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GENERAL MATHEMATICS UNITS 3 & 4

TRIAL EXAMINATION 1

2024

Reading Time: 15 minutes
Writing time: 1 hour 30 minutes

Instructions to students

This exam consists of 40 questions.
All 40 questions should be answered.
There is a total of 40 marks available for this exam.
Students may bring one bound reference into the exam.
Students may bring into the exam one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory does not need to be cleared. For approved computer-based CAS, full functionality may be used.
Unless otherwise stated, the diagrams in this exam are not drawn to scale.
An answer sheet appears on page 24 of this exam.
A formula sheet can be found on the last page of this exam.

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Data analysis

Use the following information to answer Questions 1 and 2.

The stem plot below shows the distribution of *garden areas*, in square metres, for 34 new homes.

Key: 11 |5 = 115 $n = 34$

11	5						
12							
13	2						
14	3	5					
15	2	3	4	7			
16	0	3	5	5	8	9	9
17	1	4	6	6	6	8	8
18	1	2	6	6	7	7	8
19	4	4	5	6	6		

Question 1

For these 34 new homes, the range of *garden areas* is

A. 43
B. 64
C. 71
D. 81

Question 2

The shape of the distribution is best described as

A. negatively skewed.
B. negatively skewed with one or more possible outliers.
C. positively skewed.
D. positively skewed with one or more possible outliers.

Question 3

In a study of online supermarket shoppers, data relating to the following five variables was collected:

- *latest spend* (in dollars)
- *age* (18-30 years, 31-50 years, over 50 years)
- *postcode*
- *repeat shopper* (yes, no)
- *shopping type* (delivery, click and collect)

The number of nominal variables in the study is

A. 1
B. 2
C. 3
D. 4

Use the following information to answer Questions 4 and 5.

The weights of a population of koalas are approximately normally distributed with a mean of 8.2 kg and a standard deviation of 0.9 kg.

Question 4

A randomly selected koala from this population has a standardised z-score of -1.5.
The actual weight, in kg, of this koala is

A. 6.06
B. 6.85
C. 7.44
D. 10.77

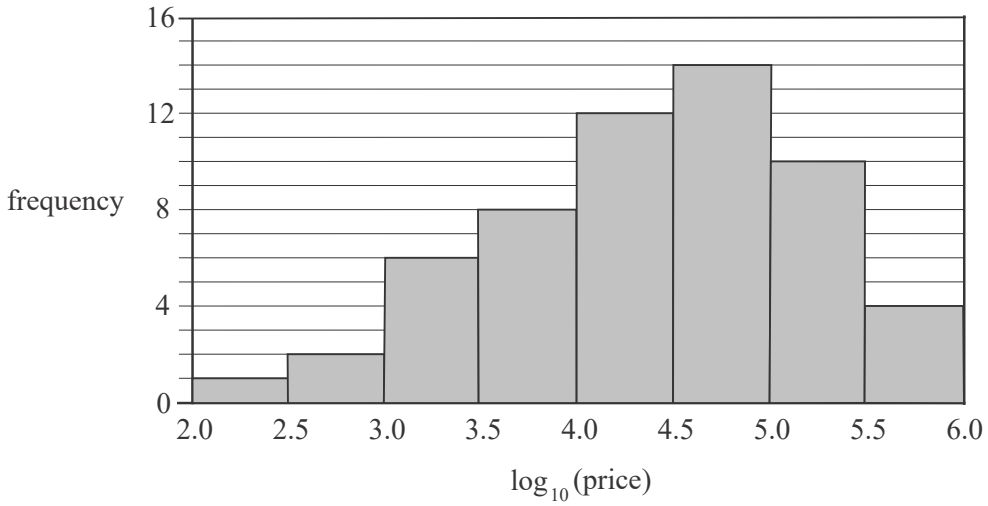
Question 5

Park rangers randomly came in contact with 268 of these koalas last year.
Using the 68-95-99.7% rule, the number of koalas that rangers came in contact with, that are expected to weigh between 5.5 kg and 8.2 kg, is closest to

A. 121
B. 130
C. 133
D. 134

Question 6

The histogram below displays the distribution of prices, in dollars, of horses at a horse sale.
The histogram has a logarithmic (base 10) scale.



