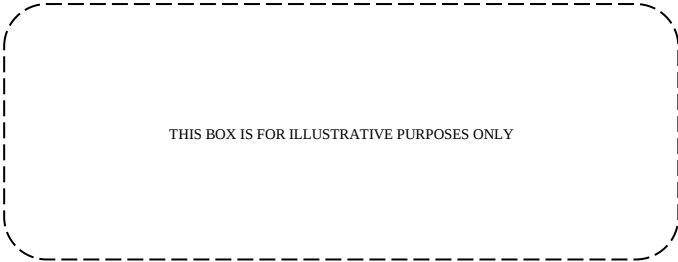




2024 Trial Examination



THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT
NUMBER

--	--	--	--	--	--	--	--

Letter

--

GENERAL MATHEMATICS

Units 3 & 4 – Written examination 2

Reading time: 15 minutes

Writing time: 1 hour and 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
15	15	60
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 19 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

Instructions

Answer all questions in the spaces provided.

In all questions where a numerical answer is required, you should only round your answer when instructed to do so.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Data analysis**Question 1. (8 marks)**

Data was collected to study newborn babies at a local hospital in a town.

The variables in this study were:

- *ID*: identity number of the baby
- *weight*: weight of the baby (g)
- *length of stay*: number of days in the hospital
- *gestational age*: pre-term (< 37 weeks), term (37-41 weeks), post-term (> 42 weeks)
- *neonatal care*: yes or no

The data collected for a sample of 15 newborn babies is displayed in Table 1.

Table 1

<i>ID</i>	<i>Weight (g)</i>	<i>Length of stay (days)</i>	<i>Gestational age (weeks)</i>	<i>Neonatal care</i>
1	2568	3	36	no
2	3100	4	45	yes
3	3450	2	38	no
4	4381	1	40	no
5	4010	4	42	yes
6	2991	3	38	no
7	3290	2	39	no
8	3176	2	38	no
9	4270	1	37	no
10	4120	5	33	yes
11	3942	6	32	yes
12	2904	3	40	no
13	4122	5	41	yes
14	3481	3	39	no
15	4400	4	44	no

a. Write down the number of categorical variables in Table 1.

1 mark

- b.** Determine the percentage of newborn babies with *length of stay* of at least 3 days.

1 mark

- c.** Determine, in grams, correct to one decimal place

- i.** the mean *weight* of all the newborn babies in this sample.

1 mark

- ii.** the median *weight* of all the newborn babies in this sample.

1 mark

- iii.** the range of the middle 50% of *weight* of all the newborn babies in this sample.

1 mark

- d.** The *gestational age*, in weeks, of this sample of newborn babies is approximately normally distributed.

Use the 68–95–99.7% rule to determine the minimum *gestational age* for a newborn baby to be included in the top 2.5% of the sample.

2 marks

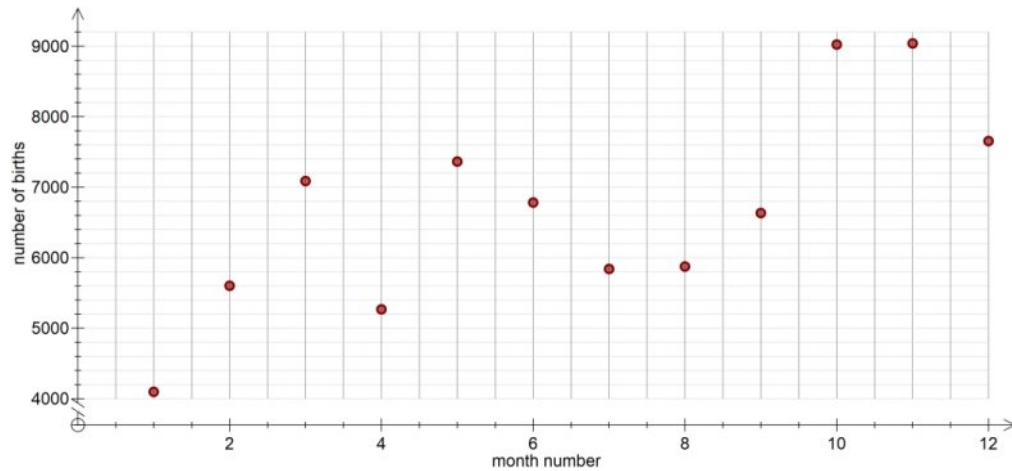
- e.** Show that the maximum *gestational age*, in weeks, of this sample of newborn babies is not an outlier.

