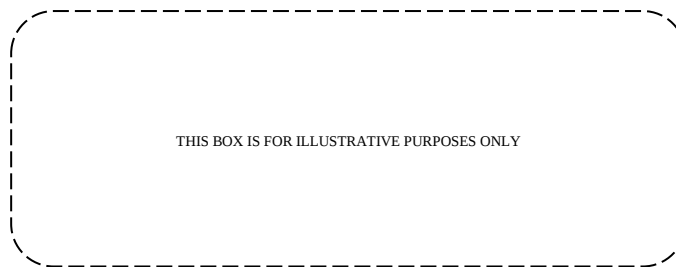




2023 Trial Examination



THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT
NUMBER

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Letter

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GENERAL MATHEMATICS

Units 3 & 4 – Written examination 2

Reading time: 15 minutes

Writing time: 1 hour and 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Section A - Core	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
	7	7	36
Section B - Modules	<i>Number of topics</i>	<i>Number of topics to be answered</i>	<i>Number of marks</i>
	2	2	24
			Total 60 marks

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 19 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION A – Data Analysis and Recursion and financial modelling**Instructions for Section A**

Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π , surds or fractions.

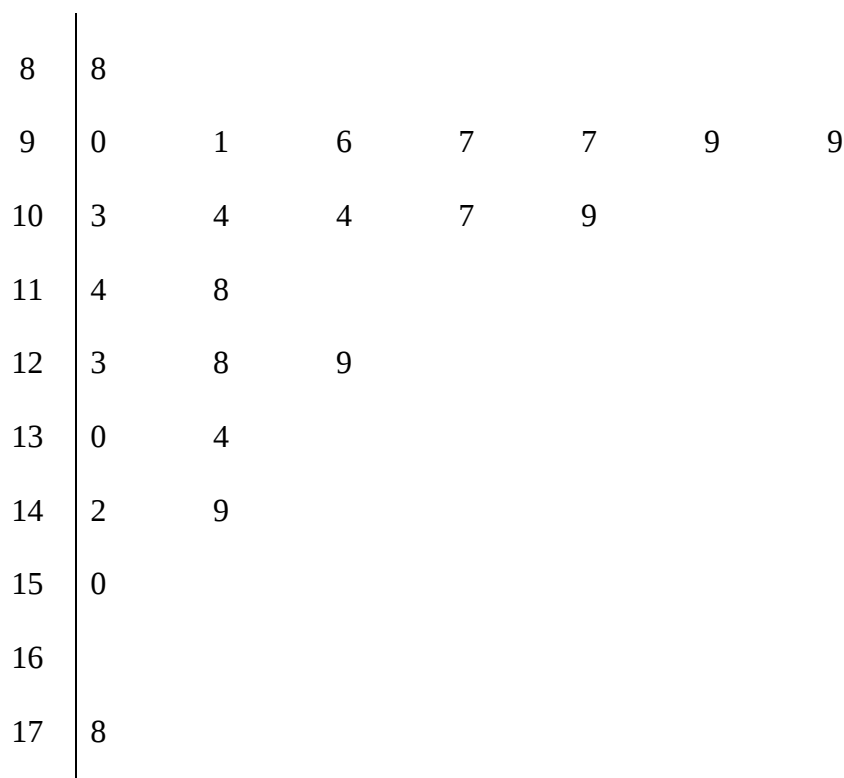
In ‘Recursion and financial modelling’, all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis**Question 1. (7 marks)**

The stem plot below displays the distribution of *unemployment rate*, in %, of females recorded over the last 24 months in Australia.

Key: 8|8 = 8.8%



a. Determine the median *unemployment rate*, in %.

1 mark

SECTION A – Question 1 - continued

- b. Determine the percentage of months for which the *unemployment rate* was less than 12.3%.

