

MATHEMATICAL METHODS

TRIAL EXAMINATION 1

2000

Reading Time: 15 minutes
Writing Time: 90 minutes

Instructions to Students

This exam consists of Part I and Part II. All questions in Part I and Part II should be answered.
Part I consists of 28 multiple-choice questions, which should be answered on the detachable answer sheet which can be found on page 20 of this exam.
Part II consists of 8 short-answer questions which should be answered in the spaces provided.
Part I begins on page 2 of this exam.
Part II begins on page 15 of this exam.
All questions in Part I and in Part II should be attempted.
Part I is worth **28 marks**.
Part II is worth **22 marks**.
Students may bring up to two A4 pages of pre-written notes into the exam.

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PART I Multiple-choice questions

Question 1

A random variable X has the probability distribution shown in the table below.

x	0	1	2	3
$\Pr(X=x)$	0.4	0.3	0.2	0.1

The mean and variance respectively of x are

- A. 1 and 1
- B. 1 and 1.4
- C. 1.4 and 1
- D. 1.4 and 1.4
- E. 1.5 and 2

Question 2

Which one of the following random variables is not discrete?

- A. the number of goals scored in a basketball match
- B. the scores received by students in a class for a maths test
- C. the weight of a footballer during a season
- D. the population of a country town over a decade
- E. the price of "Moggy" 1kg packets of cat food over a period of a week at a certain supermarket

Question 3

The weights of packets of 6 Easter buns are normally distributed with a mean of 420 grams and a standard deviation of 8 grams.

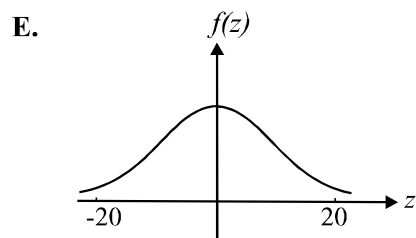
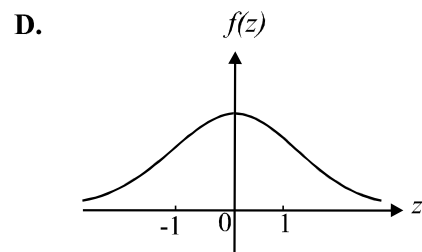
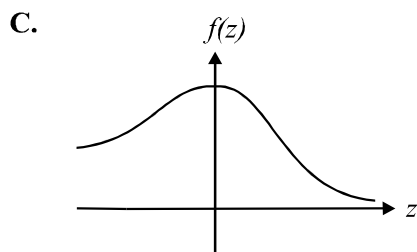
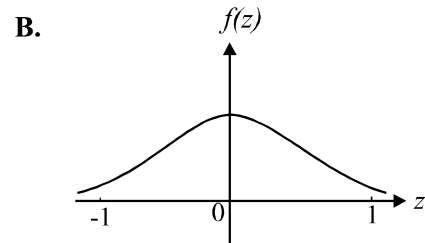
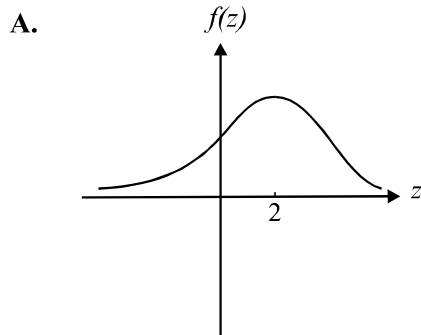
The proportion of packets which weigh between 400 and 430 grams is closest to

- A. 1%
- B. 50%
- C. 89%
- D. 93%
- E. 99%

Question 4

The random variable Z has a standard normal distribution.

Which one of the following diagrams could show the distribution of Z ?



Question 5

A large jar contains "traffic light" lollies. There are 80 red, 50 green and 70 orange lollies in the jar.

A child is allowed to choose 10 lollies from the jar and these lollies are placed in a bag to be taken home. The probability that there will be 6 red lollies in the bag is given by

- A. ${}^{10}C_6 (0.4)^6 (0.6)^4$
 B. ${}^{200}C_{10} (0.4)^6 (0.6)^4$
 C. ${}^{200}C_{10} (6)^{0.4} (4)^{0.6}$
 D. $\frac{{}^{80}C_6 {}^{120}C_4}{{}^{200}C_{10}}$
 E. $\frac{{}^{0.4}C_{0.6} {}^{0.6}C_{0.4}}{{}^{200}C_6}$

Question 6

At Seaview Secondary College, a weekly detention class will contain Year 12 students on 15% of occasions. The probability that there will be Year 12s present in at least 1 of the next 3 detention classes held is