1 mark

TURN OVER

Question 2. (5 marks)

A scatterplot is drawn below showing the relationship between *month number* and *number of births* registered in that month.



A least squares regression line is fitted to the data above.

The slope of the line is found to be 297.308 and intercept is found to be 4756.167.

- a. Determine the equation of this least squares line. Use the template below to write your answer. Round the values of the intercept and slope to four significant figures.

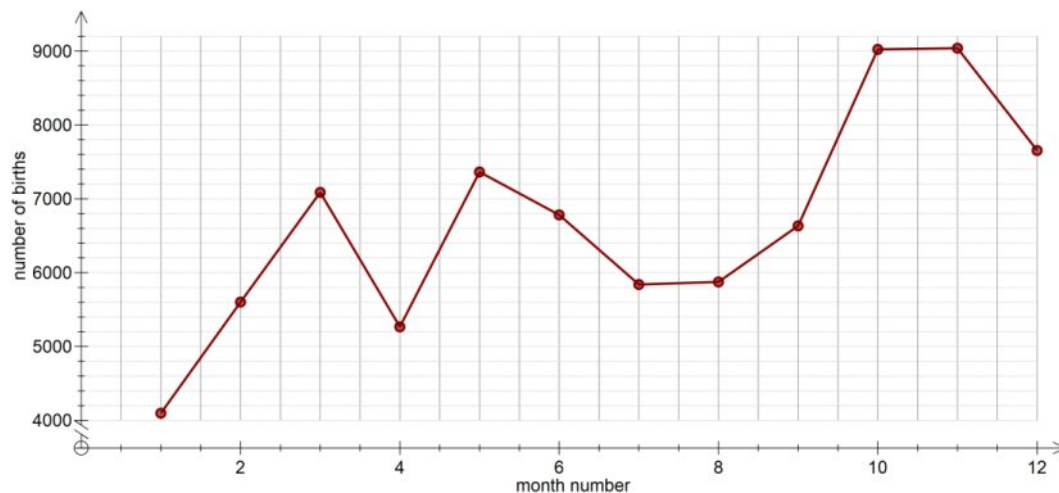
2 marks

$$\boxed{} = \boxed{} + \boxed{} \times \boxed{}$$

- b. Sketch the least squares regression line on the scatterplot provided above.

1 mark

- c. The time series plot shown below displays the trend of birth registrations over time.



A three-median smoothing technique is used to smooth the time series plot. Plot the three median smoothing value for month numbers 2, 3 and 4.

2 marks

Question 3. (6 marks)

The following data shows the maternal country of birth of a sample of 20 newborn babies.

India	Australia	India	Australia	China
Australia	Australia	Australia	India	Australia
Australia	India	China	India	India
China	Australia	China	India	Australia

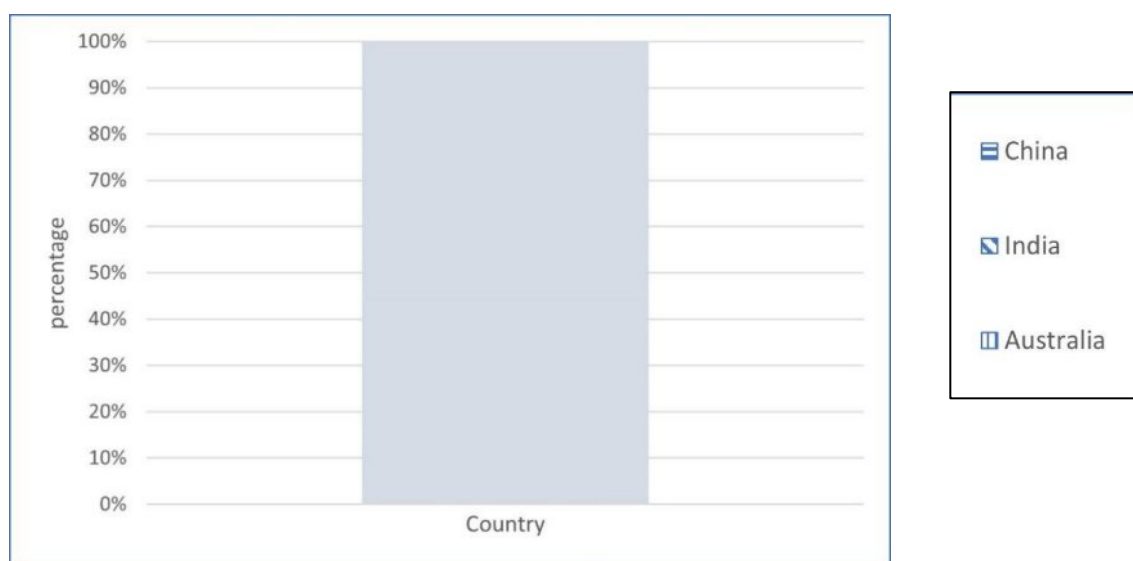
- a. Use the data above to complete the following frequency table.