Julian purchased six horses with prices of $3200, $31 500, $47 300, $75 800, $82 700 and $95 000.
The number of these six horse prices that are included in the modal class interval is

A. 1
B. 2
C. 3
D. 4

Question 7

The table below shows the *time*, in minutes, and the *age*, in years, of 12 runners who completed an eight kilometre run.

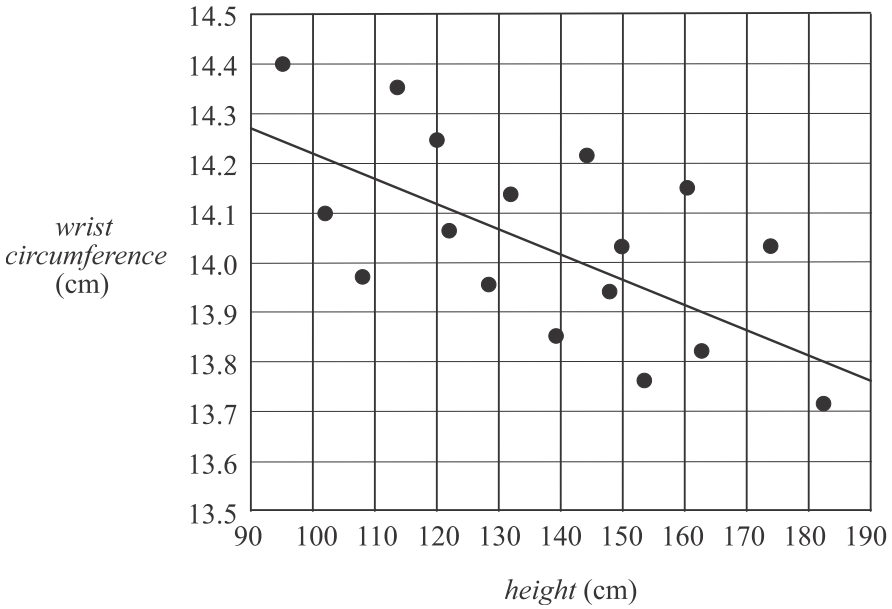
age **(years)**	***time*** **(minutes)**
22	29.3
24	33.7
26	36.2
29	32.4
30	37.5
33	36.9
37	40.1
44	45.7
65	55.2
52	49.8
58	48.3
41	39.6

The Pearson correlation coefficient, r, between *age* and *time* is closest to

A. -0.965
B. -0.931
C. 0.931
D. 0.965

Use the following information to answer Questions 8 - 10.

The scatterplot below displays the *height* and the *wrist circumference*, both in centimetres, of 17 young gymnasts.



On the assumption that the association between *wrist circumference* and *height* is linear, a least squares line has been fitted to the scatterplot which enables *wrist circumference* to be predicted from *height*.

The coefficient of determination is 0.3889.

Question 8

Which one of the following statements is **not** true?

A. The value of the correlation coefficient, *r*, is negative.
B. The explanatory variable for the least squares line is *height.*
C. There is a strong, negative, linear association between *wrist circumference* and *height*.
D. Approximately 39% of the variation in *wrist circumference* can be explained by the variation in *height.*

Question 9

The equation of the least squares line is closest to

A. $wrist\ circumference = 14.72 - 0.005 \times height$

B. $height = 14.72 - 0.005 \times wrist\ circumference$

C. $wrist\ circumference = 6882 - 76.31 \times height$

D. $height = 6882 - 76.31 \times wrist\ circumference$

Question 10

The number of young gymnasts whose actual wrist circumference was within 0.1 cm of that predicted by the least squares line is

A. 2
B. 3
C. 6
D. 8

Question 11

The association between the *weekly pay*, in dollars, and the *age*, in years, of a group of workers is non-linear.
In an attempt to linearise the data, a log (base 10) transformation is applied to the variable *age.*
The equation of the least squares line for the transformed data is given by

$$weekly\ pay = -2247.6 + 1913.9 \times \log_{10}(age)$$

According to this equation, the *age*, in years, of a worker who receives the *weekly pay* of $774.85 is closest to

A. 37.9
B. 38.0
C. 38.2
D. 38.7

Question 12

An investigation was conducted into the effects of a medication on pain relief.
As part of the investigation, the duration of *pain relief*, in hours, and the amount of *medication*, in milligrams, taken by a group of patients was recorded.
The data was used to generate the summary statistics shown below and also to fit a least squares line.

