{t}{2}$, $t = 20$ hrs **[M1]**

9 am + 20 hrs = 5 am on December 26 **[A1]**

d. Let x = distance from oil slick centre to Swino.

$x^2 = 10^2 + 5^2$

$x^2 = 125$

$x \simeq 11.18$kms $\therefore x \simeq 11.2$kms

Swino is 11.2km from the centre to 1 dp. **[A1]**

e. $r = 11.2$ km, $\therefore 11.2 = \frac{t}{2}$, $t = 22.4$ **[M1]**

9 am + 22.4 hrs = 7 am and 24 minutes

$\therefore$ 7:24 am on December 26 **[A1]**

f. Circumference $= 2\pi r$

$\therefore 2\pi r = \frac{2\pi}{5} \Rightarrow r = \frac{1}{5}$ **[A1]**

$\therefore k(2) = \frac{1}{5} \Rightarrow k = \frac{1}{10}$ **[A1]**

g. Distance travelled = Circumference **[M1]**

$\therefore \frac{t^2}{10} = 2\pi kt \Rightarrow \frac{t^2}{10} = \frac{2\pi t}{10} \Rightarrow t = 2\pi$ hrs **[A1]**

$\therefore r = kt \Rightarrow r = \frac{2\pi}{10} \Rightarrow r = \frac{\pi}{5}$ kms **[A1]**

h. Time to encircle the slick = 2π hrs

Distance boat travels $\therefore x = \frac{4\pi^2}{10} \Rightarrow x = \frac{2\pi^2}{5}$ **[A1]**

$\therefore$ Length of boom unused $= 4 - \frac{2\pi^2}{5}$

$= \frac{2(10 - \pi^2)}{5}$ km

[A1]

Question 2

a. i. $B = 15$ **[A1]**

ii. $A = -3$ **[A1]**

iii. $P = 365$

$\therefore P = \frac{2\pi}{n}$

$365 = \frac{2\pi}{n} \Rightarrow n = \frac{2\pi}{365}$ **[A1]**

$T(t) = -3\sin(\frac{2\pi}{365})t + 15$ **[A1]**

b. Max temperature = 18 °C **[A1]**

$18 = -3\sin(\frac{2\pi}{365})t + 15$

$-1 = \sin(\frac{2\pi}{365})t$

$\sin^{-1}(-1) = \frac{2\pi}{365}t \Rightarrow \frac{3\pi}{2} = \frac{2\pi}{365}t$ for $0 \le t \le 365$

$1095\pi = 4\pi t \Rightarrow t = 273.75 \Rightarrow t = 274$ **[A1]**

c. 16.5 °C or above occurs in the domain $\{t: 215 \le t \le 335\}$ to nearest day **[A2]**

d. $t = 182.5$

$P = k\cos(\pi) \Rightarrow P = -k \Rightarrow -3 = -k$

Population decreased by 3%

$\therefore P = -3$

$\therefore -3 = -k$

$\therefore k = 3$ **[A1]**

e. $3\cos(\frac{2\pi t}{365}) = \sin(\frac{2\pi t}{365})$

$3 = \tan(\frac{2\pi t}{365})$ $\quad \cos\frac{2\pi t}{365} \neq 0$ **[M1]**

$3 = \tan(\frac{2\pi t}{365}) \Rightarrow \tan^{-1}(3) = \frac{2\pi t}{365} \Rightarrow \frac{2\pi t}{365} = 1.249$

$t = 72.559 \Rightarrow t = 73$ to the nearest day

[A1]

f. $t = 255$ using a graph as calculator. **[A1]**

g. $t = 73$ is the 13th of July 1998

$t = 255$ is the 11th of January 1999 **[A2]**

Question 3

a. $3k^2 + \frac{7k}{6} + \frac{7k}{12} + \frac{k}{4} = 1$ **[M1]**

$\frac{36k^2}{12} + \frac{14k}{12} + \frac{7k}{12} + \frac{3k}{12} = 1$

$36k^2 + 24k - 12 = 0$ **[A1]**

$12(3k-1)(k+1) = 0$

$k = -1$ or $\frac{1}{3}$ $\therefore$ $k = \frac{1}{3}$ as $k \geq 0$

is due to $0 \leq Pr(X = x) \leq 1$ or must be +ve **[A1]**

b. $\Pr(x \leq 2) = \Pr(x = 1) + \Pr(x = 2)$

$\Pr(x \leq 2) = 3 \times \frac{1}{9} + \frac{7}{6} \times \frac{1}{3}$ **[M1]**

$\Pr(x \leq 2) = \frac{1}{3} + \frac{7}{18} = \frac{13}{18}$ **[A1]**

c. $\frac{dC}{dt} = \frac{1}{2}(-3t^2 + \frac{16t}{3} + 11) = 0$, for stationary point **[M1]**

$-3t^2 + \frac{16t}{3} + 11 = 0$ **[A1]**

$-9t^2 + 16t + 33 = 0$

$9t^2 - 16t - 33 = 0$

$(9t + 11)(t - 3) = 0$

$t = -\frac{11}{9}$, or 3 $\therefore$ $t = 3$ hrs as $0 \leq t \leq 4.9$ **[A1]**

Sub into C(t)