2 marks

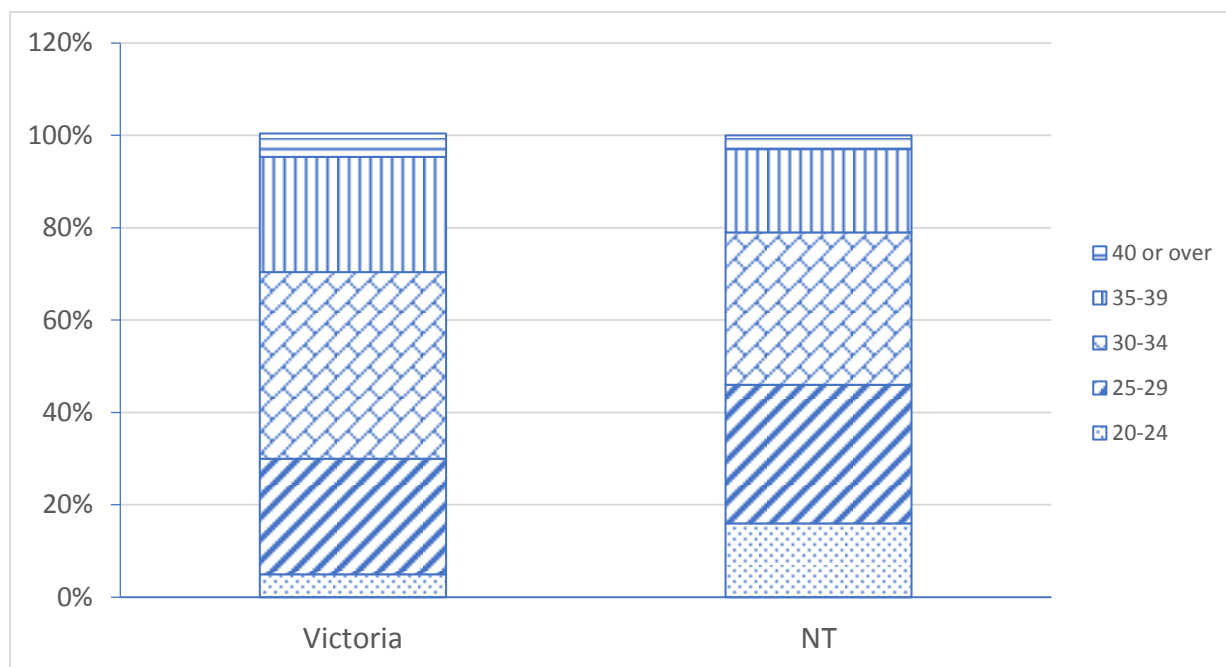
	<i>Frequency</i>	
Country	Number	Percentage (%)
Australia		45
India		35
China		20
Total		100

- b. Use the percentages in the table above to construct a percentage segmented bar chart below. A key has been provided.

1 mark



- c. The segmented bar chart below displays the percentage distribution of maternal age group (20-24, 25-29, 30-34, 35-39, 40 or over) at the time of birth for newborn babies born in 2021, in Victoria and Northern Territory (NT).



- i. Identify the response variable in this distribution.