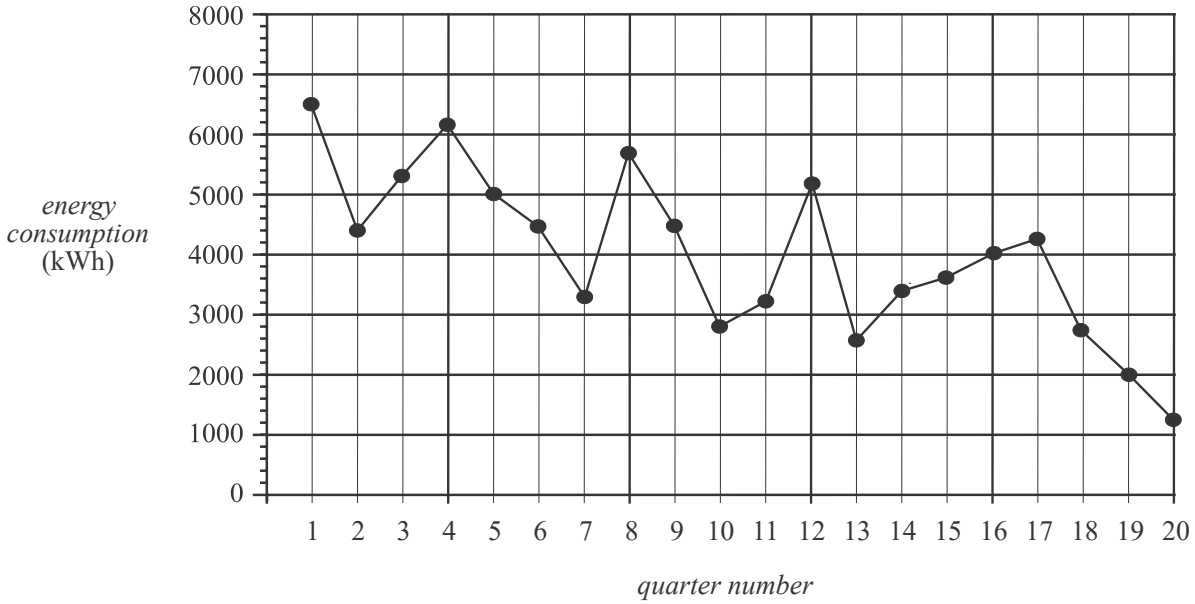
	Pain relief **(hours)**	***Medication*** **(mg)**
Mean	3.17	5.37
Standard deviation	1.64	2.83
Correlation coefficient (*r*)	0.974	

On average, for every extra milligram of *medication* taken, the duration of *pain relief* is

A. decreased by 0.16 hours.
B. decreased by 2.8 hours.
C. increased by 0.56 hours.
D. increased by 0.97 hours.

Use the following to answer Questions 13 – 15.

The time series plot below shows the quarterly *energy consumption*, in kilowatt hours, of a small business for the period quarter 1 in 2019 to quarter 4 in 2023.



Question 13

The time series plot is best described as having

A. seasonality with a decreasing trend.
B. seasonality with irregular fluctuations.
C. a decreasing trend with irregular fluctuations.
D. a decreasing trend.

Question 14

Five-median smoothing is used to smooth the time series.
The smoothed value for the energy consumption, in kilowatt hours, in quarter number 12 is closest to

A. 2800
B. 3200
C. 3440
D. 3600

Question 15

The long term seasonal index for the first quarter of each year is 1.12
The deseasonalised value for the quarterly *energy consumption*, in kWh, in the first quarter of 2020 (quarter 5) is closest to

A. 4464
B. 5000
C. 5536
D. 5600

Question 16

On a busy road, traffic lights can be activated by pedestrians at a crossing.
The table below shows the seasonal indices for the daily number of activations of these traffic lights.
The seasonal index for Saturday is missing.