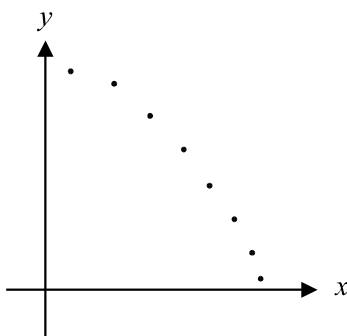
- A. ${}^3C_1 (0.15)^1 (0.85)^2$
 B. $1 - {}^3C_0 (0.15)^0 (0.85)^3$
 C. ${}^3C_1 (0.85)^1 (0.15)^2 + {}^3C_2 (0.85)^2 (0.15)^1 + {}^3C_3 (0.85)^3 (0.15)^0$
 D. $\frac{{}^{0.15}C_1 {}^{0.85}C_2}{{}^{100}C_3}$
 E. $\frac{{}^{15}C_1 {}^{85}C_2}{{}^{100}C_3}$

Question 7

A graph showing the relationship between the variables x and y is shown below.

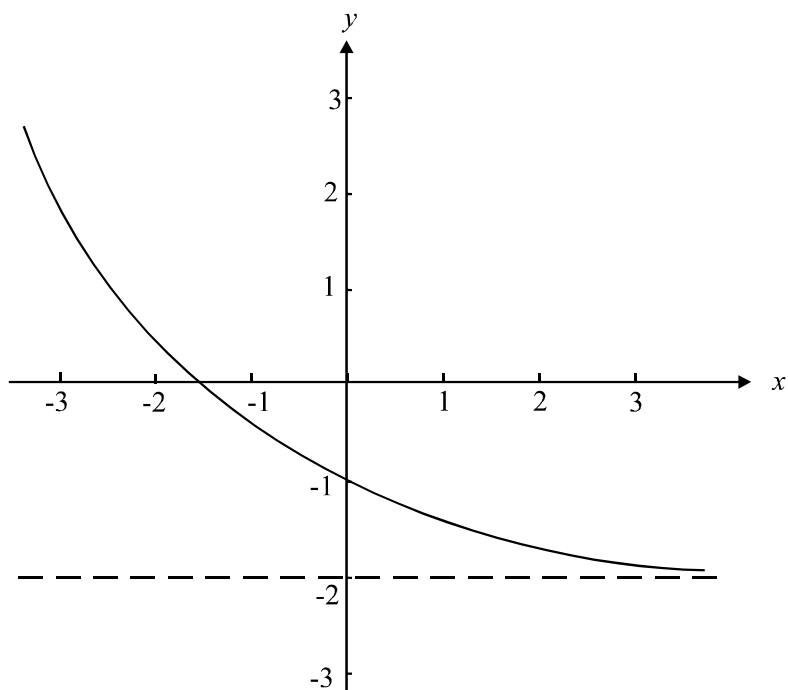
Given that a is a positive constant, the equation relating x and y could be of the form

- A. $y = \frac{a}{x^2}$
 B. $y = -ax^2$
 C. $y = ax^{\frac{1}{2}}$
 D. $y = \cos(ax)$
 E. $y = e^{-ax}$

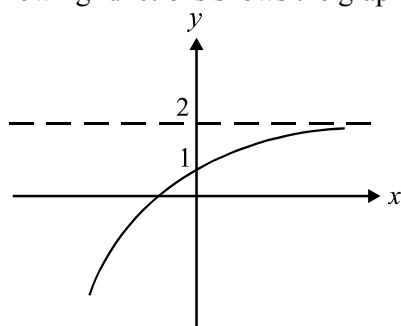
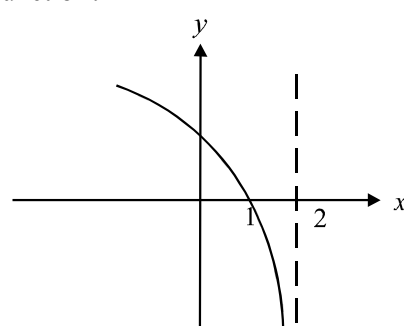
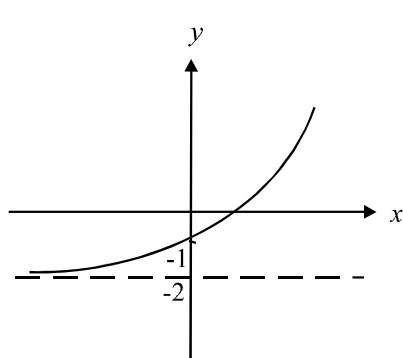
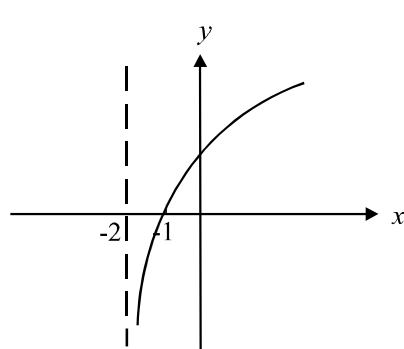
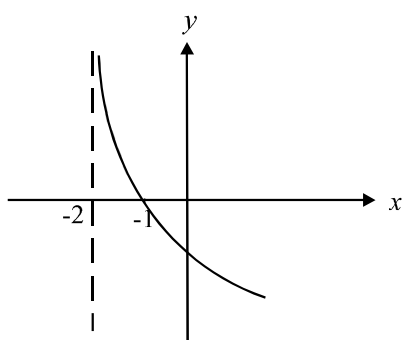