1 mark

- ii. Does the information in the segmented bar chart support the contention there is an association between maternal age groups and place of birth?

Explain your conclusion by comparing the values of two appropriate percentages.

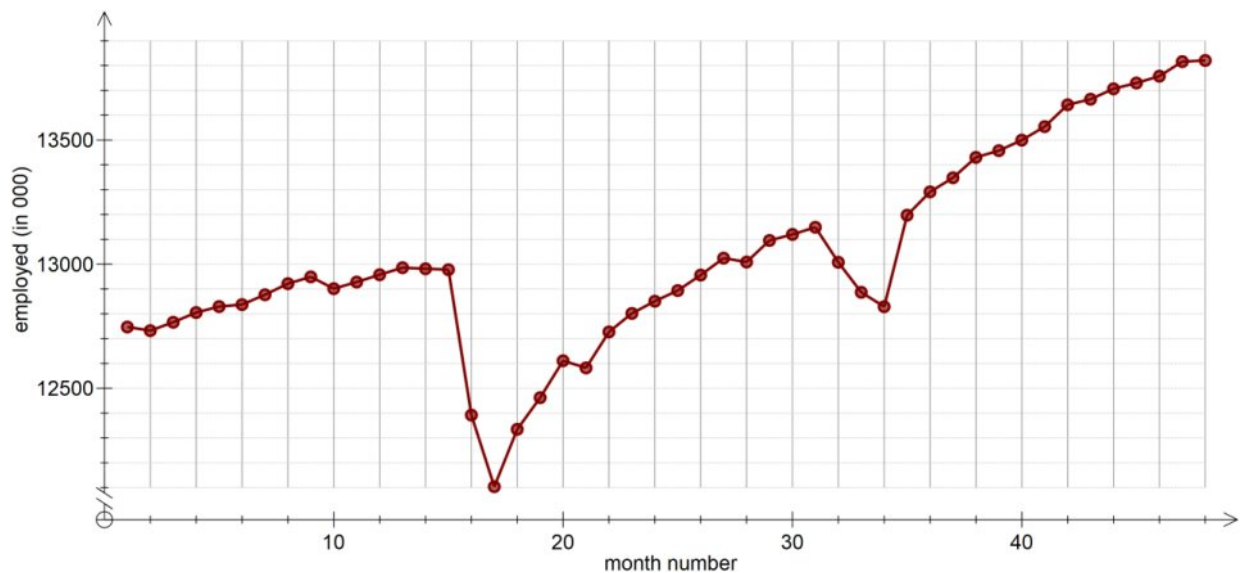
Round these percentages to the nearest whole number.

2 marks

Question 4. (3 marks)

The time series plot below shows the number of employed people each month in Australia, recorded over four years, from January 2019 to December 2022.

In this graph, month number 1 is January 2019, month number 2 is February 2019 and so on.



a. The coefficient of determination for this data set is 0.5820.

i. Determine the value of coefficient of correlation, correct to four decimal places.

1 mark

ii. Hence, describe the association between month number and number of people employed in terms of direction, form and strength.

1 mark

b. The least squares regression line is fitted to the data set to predict the number of people employed (in 000) in February 2023.

State the type of prediction made here.

1 mark

TURN OVER

Question 5. (2 marks)

The table below shows the monthly sales of jackets at a store for 2022.

Number of jackets												
Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2022	650	758	790	810	1020	1134	1765	1468	1673	650	518	422

Show that, when rounded to two decimal places, the seasonal index for June 2022 estimated from this data is 1.17. 2 marks

Recursion and financial modelling**Question 6. (4 marks)**

Sakira borrowed \$40 000 to buy a new car.

Interest on this loan is charged at the rate of 7.2% per annum, compounding quarterly.

Sakira will repay the loan in full with quarterly repayments over six years.

- a.** What is the quarterly interest rate for this loan? 1 mark

The balance of the loan, in dollars, after n quarters, L_n , can be modelled by the recurrence relation

$$L_0 = 40\,000, \quad L_{n+1} = 1.018L_n - 2067.33$$

- b.** Using recursive calculations, show that the balance of the loan after two quarters is \$37 281.09, to the nearest cent. 1 mark

- c.** The final repayment required will differ slightly from all the earlier repayments of \$2067.33.