Day	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.
Day number	1	2	3	4	5	6	7
Seasonal indices	1.28	1.12	1.01	1.05	0.93		0.79

The daily deseasonalised number of activations for each day can be forecast using the equation

$$deseasonalised\ number = 439.57 - 11.392 \times day\ number$$

The actual number of activations for Saturday is predicted to be closest to

A. 304
B. 313
C. 371
D. 452

Recursion and financial modelling

Question 17

The first five terms of a sequence are 10, 18, 42, 114, 330.

A recurrence relation that could be used to generate this sequence is

A. $T_0 = 10, \quad T_{n+1} = T_n + 8$

B. $T_0 = 10, \quad T_{n+1} = 2T_n + 2$

C. $T_0 = 10, \quad T_{n+1} = 3T_n - 12$

D. $T_0 = 10, \quad T_{n+1} = 4T_n - 22$

Question 18

Cam purchases a work van and depreciates its value using the unit cost method.
The recurrence relation below models its value, V_n, in dollars, after it has travelled n kilometres.

$$V_0 = 50\,000, \qquad V_{n+1} = V_n - 0.8$$

Cam will sell the van when its value reaches $35 000.
The total number of kilometres the van will have travelled by the time it is sold is

A. 12 000
B. 18 750
C. 31 250
D. 43 750

Question 19

The value, D_n, in dollars, of an annuity investment after n months, can be modelled by the recurrence relation

$$D_0 = 15\,000, \qquad D_{n+1} = 1.0025D_n + 200$$

The value of this investment after six months is

A. $14 018.89
B. $16 200.19
C. $16 219.38
D. $16 433.94

Question 20

Katrina invested $30 000 into an account earning compound interest calculated quarterly. The recurrence relation shown below models the balance, B_n, in dollars, of the account after n quarters.

$$B_0 = 30\,000, \qquad B_{n+1} = 1.016B_n$$

The effective annual rate of interest for Katrina's investment is

A. 6.28%
B. 6.40%
C. 6.56%
D. 6.64%

Question 21

If P_n represents the value of a perpetuity investment after n years, which one of the following recurrence relations could be used to model the investment?

A. $P_0 = 65\,000, \qquad P_{n+1} = 0.09P_n - 5850$
B. $P_0 = 65\,000, \qquad P_{n+1} = 1.09P_n - 5850$
C. $P_0 = 65\,000, \qquad P_{n+1} = 1.001P_n - 5850$
D. $P_0 = 65\,000, \qquad P_{n+1} = 1.01P_n - 6500$

Question 22

Jemma invests $320 000 in an annuity. She will receive a fixed amount each month from the annuity for the next 10 years. The monthly balance for this investment, B_n, in dollars, after n months, can be modelled by the recurrence relation

$$B_0 = 320\,000, \qquad B_{n+1} = RB_n - 3750$$

If the balance of the investment at the end of 10 years is zero dollars, the value of R is closest to

A. 1.006
B. 1.0072
C. 1.06
D. 1.072

Question 23

The section of the amortisation table shown below outlines the final four payments for a reducing balance loan. The loan was for 15 years and the interest was calculated just before each payment was made.

Payment number	Monthly payment	Interest	Principal reduction	Balance
177	6400.00	113.61	6286.39	15 702.97
178	6400.00	81.13	6318.87	9384.10
179	6400.00	48.48	6351.52	3032.58
180				0

The last payment had to be adjusted to achieve a zero balance. The value of the last monthly payment is closest to

A. $3016.91
B. $3032.58
C. $3048.25
D. $3367.42

Question 24

Elisse borrows $50 000 to start her own business. The interest on the loan is 5.8% per annum compounding monthly. She plans to fully repay the loan by making monthly payments of $1516.57 for three years.

At the end of the first year of the loan, her grandmother gives her a gift of $5000, which Elisse pays straight into the loan account to reduce the balance owed. She will now make adjusted monthly payments for the remaining two years in order to fully repay the loan.