1 mark

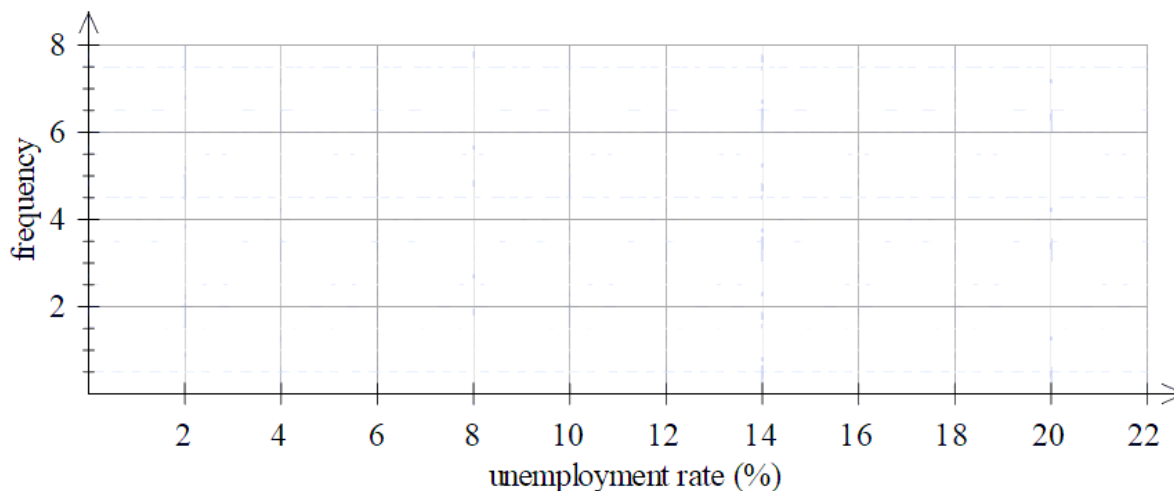
- c. Show that the *unemployment rate* of 17.8% is an outlier.

2 marks

d.

- i. On the grid below, use the data from the stem plot on page 3 to construct a histogram that displays the distribution of *unemployment rate*.
Use class intervals of widths of two, starting at 8%.

2 marks



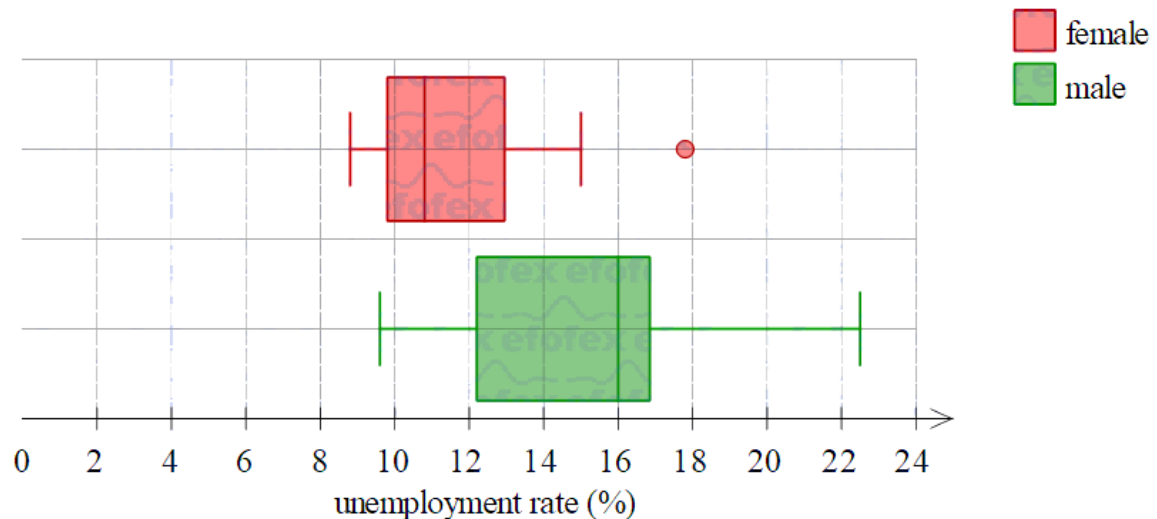
- ii. Hence, describe the shape of the distribution of *unemployment rate*.