Determine the value of the final repayment. Round your answer to the nearest cent.

1 mark

- d.** By what percentage did the value of the car increase after six years?

Give your answer to the nearest whole number.

1 mark

TURN OVER

Question 7. (4 marks)

Sakira invests \$25 000 to save for an overseas holiday.

Interest on this loan compounds weekly.

She makes an additional payment of \$ d at the end of each week, after interest is credited to her account.

The balance of the investment, in dollars, after n weeks, V_n , can be determined using a recurrence relation of the form.

$$V_0 = 25\,000, \quad V_{n+1} = 1.0006V_n + d$$

- a.** Show that the annual interest rate of Sakira's investment is 3.12%. 1 mark

- b.** Determine the value of d if Sakira has \$50 339.81 in her account after three years. 1 mark

- c.** Let $d = 120$.
Determine the total amount of interest earned by Sakira in three years of her investment. 2 marks

Question 8. (4 marks)

Sakira invests \$500 000 in an annuity that provides her with a monthly payment of \$3973.00. Interest is calculated monthly.

Three lines of the amortisation table for this annuity are shown below.

Payment number	Payment (\$)	Interest (\$)	Principal reduction (\$)	Balance (\$)
0	0.00	0.00	0.00	500 000.00
1	3442.00	2300.00	1142.00	498 858.00
2	3442.00	2294.75	1147.25	497 710.75
3				

- a. Write a calculation to show the monthly interest rate for this annuity is 0.46%.

1 mark

- b. Using the values in the table, complete the next line of the amortisation table. Write your answers in the spaces provided in the table above. Round all values to the nearest cent.

2 marks

- c. How long will this annuity provide a monthly payment of \$3442?

1 mark

TURN OVER

Matrices**Question 9. (3 marks)**

A new theme park is opened in a town.

The entry ticket sells in three forms: child (C), adult (A) and family (F).

The cost of admission, in dollars, for each ticket type is presented in matrix P below.

$$P = \begin{bmatrix} 28 \\ 36 \\ 102 \end{bmatrix} \begin{matrix} C \\ A \\ F \end{matrix}$$

A family ticket allows two adults and two children.

- a.** The element in row i and column j of matrix P is p_{ij} .

Which element shows the cost for one family ticket?

1 mark

- b.** On a particular day, 182 child tickets, 145 adult tickets and 218 family tickets are sold. The total revenue collected by the theme park that day by selling entry tickets can be calculated using the product matrix $N \times P$, where matrix N represents the number of tickets of each type sold.

Write down the matrix N .

1 mark

$N =$

- c.** A family of five, two adults and three children, buy tickets in a way to pay the minimum amount.

The total cost to the family can be calculated using the matrix equation below.

Complete the matrices below.

1 mark

$$\begin{bmatrix} \text{ } & \text{ } & 1 \end{bmatrix} \begin{bmatrix} 28 \\ 36 \\ 102 \end{bmatrix} = [130]$$

Question 10. (5 marks)

The table below shows the total revenue for the ticket sales, rounded to the nearest hundred dollars, for the first five days from its opening.

Day	1	2	3	4	5
Revenue	\$830 000	\$945 600	\$902 300	\$876 000	\$989 500

Let matrix $R = [830000 \ 945600 \ 902300 \ 876000 \ 989500]$ represent the ticket sales for the first five days.

- a. Calculate $R \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ and hence determine what this matrix calculation represents in the context of the problem. 2 marks

- b. Complete the matrix equation below that calculates the average ticket sales per day, based on sales over the first five days. 2 marks

$$[\text{ }] \times R \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = [\text{ }]$$

Let $S = k \times R \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, where k is a real number.

- c. The theme park owner decides to donate 10% of the first week's of earning to an orphanage.
Find the value of k such that S calculates the total revenue collected from ticket sales in the first five days, after donating to the orphanage. 1 mark

TURN OVER

Question 11. (2 marks)

There are five water rides (A to E) at the theme park. The lifeguards rotate through each of the five rides each week, before starting the cycle again.

The following matrix displays the movement of lifeguards between the five rides.

$$M = \begin{matrix} & \begin{matrix} \text{this week} \\ A & B & C & D & E \end{matrix} \\ \begin{matrix} \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right] \end{matrix} & \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} \end{matrix} \text{ next week}$$

- a. Lewis started as a lifeguard at ride A.