The difference between the total of the monthly payments over the three years, that Elisse would have paid with or without the $5000 gift, is closest to

A. $1516.57
B. $4710.00
C. $5307.60
D. $7961.40

Matrices

Question 25

The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is an example of

A. an identity matrix.
B. a binary matrix.
C. a diagonal matrix.
D. a permutation matrix.

Question 26

The matrix below shows how five people, Val (V), Wes (W), Xander (X), Yolanda (Y) and Zoe (Z) can communicate with each other.

$$\begin{array}{cc} & \begin{array}{ccccc} \multicolumn{5}{c}{\textit{receiver}} \\ V & W & X & Y & Z \end{array} \\ \begin{array}{c} \\ V \\ W \\ \textit{sender}\ X \\ Y \\ Z \end{array} & \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

In this matrix

- the '1' in row V, column X, indicates that Val can communicate directly with Xander
- the '0' in row W, column V indicates that Wes cannot communicate directly with Val

Zoe needs to communicate with Val.
The shortest possible communication link that enables this to happen is

A. $Z - V$
B. $Z - Y - V$
C. $Z - Y - W - V$
D. $Z - W - X - V$

Question 27

Matrix M is a 1×3 matrix.
Matrix N is a 1×2 matrix.
Matrix O is a 2×3 matrix.

The transpose of each of the matrices M, N and O are written as M^T, N^T and O^T respectively.
The order of the matrix expression $(M \times O^T)^T \times N$ is

A. 1×2
B. 2×2
C. 2×3
D. 3×2

Question 28

The movement of a population of wallabies between three different locations *P, Q* and *R* from one evening to the next is modelled by the transition matrix *T* below.

$$T = \begin{array}{c} \\ P \\ Q \\ R \end{array} \begin{array}{c} \begin{array}{ccc} P & Q & R \end{array} \\ \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0 & 0.6 & 0 \\ 0.1 & 0.3 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ P \\ Q \\ R \end{array}$$

this evening / *next evening*

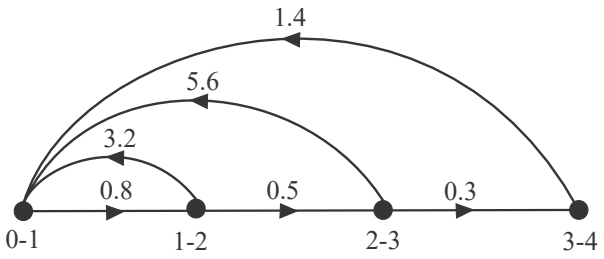
Which one of the following statements best describes what will happen in the long term?

A. Some of the wallabies will end up at location *P*.
B. All of the wallabies will end up at location *R*.
C. All of the wallabies will end up at location *P*.
D. Some of the wallabies will end up at location *Q*.

Question 29

A Leslie matrix, L, is used to model how a population of female fish changes over time. These fish have a life expectancy of four years and the population can be divided into four age groups, 0-1, 1-2, 2-3, 3-4.

The information in L is presented in the transition diagram below.



The matrix L is

A. $\begin{bmatrix} 0 & 3.2 & 5.6 & 1.4 \\ 0.8 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 0.8 & 0.5 & 0.3 \\ 3.2 & 0 & 0 & 0 \\ 0 & 5.6 & 0 & 0 \\ 0 & 0 & 1.4 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 0.8 & 3.2 & 5.6 & 1.4 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 3.2 & 5.6 & 1.4 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.3 \end{bmatrix}$

Question 30

The matrix A has a determinant equal to one.

Consider the following four statements:

- Matrix A could be a row matrix.
- The inverse matrix A^{-1} does not exist.
- Matrix A must be an identity matrix.
- Matrix A and its inverse matrix, A^{-1}, could be equal to one another.