1 mark

SECTION A – continued
TURN OVER

Question 2. (5 marks)

The boxplots below show the distribution of *unemployment rate (%)* of females and males over the last 24 months.



a.

- i.** Write down the percentage of months for which the *unemployment rate* of males was less than 16%.

1 mark

- ii.** Determine whether the *unemployment rate* is more variable for males or females. Give the approximate values of both the interquartile ranges in your answer.

2 marks

SECTION A – Question 2 - continued

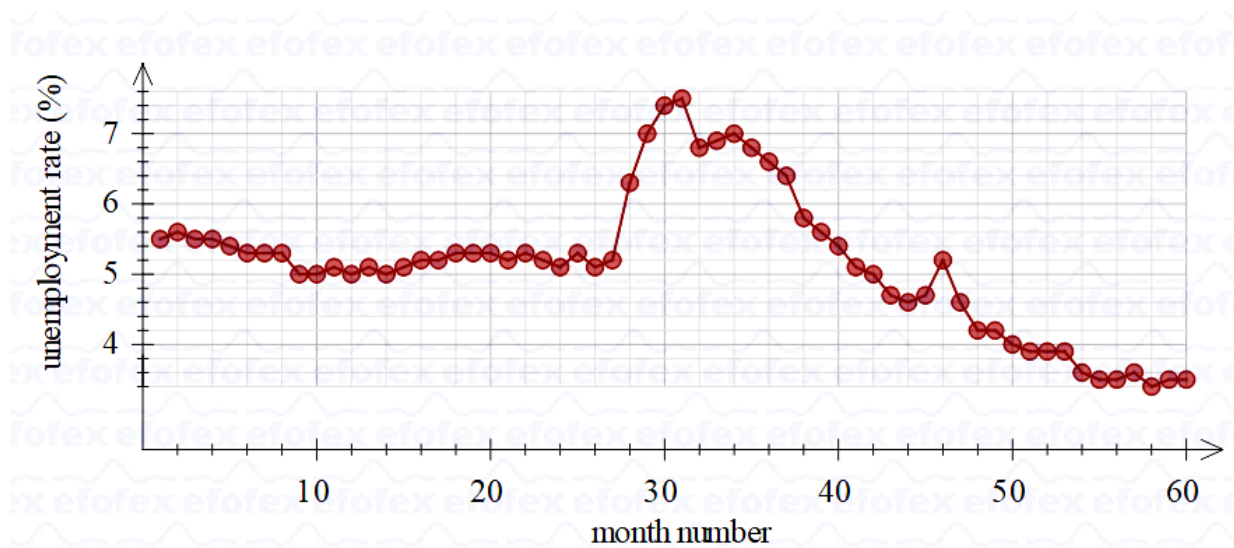
- b. Do the boxplots support the contention that there is an association between *unemployment rate* of gender?

Refer to the values of an appropriate statistic in your answer.

2 marks

Question 3. (8 marks)

The time series plot below shows the unemployment rate (%), in Australia over the last 60 months.



A least squares regression line has been fitted to the plot to model the trend in the unemployment rate over this period.

The equation of the least squares line is

$$\text{unemployment rate}(\%) = 5.976723 - 0.026832 \times \text{month number}$$

- a. Describe the time series trend from the graph above.