$\frac{1}{2}(-3^3 + \frac{72}{3} + 33) = \frac{1}{2}(-27 + \frac{72}{3} + 33)$

$\frac{1}{2}(\frac{90}{3}) = 15$

$\therefore$ maximum concentration = 15% **[A1]**

d. $\frac{dC}{dt} = \frac{1}{2}(-3t^2 + \frac{16t}{3} + 11)$

At $t = \frac{8}{9}$ hrs, $\frac{dc}{dt} \approx 6.82\%$

$\therefore$ 7% to the nearest per cent **[A1]**

e. Let X = conc. of petalene

$\Pr(X < (p - 4)) = 0.1587$

$\Pr(X < (p - 4)) = \Pr(Z < \frac{p - 4 - p}{d})$

$\Pr(Z < \frac{-4}{d}) = 0.1587$

$1 - \Pr(Z > \frac{-4}{d}) = 0.1587$ **[M2]**

$\therefore \Pr(Z > \frac{4}{d}) = 0.1587$ using symmetry

$\therefore \Pr(Z < \frac{4}{d}) = 0.8413$

$\therefore$ from tables $\frac{4}{d} = 1$

$d = 4$ **[A1]**

f. $\Pr\left(X < \left(\frac{3p}{2} - 10\right)\right) = 0.5987$

$\Pr\left(X < \left(\frac{3p}{2} - 10\right)\right) = 0.5987 = \Pr\left(Z < \frac{\frac{3p}{2} - 10 - p}{4}\right)$

$\Pr\left(Z < \frac{\frac{p}{2} - 10}{4}\right) = 0.5987$

$\frac{\frac{p}{2} - 10}{4} = 0.25$ **[M2]**

$\therefore p = 22$ **[A1]**

Question 4

a. i. $t = 0, P = 20$ $\therefore 20 = Ae^0 + B$

$20 = A + B$ **[A1]**

ii. $t = 3, P = 25$ $\therefore 25 = Ae^3 + B$ **[A1]**

iii. Using $25 = Ae^3 + B$ and $20 = Ae^0 + B$ **[M1]**

$25 = Ae^3 + B$

Subtract $20 = Ae^0 + B$

$5 = A(e^3 - 1)$

$A = \frac{5}{(e^3 - 1)} \approx 0.262$ **[A1]**

Substitute into $20 = Ae^0 + B$

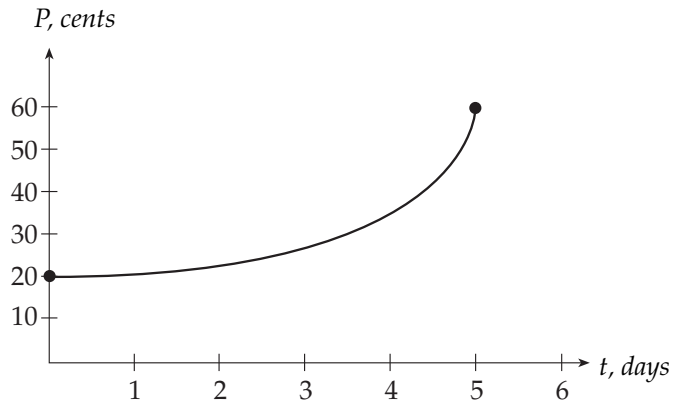
$20 = 0.262 + B$

$B = 19.738$ to three d.p.s.

$A = 0.262$ **[A1]**

b. $t = 5,\ \therefore P = 0.262e^5 + 19.738 = 58.6$ cents to one d.p. **[A1]**

c.



[A2]

It would be an advantage to use a graphics calculator for the questions from here onwards.

d. i. $\int_1^2 (0.262e^t + 19.738)\,dt$

$= \left[0.262e^t + 19.738t\right]_1^2$ **[A1]**

$= \left[0.262e^2 + 19.738(2)\right] - \left[0.262e^1 + 19.738(1)\right]$

$\simeq$ 20.96 sq units to 2 dps. **[A1]**

ii. $\int_2^3 (0.262e^t + 19.738)\,dt$

$\left[0.262e^t + 19.738t\right]_2^3$ **[A1]**

$\left[0.262e^3 + 19.738(3)\right] - \left[0.262e^2 + 19.738(2)\right]$

= 23.06 sq units to 2 dps. **[A1]**

iii. 23.06 – 20.96 = 2.1

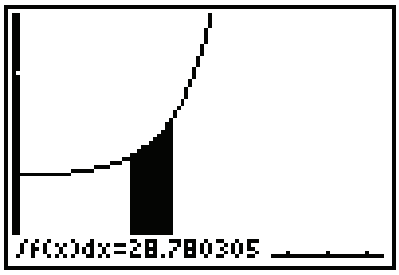
The percentage increase

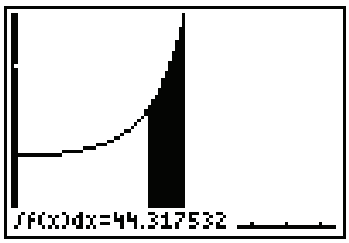
$= \frac{2.1}{20.96} \times \frac{100\%}{1} = 10\%$

to the nearest per cent.

[A1]

e. i. Using the graphic calculator





Thursday to Friday = 28.78 sq units
Friday to Monday = 44.32 sq units **[A1]**
(to 2 dps)
Percentage Increase
Wednesday to Thursday area and
Thursday to Friday area increase is 24.8%
Thursday to Friday area and Friday to
Monday area increase is 54% to nearest %.
$\therefore$ At the end of Friday 18th August or
at the start of Monday 21st August **[A1]**

ii. 54% increase in one day **[A1]**