The number of these statements that are true is

A. 0
B. 1
C. 2
D. 3

Question 31

Five workmates, Erin (*E*), Fred (*F*), Georgie (*G*), Harry (*H*) and Ivo (*I*) played a round robin darts tournament one lunchtime. All five played each of their four workmates once. In every match there was a winner and a loser.
The matrix below shows the one-step dominance matrix which displays the results of the tournament.

		loser				
		E	*F*	*G*	*H*	*I*
	E	0	0	0	0	1
	F	1	0	0	1	0
winner	*G*	1	1	0	1	0
	H	1	0	0	0	0
	I	0	1	1	1	0

A '1' in the matrix indicates that the player named in that row defeated the player named in that column.
For example, in the second row and fourth column, the '1' indicates that Fred defeated Harry.
The players are ranked using the sum of their two-step dominances.
The winner of the tournament has the highest two-step dominance sum.
Which one of the following tables gives the correct player ranking for the tournament?

A.

Ranking	Player
1	Erin
2	Harry
3	Fred
4	Georgie
5	Ivo

B.

Ranking	Player
1	Georgie
2	Ivo
3	Erin
4	Harry
5	Fred

C.

Ranking	Player
1	Fred
2	Erin
3	Georgie
4	Ivo
5	Harry

D.

Ranking	Player
1	Ivo
2	Georgie
3	Erin
4	Fred
5	Harry

Question 32

Guests on a five night cruise have their evening meal at the Ark Restaurant (*A*), the Bay Bistro (*B*), the Captains Table (*C*) or the Deck Bar (*D*)
The transition matrix, *T*, below, shows the proportion of guests who change location from one night to the next. Matrix *T* is incomplete.

$$T = \begin{array}{c} \\ \\ \\ \\ \\ \end{array} \begin{array}{c} \textit{this night} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0.6 & 0.3 & - & 0.1 \\ - & 0.5 & 0.1 & - \\ 0.2 & - & 0.4 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.3 \end{bmatrix} \end{array} \begin{array}{l} A \\ B \\ C \\ D \\ \textit{next night} \end{array}$$

On the first night of the cruise, 80 guests dined at the Ark Restaurant, 40 dined at the Bay Bistro, 10 dined at the Captains Table and 50 dined at the Deck Bar.

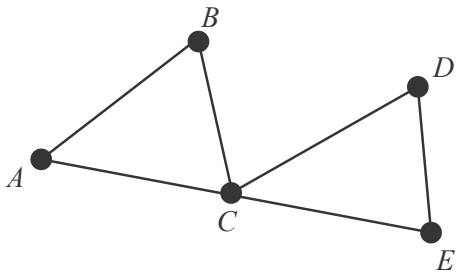
Of those guests who dined at the Ark Restaurant on the third night of the cruise, the percentage of these who also dined at the Ark restaurant on the second night of the cruise is closest to

A. 57.5%
B. 60%
C. 67%
D. 95.8%

Networks and decision mathematics

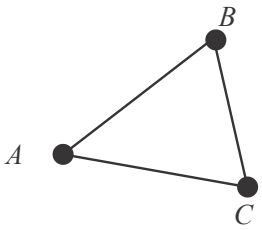
Question 33

Consider the graph below.

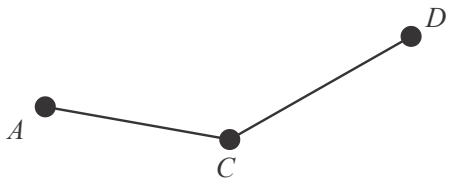


Which one of the following is **not** a subgraph?

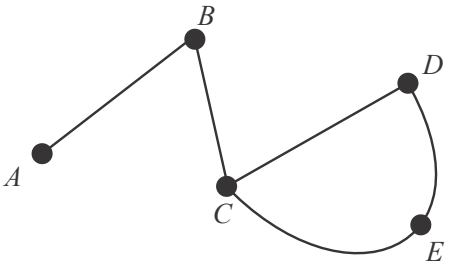
A.



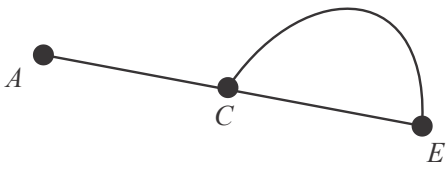
B.



C.