2 marks

SECTION A – Question 3 - continued
TURN OVER

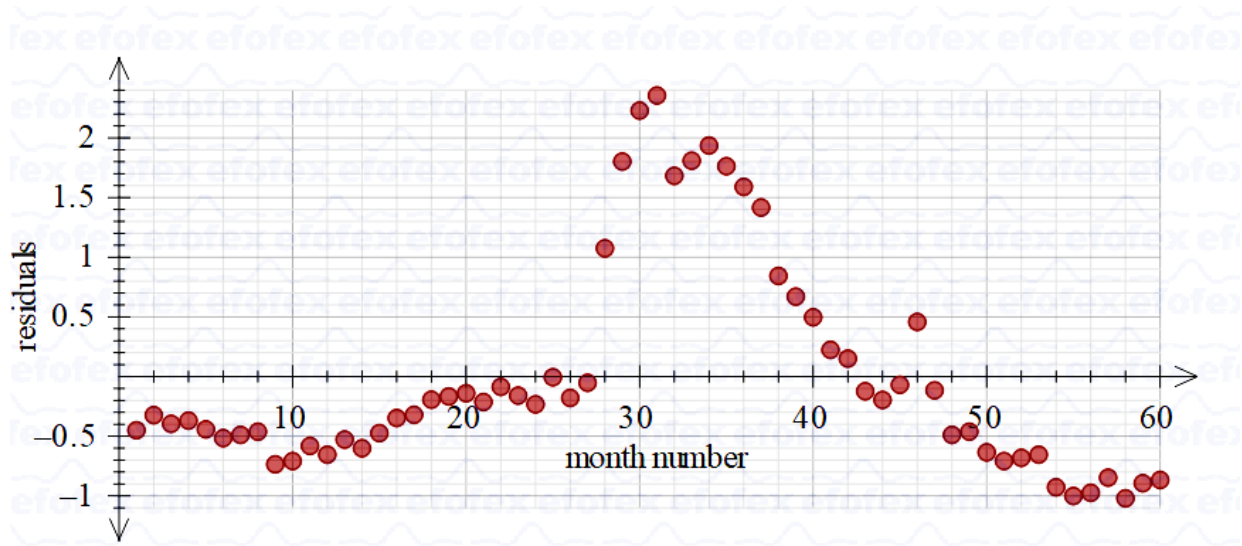
- b. Interpret the slope of the least squares line in terms of the variables unemployment rate and month number. 1 mark

- c. Round the intercept and slope to four significant figures.
Write your answers in the boxes provided.

2 marks

$$\text{unemployment rate}(\%) = \boxed{} - \boxed{} \times \text{month number}$$

The residual plot of unemployment rate over the last 60 months is shown below.



- d. The residual plot shows a residual of 0.5 for month number 46.
Determine the unemployment rate, in %, in month number 46.
Round your answer to one decimal place.

2 marks

SECTION A – Question 3 - continued

- e. The residual plot above shows a clear pattern.

What information does this provide about the relationship between the unemployment rate and month number? 1 mark

Question 4. (4 marks)

The time series data below shows the daily *maximum temperature*, in degrees Celsius, recorded in Melbourne for a period of two weeks.

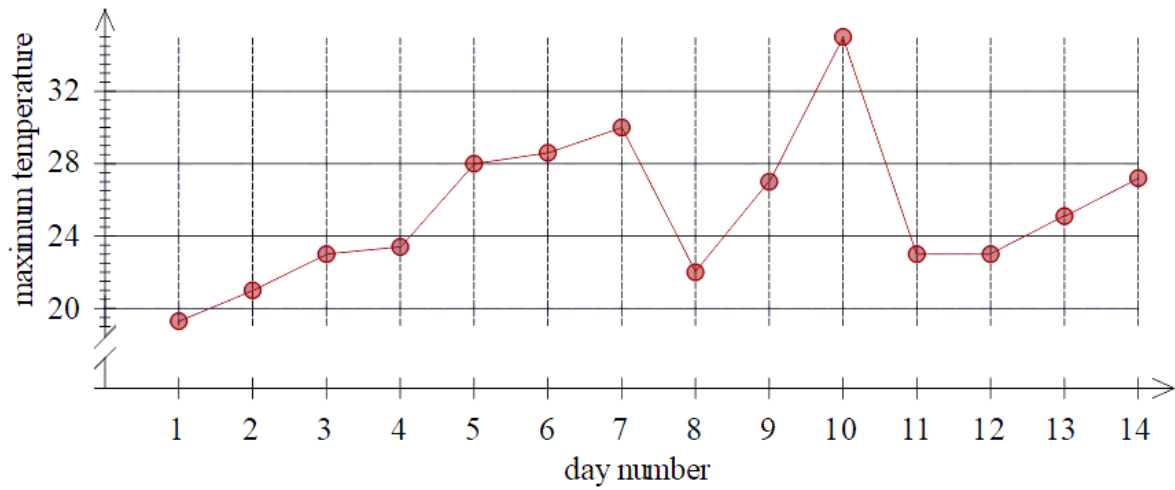
Day number	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Maximum temperature (°C)	19.3	21.0	23.0	23.4	28.0	28.6	30.0	22.0	27.0	35.0	23.0	23.0	25.1	27.2