What is the order in which Lewis will be located in the first five weeks of his job?

1 mark

- b. The location for each of the five lifeguards in week 3 can be found using the matrix calculation below.

$$M^r \times \begin{bmatrix} \text{Lewis} \\ \text{Chan} \\ \text{Daisy} \\ \text{Manoj} \\ \text{Skev} \end{bmatrix}$$

where $\begin{bmatrix} \text{Lewis} \\ \text{Chan} \\ \text{Daisy} \\ \text{Manoj} \\ \text{Skev} \end{bmatrix}$ represents the location of the lifeguards in week 1.

What is the value of r ?

1 mark

Question 12. (2 marks)

The following matrix shows the proportion of students at a school who will attend/not attend the theme park next year based on whether they attended/not attended this year.

$$T = \begin{matrix} & \begin{matrix} \text{this year} \\ A & N \end{matrix} \\ \begin{matrix} \text{next year} \\ A \\ N \end{matrix} & \begin{bmatrix} 0.85 & 0.80 \\ 0.15 & 0.20 \end{bmatrix} \end{matrix}$$

The matrix recurrence relation below can be used to calculate the number of students attend/not attend the theme park each year.

$$S_{n+1} = TS_n + B, \text{ where } S_{2022} = \begin{bmatrix} 790 \\ 410 \end{bmatrix} \begin{matrix} A \\ N \end{matrix}$$

$$\begin{bmatrix} 790 \\ 410 \end{bmatrix} = \begin{bmatrix} 0.85 & 0.80 \\ 0.15 & 0.20 \end{bmatrix} \begin{bmatrix} 760 \\ 395 \end{bmatrix} + B$$

Assuming new students are enrolled and some exit the school in 2022, determine the matrix B .

2 marks

TURN OVER

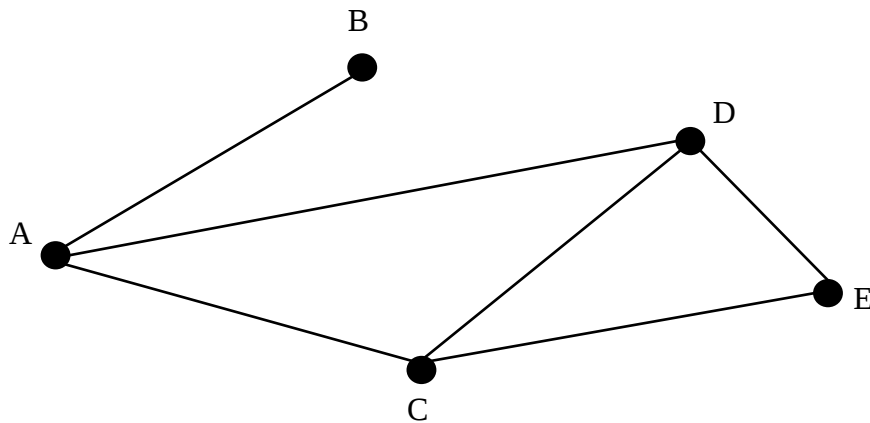
Networks and decision mathematics

Question 13. (4 marks)

A large construction site five toilet blocks, A , B , C , D and E .

A graph can be drawn with vertices to represent each of the toilet block.

Edges represent a road connection between two toilet blocks.



a.

- i. Complete the Euler's formula by writing the appropriate numbers in the boxes provided below.

1 mark

$$\boxed{} + \boxed{} = \boxed{} + \boxed{2}$$

v
 f
 e

- ii. Euler's formula confirms two properties of the graph. One of them is about the graph being connected.
State the second property.

1 mark

- b. What is the sum of the degrees of the vertices of the graph above?

1 mark

- c. Name the edge on the graph which is a bridge.