Question 8

The graph of a function is shown below.



Which one of the following functions shows the graph of its inverse function?

A.**B.****C.****D.****E.**

Question 9

The equations of the vertical and horizontal asymptotes of the graph with equation $y = \frac{A}{x+b} + B$ are respectively

- A. $x = A$ and $y = B$
- B. $x = b$ and $y = A$
- C. $x = -b$ and $y = B$
- D. $x = A$ and $y = b$
- E. $x = B$ and $y = -b$

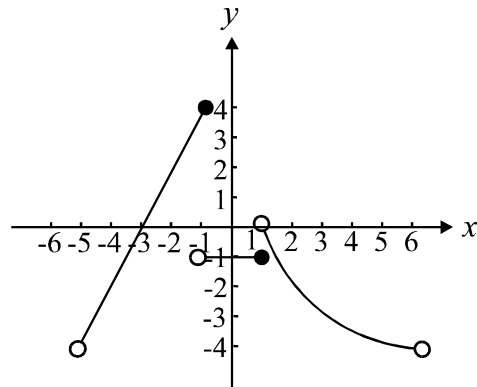
Question 10

The domain and range of the function $y = 2x^{-1} + 4$ are given respectively by

- A. $(-\infty, 0) \cup (0, \infty)$ and $(-\infty, 4) \cup (4, \infty)$
- B. $(-\infty, 2) \cup (2, \infty)$ and $(-\infty, 0) \cup (0, \infty)$
- C. $[-\infty, 0) \cup (0, \infty]$ and $[-\infty, 4) \cup (4, \infty]$
- D. $(-\infty, 4) \cup (4, \infty)$ and $(-\infty, 0) \cup (0, \infty)$
- E. $(-\infty, 0.5) \cup (0.5, \infty)$ and $(-\infty, 0) \cup (0, \infty)$

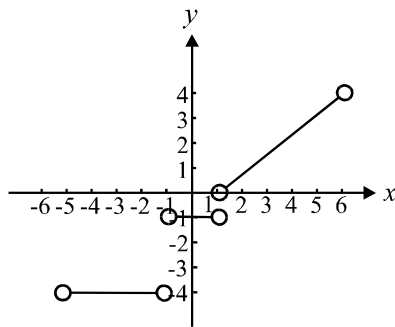
Question 11

The graph of the function g is shown below.

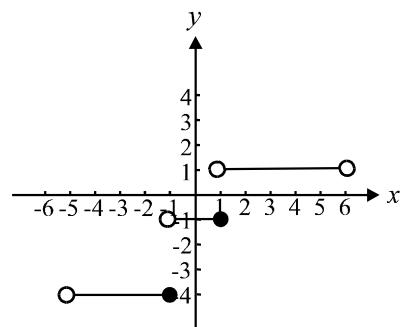


The gradient function of g is closest to

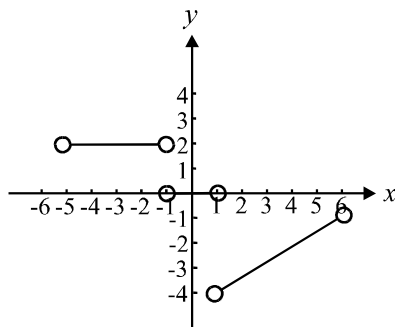
A.



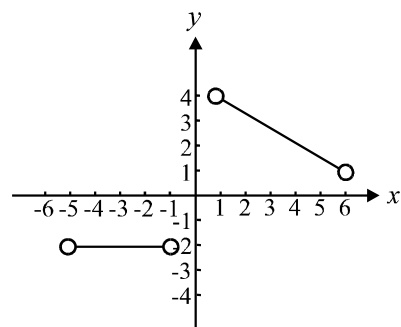
B.



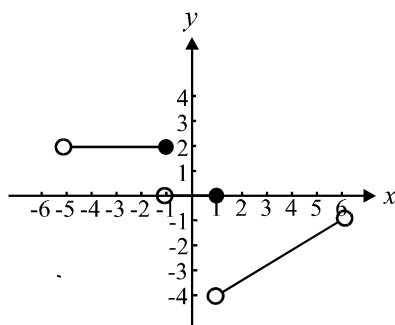
C.