- a. Write down the value of the five-mean smoothed temperature for day 10? 1 mark

- b. What would be the number of points in the smoothed plot if nine-mean smoothing is applied for the full two-week period. 1 mark

SECTION A – Question 4 - continued
TURN OVER

The time series plot below shows the maximum temperature, in degrees Celsius, over the two-week period.



- c. Write down the value of the five-median smoothed rainfall for day 6. 1 mark

- d. Determine the standard deviation of maximum temperature over the two-week period.
Give your answer correct to two decimal places.

1 mark

SECTION A – continued

Recursion and financial modelling**Question 5. (5 marks)**

Mahi owns workplace equipment, which she depreciates in value using flat rate depreciation. The value of the equipment, in dollars, after n years, V_n ,

$$V_n = 180\,000, \quad V_n = 180\,000 - 10\,500n$$

- a. Determine V_1 , the value of the equipment after one year. 1 mark

- b. The value of the equipment, in dollars, after n years, V_n , can also be modelled by a recurrence relation.

Write this recurrence relation in terms of V_0 , V_{n+1} and V_n . 1 mark

- c. After how many years will the equipment first have a value below \$100 000? 1 mark

- d. By what percentage does Mahi's workplace equipment reduce during the second year?

Give your answer correct to two decimal places. 1 mark

SECTION A – continued
TURN OVER

Question 6. (4 marks)

Mahi invests \$360 000 in an annuity paying 3% interest per annum, compounding monthly.

Her annuity will provide a monthly payment of \$2980.03 for 12 years.

The first four lines of the amortisation table for Mahi's annuity are shown below.

The information for payment number 2 is missing.

Payment number	Payment (\$)	Interest (\$)	Principal reduction (\$)	Balance (\$)
0	0.00	0.00	0.00	360 000.00
1	2980.03	900.00	2080.03	357 919.97
2				

- a. What is the value of payment number 2? 1 mark

- b. Calculate the interest associated with payment number 2? 1 mark

- c. Complete the recurrence relation below, in terms of A_0 , A_{n+1} and A_n , that could be used to model the value of Mahi's annuity from month to month.

1 mark

- d. If Mahi had invested the original \$360 000 annuity as a perpetuity at the same annual interest rate, what monthly payment would she have drawn?

1 mark

SECTION A – continued

Question 7. (3 marks)

To purchase furniture for her workplace, Mahi took out a reducing balance loan of \$540 000 with interest calculated monthly.

The balance of the loan, in dollars, after n months, L_n , can be modelled by the recurrence relation

$$L_0 = 540\,000, \quad L_{n+1} = 1.00175L_n - 2305.80$$

- a.** Showing recursive calculations, determine the balance of the loan after two months.

1 mark

- b.** Determine the annual compound interest rate for this loan.

1 mark

- c.** Mahi wants to pay off the loan one year earlier than the total time taken to pay off the loan using the recursion relation below.

$$L_0 = 540\,000, \quad L_{n+1} = 1.00175L_n - a$$

Determine the value of a to the nearest cent.

1 mark

END OF SECTION A
TURN OVER

SECTION B – Matrices and Networks and decision mathematics**Instructions for Section B**

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example π , surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Matrices**Question 1. (3 marks)**

Matrix R shows the cost, in dollars, of three modes of transport from Town A to Town B: airplane (A), bus (B) and train (T).

$$R = \begin{bmatrix} & A & B & T \\ 110 & 80 & 65 \end{bmatrix}$$

The Kumar family of five adults is planning to travel from Town A to Town B.