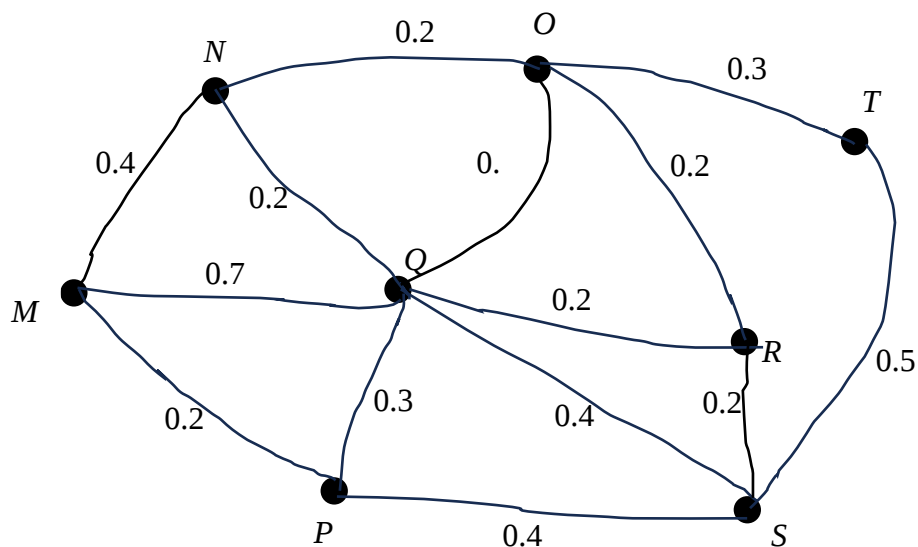
1 mark

Question 14. (5 marks)

The site has eight houses being built, M, N, O, P, Q, R, S and T .

The edges on the graph represent the roads between the houses.

The numbers on each edge represent the length, in kilometres, along each road.



Three council employees, Jobun, Lui and Vinni, are inspecting house located at M .

- a.** Jobun has an appointment with an engineer at house R .

What is the minimum distance Jobun could travel from M to R ?

1 mark

- b.** Lui would like to visit each house, for an inspection, and return at M .

- i.** Find a possible route Lui could take.

1 mark

- ii.** What is the mathematical name given to this walk?

1 mark

TURN OVER

c. Vinni would like to inspect each road connecting these houses.

i. What is the mathematical name given to this walk?

1 mark

ii. Explain why Vinni is unable to complete this walk?

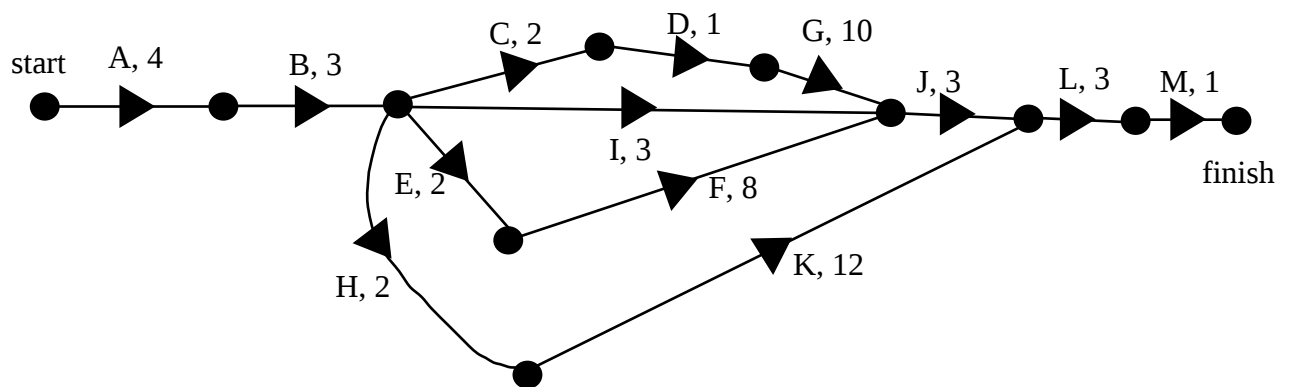
1 mark

Question 15. (3 marks)

Another site in the area is getting ready for housing.

This project involves 13 activities, A to M.

The directed network below shows these activities and their completion times, in days.



a. Which activity has two immediate predecessors?

1 mark

b. Determine the float time for activity E.

1 mark

- c. Explain why crashing activity K by 2 days will not reduce the total completion time of the project.

1 mark

END OF QUESTION AND ANSWER BOOK