D.



E.



Question 12

If $f(x) = x^2 \tan(4x)$ then $f'(x)$ is equal to

- A. $2x \sec^2(4x)$
- B. $8x \sec^2(4x)$
- C. $2x \tan(4) + x^2 \sec^2(4x)$
- D. $2x \tan(4x) + 4x^2 \sec^2(4x)$
- E. $8x \tan(4x) + 2x^2 \sec^2(4x)$

Question 13

If $y = \frac{e^{\sin(3x)}}{2}$ then $\frac{dy}{dx}$ is equal to

- A. $\frac{\sin(3x)}{2} e^{\sin(3x)}$
- B. $\frac{\cos(3x)}{2} e^{\sin(3x)}$
- C. $\frac{3}{2} \sin(3x) e^{\sin(3x)}$
- D. $\frac{3}{2} \cos(3x) e^{\sin(3x)}$
- E. $\frac{3}{2} \sin(3x) \cos(3x) e^{\sin(3x)}$

Question 14

The derivative of $\frac{\log_e(x^2 - 4x)}{x}$ is given by

- A. $\frac{1}{x-4}$
- B. $\frac{2x-4}{x(x-4)}$
- C. $\frac{2x-4}{x-4} - \log_e(x^2 - 4x)$
- D. $\frac{2x-4}{x^2(x-4)} - \log_e(x^2 - 4x)$
- E. $\frac{2x-4}{x^2(x-4)} - \frac{1}{x^2} \log_e(x^2 - 4x)$

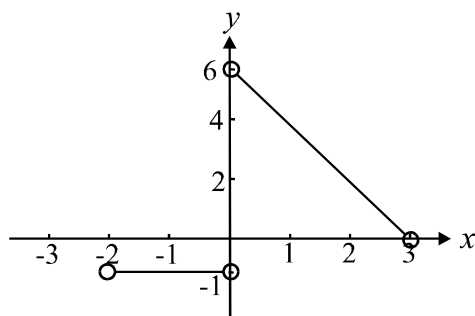
Question 15

The stationary point/s of the curve with equation $y = x^4 + 8x^3 + 5x^2 + x$ is/are closest to

- A. (0, 0)
- B. (0, 1)
- C. (0, 0) and (-5.558, -270.378)
- D. (-0.170, -0.027) and (-3.744, 90.055)
- E. (0, 0) and (-7.319, 0)

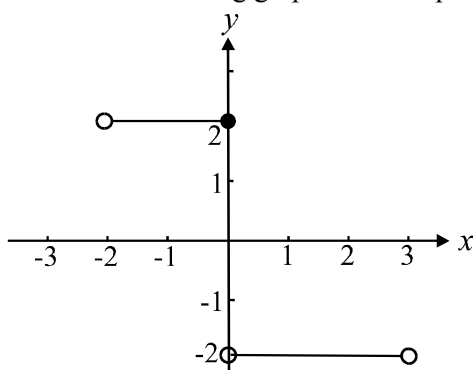
Question 16

The graph of the function g over the domain $x \in (-2, 3)$ is shown below.

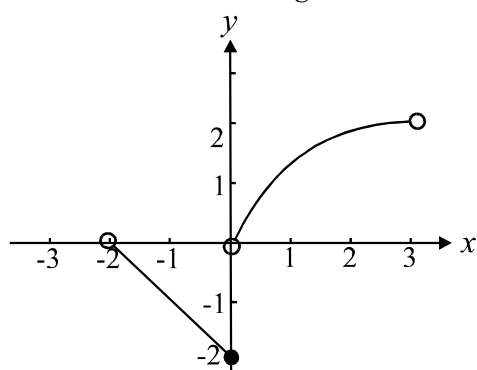


Which one of the following graphs could represent the antiderivative function of g ?

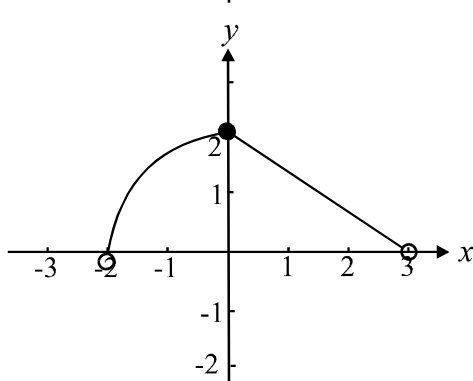
A.