- a. The matrix equation below can be used to calculate the cost of travelling from town A to Town B by each mode of transport for the Kumar family.

$$[k] \times \begin{bmatrix} 110 & 80 & 65 \end{bmatrix} = \begin{bmatrix} a & b & c \end{bmatrix}$$

Write down the value of k , a , b and c .

2 marks

- b. The Kumar family decides to travel by train.
The matrix product PL can be used to calculate the total cost to the Kumar family to travel from Town A to Town B.
Write down the matrix L .

1 mark

$$L =$$

SECTION B – continued

Question 2. (4 marks)

The café in the train sells boxed meals of two types- cold meal and hot meal.

Let x represent the cost, in dollars, of a cold meal and y represent the cost, in dollars, of a hot meal.

The following system of simultaneous linear equations represents the number of cold meals and hot meals sold over two days.

$$40x + 60y = 2160$$

$$70x + 50y = 2460$$

- a. Write the equations in matrix form by completing the blank matrices below. 1 mark

$$\begin{bmatrix} 40 & 60 \\ 70 & 50 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2160 \\ 2460 \end{bmatrix}$$

b. Let $A = \begin{bmatrix} 40 & 60 \\ 70 & 50 \end{bmatrix}$

- i. Explain, quoting values, why A^{-1} exists. 1 mark

- ii. The product matrix BC can be used to calculate the cost of one cold meal and one hot meal.

Write down the matrix B .

1 mark

$$B =$$

- c. Determine the cost of two cold meals and three hot meals. 1 mark

SECTION B – continued
TURN OVER

Question 3. (3 marks)

Some employees of a company travel weekly to three locations- Sydney, Adelaide or Perth.

Each week, these employees must travel to one of these locations for work.

Matrix T below contains the proportion of employees who are expected to change their location from week to week.

$$T = \begin{array}{ccc|c} & \text{this week} & & \\ & S & A & P \\ \begin{array}{l} S \\ A \\ P \end{array} & \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.7 & 0.1 \\ 0 & 0.1 & 0.8 \end{bmatrix} & \begin{array}{l} S \\ A \\ P \end{array} & \text{next week} \end{array}$$

The initial state matrix S_1 below shows the number of employees who travelled to each location in week 1.

$$S_1 = \begin{array}{l} \begin{bmatrix} 80 \\ 50 \\ 40 \end{bmatrix} \begin{array}{l} S \\ A \\ P \end{array} \end{array}$$

Let S_n be the matrix that shows the number of employees who travel to each location in week n .

- a. What does the number 0 in the third row and first column represent? 1 mark

- b. How many employees do not change their location of travel from week 1 to week 2? 1 mark

- c. Determine the number of employees who travel to Sydney for work in week 5.
Give your answer to the nearest whole number.

1 mark

SECTION B – continued

Question 4. (2 marks)

The matrix below is a Leslie matrix that models the change in population of a particular species from one age group to another.

$$L = \begin{array}{ccc|c} & \begin{array}{ccc} \text{from one age group} \\ 1 & 2 & 3 \end{array} & \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 2.4 & 0.3 \\ 0.6 & 0 & 0 \\ 0 & 0.2 & 0 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{array}{c} \\ \text{to the next age group} \\ \end{array} \end{array}$$

The initial state matrix S_0 below shows the number of females in each age group at the start.

$$S_0 = \begin{bmatrix} 200 \\ 200 \\ 200 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

- a.** Explain what the number 0.6 represents in the matrix L . 1 mark

- b.** Determine the number of females in the population in age group 2 after one time period. 1 mark