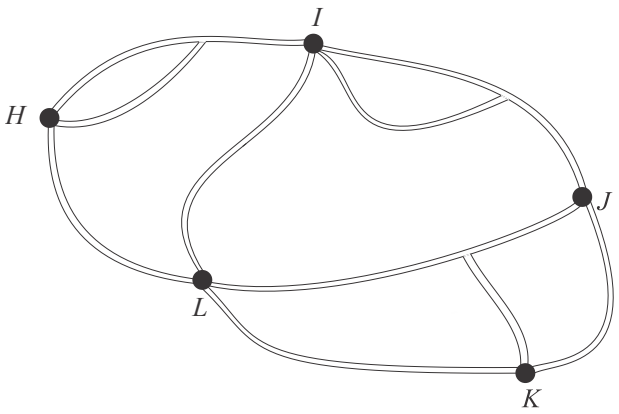


D.



Question 34

The map below shows all the path connections between five markers, *H, I, J, K* and *L* on an orienteering course.



The adjacency matrix that could represent these path connections is given by

A.

	H	*I*	*J*	*K*	*L*
H	1	2	0	0	1
I	2	1	2	0	1
J	0	2	0	2	1
K	0	0	2	0	2
L	1	1	1	2	0

B.

	H	*I*	*J*	*K*	*L*
H	0	2	0	0	1
I	2	0	2	0	1
J	0	2	0	2	1
K	0	0	2	0	2
L	1	1	1	2	0

C.

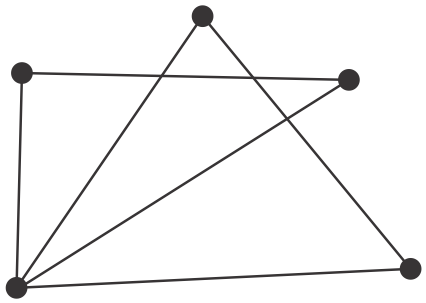
	H	*I*	*J*	*K*	*L*
H	0	3	0	0	1
I	3	0	2	0	1
J	0	2	0	2	1
K	0	0	2	0	2
L	1	1	1	2	0

D.

	H	*I*	*J*	*K*	*L*
H	1	3	0	0	1
I	3	1	2	0	1
J	0	2	0	3	1
K	0	0	2	0	2
L	1	1	1	2	0

Question 35

Consider the graph below.

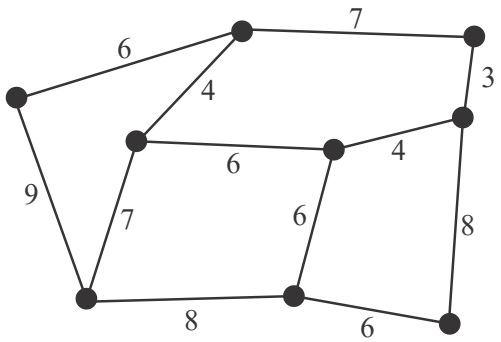


The number of faces is

A. 3
B. 4
C. 5
D. 6

Question 36

A weighted graph is shown below.

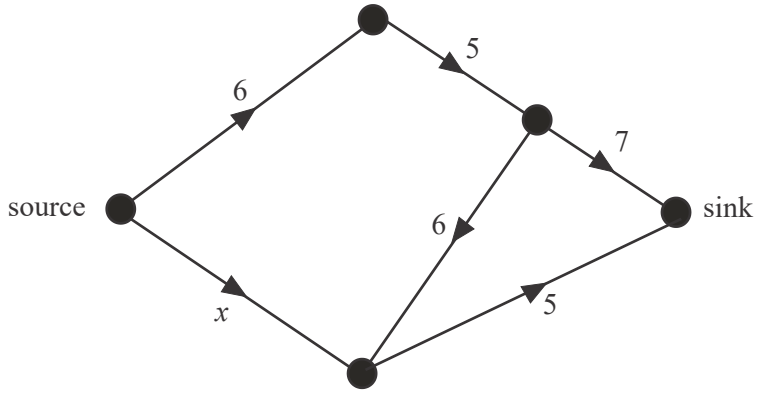


The minimum spanning tree for this graph has a weight of

A. 35
B. 36
C. 39
D. 42

Question 37

The following network shows the flow of water, in litres per minute, through a system of pipes that connect the source to the sink.