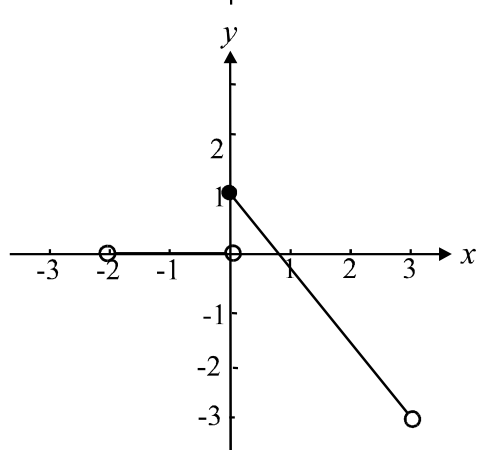
B.



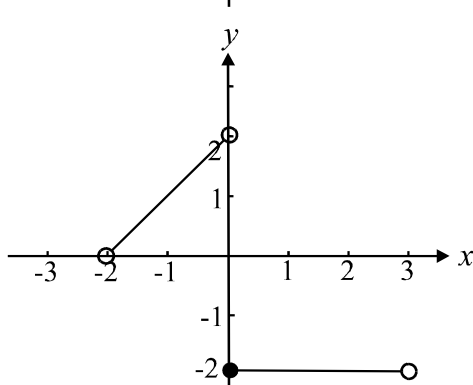
C.



D.



E.

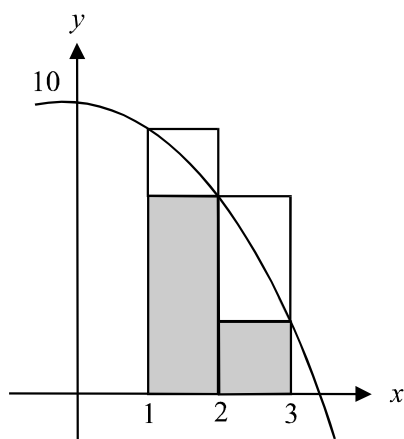


$\int (\sin \frac{x}{2} + \frac{1}{(2x+1)^2}) dx$ is equal to

- A. $\cos \frac{x}{2} + \frac{1}{2x+1} + c$
- B. $-\cos \frac{x}{2} - \frac{1}{(2x+1)^3} + c$
- C. $-2 \cos \frac{x}{2} - \frac{1}{2(2x+1)} + c$
- D. $-\frac{1}{2} \cos \frac{x}{2} + \frac{2}{(2x+1)^3} + c$
- E. $-\cos \frac{x}{2} + \frac{2}{2x+1} + c$

Question 18

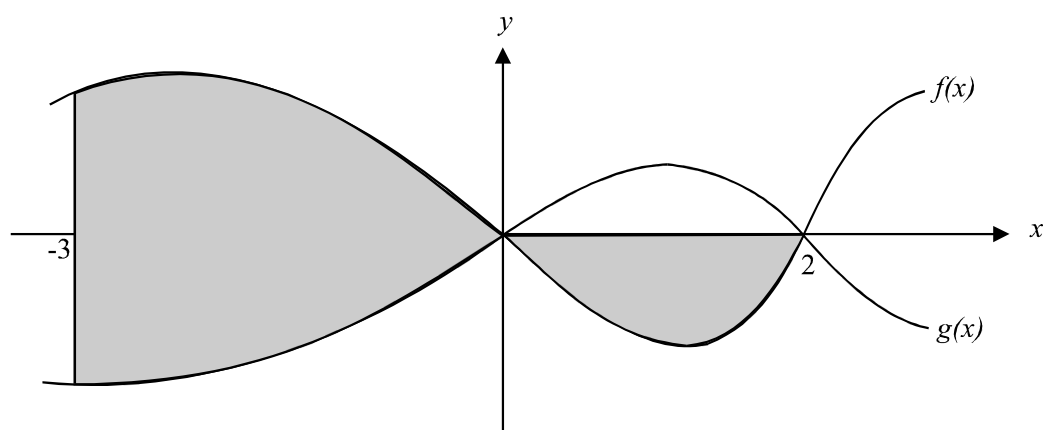
An approximate value of $\int_1^3 (-x^2 + 10) dx$ can be calculated using the rectangles shown in the diagram below.