SECTION B – continued
TURN OVER

Networks and decision mathematics

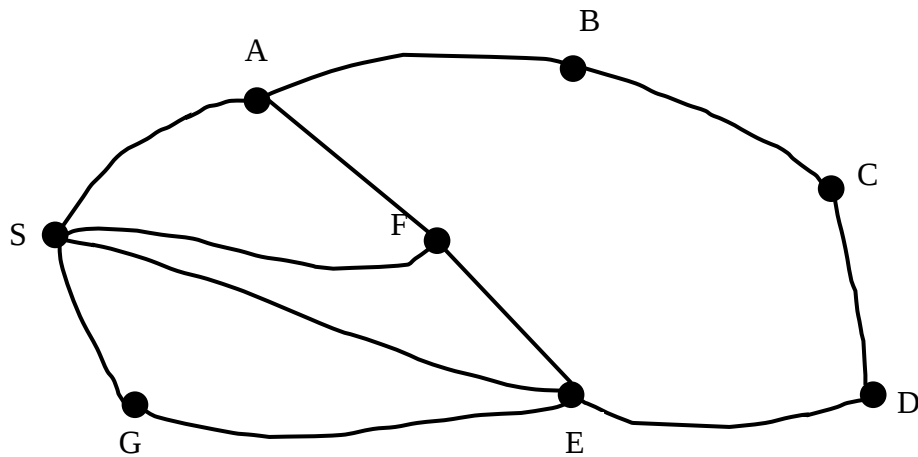
Question 1. (4 marks)

Josie is a head supervisor of a cleaning company.

She visits a small hospital in town to audit cleaning practices.

The diagram below shows a network of paths in the hospital that lead to six different wards- A, B, C, D, E, F, G .

The vertex S represents the entry to the hospital.



- a. On a particular day, Josie starts at the entry (S) and visits each ward once only to audit before returning to the main entry.

i. Write down a route that Josie could follow.

1 mark

ii. What is the mathematical term given to the route found in part i.?

1 mark

- b. Later in the day, Josie will start from ward F and travel along each path once only to complete the cleaning audit.

i. Which vertex will Josie finish on?

1 mark

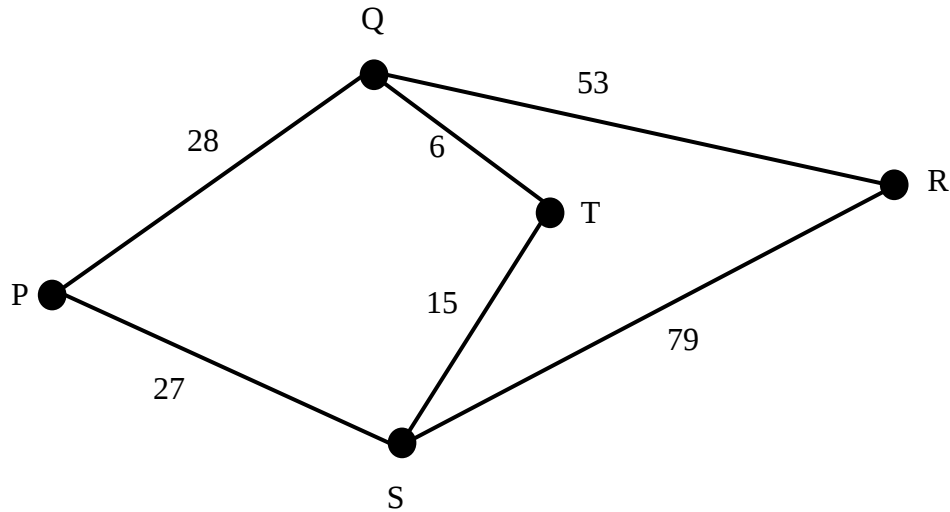
ii. What is the mathematical term given to the route found in part i.?

1 mark

Question 2. (3 marks)

Josie also travels to different towns to complete cleaning audits.

The following weighted network shows the roads connecting each pair of town and distances, in kilometres, between five towns- *P*, *Q*, *R*, *S* and *T*



a.

i. Draw a minimum spanning tree for the network above.

1 mark

ii. Determine the minimum distance that Josie will have to travel.

1 mark

b. Josie would like to travel on each road exactly once to inspect the condition of each road. Explain why Josie is unable to travel along each road exactly once.