The maximum possible flow through the network, from source to sink, is 9 litres per minute.

The value of x is

A. 2
B. 3
C. 4
D. 5

Question 38

A connected planar graph consists of seven vertices and six edges.
Consider the following four statements:

- The graph could be bipartite.
- The graph could contain a path.
- The graph could contain a cycle.
- The graph could contain two or more bridges.

How many of these four statements are true?

A. 1
B. 2
C. 3
D. 4

Question 39

A project involves nine activities, *A* to *I*.
The table below shows the immediate predecessor(s) and the duration, in days, for each activity.

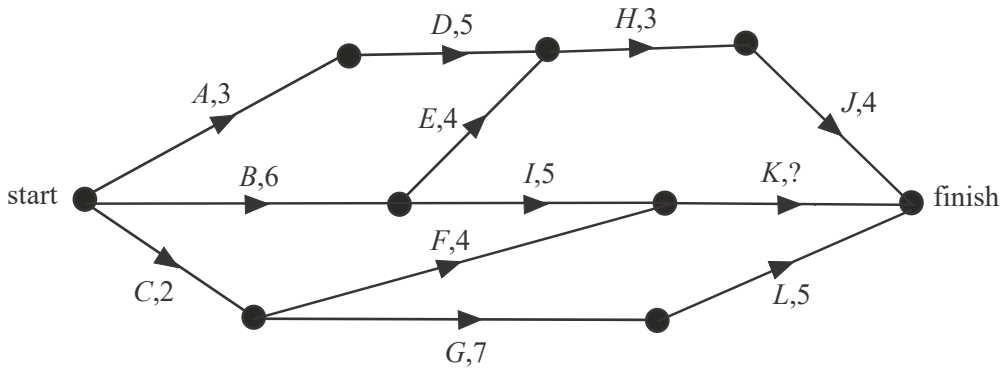
Activity	Immediate predecessor	Time (days)
A	-	8
B	-	20
C	-	33
D	*A*	18
E	*A*	20
F	*B*	9
G	*B, C*	10
H	*D*	8
I	*E, F*	4

A directed network for this project will require a dummy activity.
The dummy activity will be drawn from the end of

A. activity *A* to the start of activity *F*.
B. activity *B* to the start of activity *E*.
C. activity *C* to the start of activity *F*.
D. activity *B* to the start of activity *G*.

Question 40

The directed network below shows a sequence of activities required to complete a project. The duration, in days, for each activity is shown next to the activity except for activity *K*.



The project has a minimum completion time of 17 days.
If activity *K* has a float time of 5 days, then the duration, in days, of activity *K* is

A. 1
B. 3
C. 4
D. 6

GENERAL MATHEMATICS UNITS 3 & 4

TRIAL EXAMINATION 1

MULTIPLE-CHOICE ANSWER SHEET

STUDENT NAME ……..

INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example: (A) ■ (C) (D)

The answer selected is B. Only one answer should be selected.

1.	A	B	C	D	21.	A	B	C	D
2.	A	B	C	D	22.	A	B	C	D
3.	A	B	C	D	23.	A	B	C	D
4.	A	B	C	D	24.	A	B	C	D
5.	A	B	C	D	25.	A	B	C	D
6.	A	B	C	D	26.	A	B	C	D
7.	A	B	C	D	27.	A	B	C	D
8.	A	B	C	D	28.	A	B	C	D
9.	A	B	C	D	29.	A	B	C	D
10.	A	B	C	D	30.	A	B	C	D
11.	A	B	C	D	31.	A	B	C	D
12.	A	B	C	D	32.	A	B	C	D
13.	A	B	C	D	33.	A	B	C	D
14.	A	B	C	D	34.	A	B	C	D
15.	A	B	C	D	35.	A	B	C	D
16.	A	B	C	D	36.	A	B	C	D
17.	A	B	C	D	37.	A	B	C	D
18.	A	B	C	D	38.	A	B	C	D
19.	A	B	C	D	39.	A	B	C	D
20.	A	B	C	D	40.	A	B	C	D