That approximate value is

- A. 6
- B. 11
- C. 22
- D. 25
- E. 60

Question 19



The total shaded area shown in the diagram above is given by

- A. $\int_{-3}^0 (f(x) - g(x))dx - \int_0^2 f(x)dx$
- B. $\int_0^{-3} (f(x) - g(x))dx + \int_0^2 f(x)dx$
- C. $\int_0^{-3} (f(x) + g(x))dx - \int_0^2 g(x)dx$
- D. $\int_{-3}^0 (g(x) - f(x))dx - \int_2^0 f(x)dx$
- E. $\int_{-3}^0 (g(x) + f(x))dx - \int_2^0 g(x)dx$

Question 20

A trigonometric function is given by $f(x) = 3 \sin(4x - 2\pi) + 1$

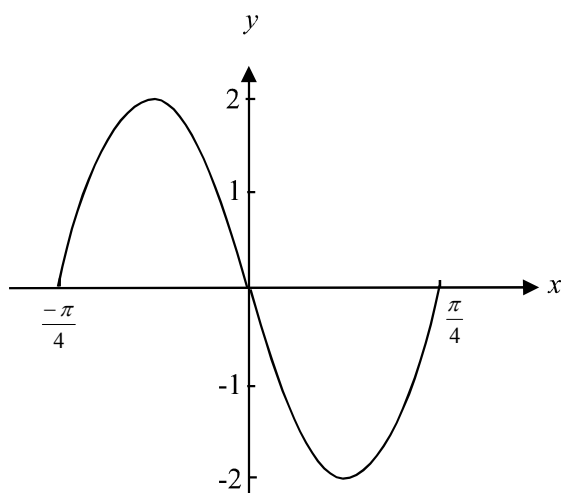
The "translation" of this function f is the number of units the basic graph of $y = \sin x$ has been translated horizontally to obtain f .

The amplitude, period and translation of f are given respectively by

	Amplitude	Period	Translation
A.	3	$\frac{\pi}{2}$	2π right
B.	3	$\frac{\pi}{2}$	$\frac{\pi}{2}$ right
C.	3	2π	2π right
D.	6	$\frac{\pi}{2}$	$\frac{\pi}{2}$ right
E.	6	2π	2π right

Question 21

One cycle of a trigonometric function f is shown below.



The rule for this function f could be

- A. $y = -2 \sin(x - \frac{\pi}{4})$
- B. $y = 2 \cos 2(x + \frac{\pi}{4})$
- C. $y = 2 \sin 4(x + \frac{\pi}{4})$
- D. $y = 4 \sin 2(x - \frac{\pi}{4})$
- E. $y = 4 \cos 4(x - \frac{\pi}{4})$

Question 22

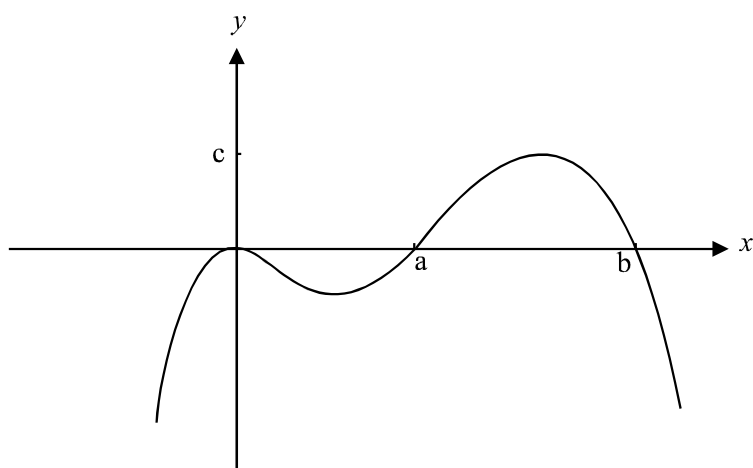
The sum of the solutions to the equation $10 \cos(3x) = 5$ over the domain $[0, \pi]$ is

- A. $\frac{\pi}{9}$
- B. $\frac{\pi}{3}$
- C. $\frac{2\pi}{3}$
- D. $\frac{13\pi}{18}$
- E. $\frac{13\pi}{9}$

Question 23

Let $f(x) = a \sin(x - b)$ where a and b are positive constants. The equation $f(x) = c$, $0 < c < a$, will certainly have no solutions over the interval $[0, \pi]$ if b equals

- A. 0
- B. $\frac{\pi}{4}$
- C. $\frac{\pi}{2}$
- D. π
- E. 2π

Question 24

The equation of the graph shown above is

- A. $y = -x(x - a)(x - b)$
- B. $y = x^2(ax - b)^2 + c$
- C. $y = x(x - a)(x - b)$
- D. $y = -x^2(x - a)(x - b)$
- E. $y = -x(x^2 - a)(x - b)$

Question 25

In the expansion of $(3x - a)^6$, $a > 0$, the coefficient of x^4 is 4860. The value of a is

- A. -2
- B. 2
- C. -2 or 2
- D. 4
- E. $\sqrt{60}$

Question 26

The function $f : [a, b] \rightarrow R$ where $f(x) = x^4 - x^2$ has an inverse function f^{-1} if