1 mark

SECTION B – continued
TURN OVER

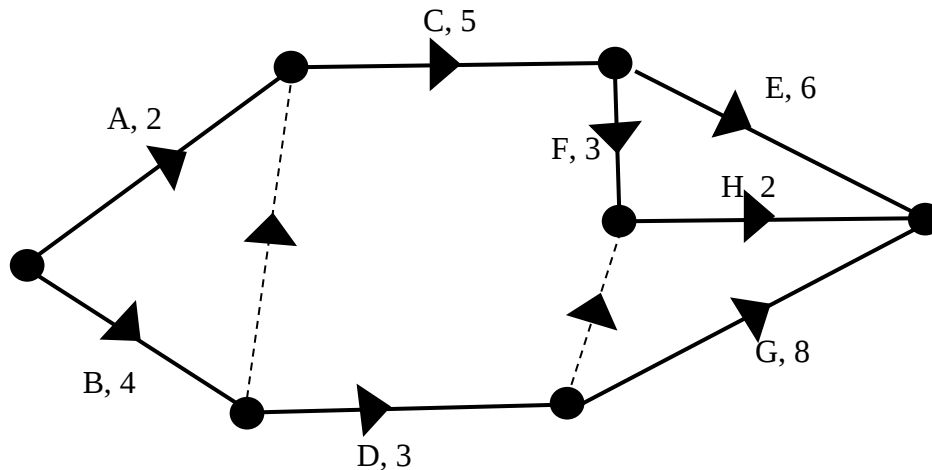
Question 3. (3 marks)

The hospital decides to add another ward for patients.

This project requires 8 activities to be completed.

The network below shows these 8 activities and their completion times in weeks.

The dotted lines represent the dummy activities.



- a. How many activities in this project have two immediate predecessors?

1 mark

- b. What is the minimum completion time, in weeks, for the project.

1 mark

- c. Some extra workers are employed to shorten the duration of activity B by one day.
Explain why reducing E or F by one day will not reduce the minimum completion time.

1 mark

SECTION B – continued

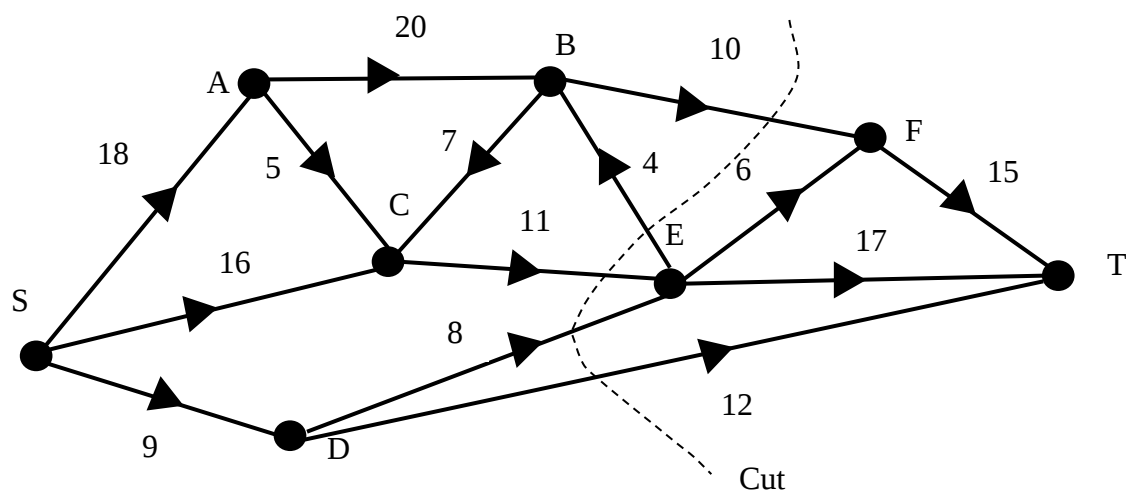
Question 4. (2 marks)

The graph below shows a network of water pipes in the hospital.

The water source is located at S and the distribution centre is at T.

The other vertices represent pumping stations.

The weights on the edges show the capacities in kilolitres per hour that can flow through each pipe.



- a. Determine the value of the cut shown in the network diagram above.

1 mark

- b. What is the maximum flow of water, in kilolitres, from the source to the distribution centre?

1 mark

END OF QUESTION AND ANSWER BOOK