- A. $a = -1$ and $b = 0$
- B. $a = -1$ and $b = 1$
- C. $a = 0$ and $b = 1$
- D. $a = -\frac{1}{\sqrt{2}}$ and $b = 0$
- E. $a = -\frac{1}{\sqrt{2}}$ and $b = \frac{1}{\sqrt{2}}$

Question 27

The function $g : [1, \infty) \rightarrow R$, where $g(x) = 2(x-1)^2 + 3$ has an inverse function g^{-1} which is defined by

- A. $g^{-1} : (-\infty, 1] \rightarrow R$, where $g^{-1}(x) = 2(x-1)^2 + 3$
- B. $g^{-1} : [1, \infty) \rightarrow R$, where $g^{-1}(x) = \sqrt{\frac{x-3}{2}} + 1$
- C. $g^{-1} : [3, \infty) \rightarrow R$, where $g^{-1}(x) = \sqrt{\frac{x-1}{2}} + 3$
- D. $g^{-1} : [3, \infty) \rightarrow R$, where $g^{-1}(x) = \sqrt{\frac{x-3}{2}} + 1$
- E. $g^{-1} : [1, \infty) \rightarrow R$, where $g^{-1}(x) = \sqrt{\frac{x-1}{2}} + 3$

Question 28

If $\frac{1}{2} \log_2 2 - \log_2 \sqrt{8} - \log_2 x = 1$, then x is equal to

- A. 0
- B. $\frac{1}{4}$
- C. $\sqrt{2}$
- D. 2
- E. 4

PART II Short answer questions

Question 1

Factorise $x^4 - 6x^3 - x^2 + 6x$

2 marks

Question 2

The number of trains, X , travelling from a country town to the city in a day, has a probability distribution shown in the table below.

X	0	1	2	3	4
$p(x)$	$\frac{2}{q}$	$\frac{7}{2q}$	0.2	$\frac{3}{2q}$	$\frac{1}{q}$

a. Find the value of q .

1 mark

b. Find the expected number of trains, correct to 2 decimal places, travelling from this country town to the city in a day.

1 mark

c. Find the probability, correct to 2 decimal places, that at least 1 train will travel from this country town to the city in a day.

1 mark

Question 3

The time taken for Kerry to complete a mini-triathlon varies according to a normal distribution with a mean of 72 minutes and a standard deviation of 5 minutes.

Kerry's coach notices that at 30% of the many races in which she has competed, she has taken less than n minutes to complete the course.

Calculate the value of n to 2 decimal places.

2 marks

Question 4

- a. Find the gradient of the curve with equation $y = 10^{\frac{x}{2}}$ at the point where $x = 1.2$
Express your answer correct to 3 decimal places.

1 mark

- b. Since $10^{\frac{1.2}{2}} = 10^{0.6}$ and $10^{0.6} \approx 3.981$, use an appropriate formula to find an approximate value for $10^{0.65}$. Express your answer correct to 2 decimal places.

2 marks

Question 5

The velocity, v (m/sec), of a billycart at any time t (secs), during a race is given by

$$v = 0.5 \log_e(t^2 + 1) + 2, \quad t \geq 0$$

- a.** Calculate the average rate of change of velocity of the billycart during the first 3 seconds of the race correct to 1 decimal place.

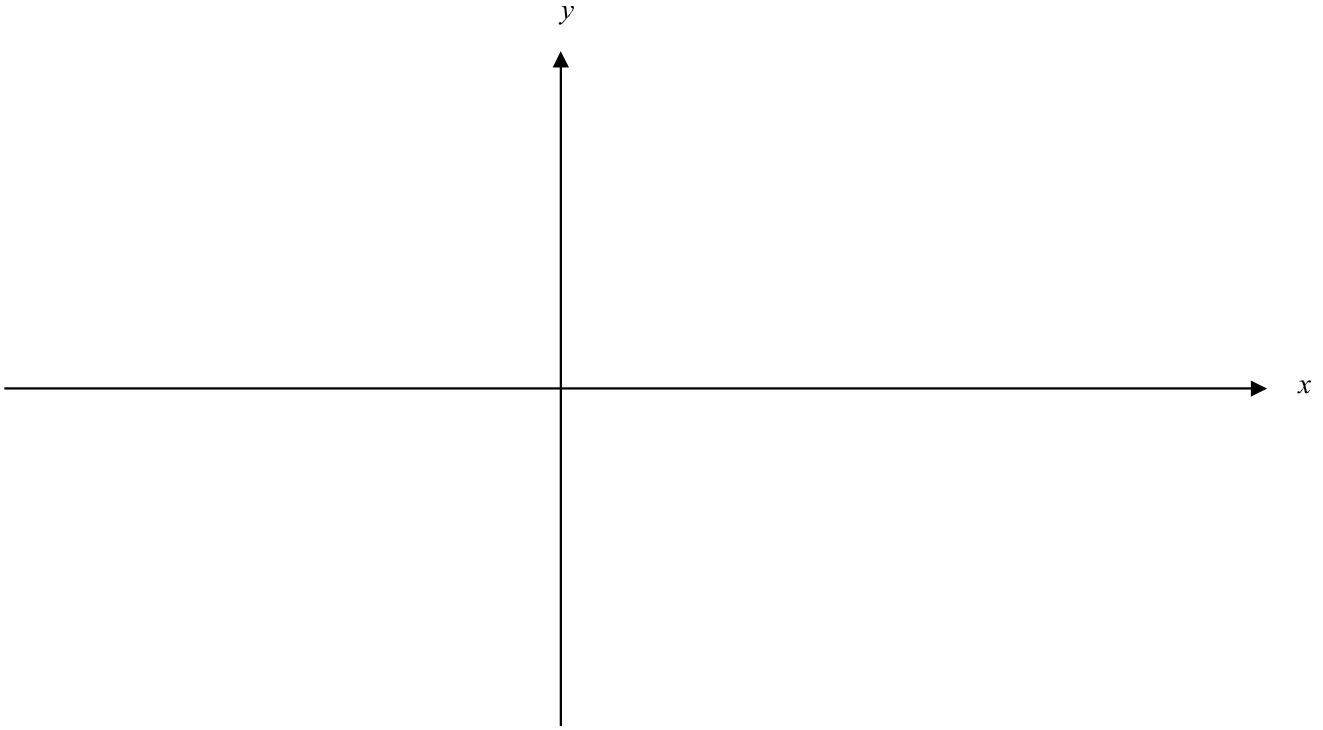
1 mark

- b.** Find the instantaneous rate of change of velocity at time $t = 3$ seconds.

2 marks

Question 6

- a.** On the set of axes below sketch the graph of $f : \left(-\frac{3\pi}{4}, \frac{3\pi}{4}\right) \rightarrow \mathbb{R}$ where $f(x) = \tan(2x)$ showing clearly the important features of the graph.



2 marks

- b.** Using your graph in part **a.** or otherwise, find the solutions to the equation

$$\sqrt{3} \sin 2x = \frac{3}{\sqrt{3}} \cos 2x \text{ over the domain } \left(-\frac{3\pi}{4}, \frac{3\pi}{4}\right)$$

1 mark

Question 7

The tangent to the curve with equation $y = 2x^2 + 1$ at the point where $x = 2$ intersects with the normal to the curve with equation $y = \sqrt{x - 2}$ at the point where $x = 3$.
Find the coordinates of this point of intersection.

3 marks

Question 8

Find the area bounded by the curves with equations $y = \frac{1}{2} \sin 2x$ and $y = -\cos \frac{x}{3}$, the line $x = \frac{\pi}{2}$ and the y axis.

3 marks
Total 22 marks

MATHEMATICAL METHODS

TRIAL EXAM 1

2000

PART I

MULTIPLE-CHOICE ANSWER SHEET

STUDENT NAME:

INSTRUCTIONS

Fill-in the letter that corresponds to your choice. Example: ☐ A ☒ B ☐ C ☐ D ☐ E

The answer selected is B. Only one answer should be selected.

- | | | |
|---|---|---|
| 1. ☐ A ☐ B ☐ C ☐ D ☐ E | 11. ☐ A ☐ B ☐ C ☐ D ☐ E | 20. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 2. ☐ A ☐ B ☐ C ☐ D ☐ E | 12. ☐ A ☐ B ☐ C ☐ D ☐ E | 21. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 3. ☐ A ☐ B ☐ C ☐ D ☐ E | 13. ☐ A ☐ B ☐ C ☐ D ☐ E | 22. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 4. ☐ A ☐ B ☐ C ☐ D ☐ E | 14. ☐ A ☐ B ☐ C ☐ D ☐ E | 23. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 5. ☐ A ☐ B ☐ C ☐ D ☐ E | 15. ☐ A ☐ B ☐ C ☐ D ☐ E | 24. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 6. ☐ A ☐ B ☐ C ☐ D ☐ E | 16. ☐ A ☐ B ☐ C ☐ D ☐ E | 25. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 7. ☐ A ☐ B ☐ C ☐ D ☐ E | 17. ☐ A ☐ B ☐ C ☐ D ☐ E | 26. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 8. ☐ A ☐ B ☐ C ☐ D ☐ E | 18. ☐ A ☐ B ☐ C ☐ D ☐ E | 27. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 9. ☐ A ☐ B ☐ C ☐ D ☐ E | 19. ☐ A ☐ B ☐ C ☐ D ☐ E | 28. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 10. ☐ A ☐ B ☐ C ☐ D ☐ E | | |