

Trial Examination 2023

VCE General Mathematics Units 1&2

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes

Writing time: 1 hour 30 minutes

Student's Name: _____

Teacher's Name: _____

Structure of booklet

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
12	12	60

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 18 pages

Formula sheet

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, you should only round your answer when instructed to do so.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

AREA OF STUDY 1

- Data analysis, probability and statistics

Question 1 (5 marks)

Aiden and Bell are auditioning for a school talent show. Their performance will be scored by seven judges, and their mean score will be used to compare their performances. The following stem plot shows the scores that Aiden and Bell received from the judges.

Aiden		Bell
8 5	7	9
8 6 x 4	8	4 4 y 7 7
2	9	3

Key
 $7 | 9 = 79$
 $n = 7$

It is known that the mean of Aiden's scores is 84 and the mode of Bell's scores is 84.

- a.** Find the value of x and y . 2 marks

- b.** The candidate who has the higher mean score will progress to the next round.
Determine whether Aiden or Bell will progress to the next round. 2 marks

- c. To avoid any outliers affecting a candidate's mean score, the judges decide to exclude the highest and lowest scores when calculating the mean score.

Find Aiden and Bell's mean score using this method.

1 mark

Question 2 (5 marks)

The football coach at Generalville High School is selecting which students will be included in the football team. One of the statistics the coach looks at while selecting students is the time it takes them to complete a 50 m sprint, in seconds. The following data set shows the sprint times of 21 students.

2.1 5.7 5.7 5.9 6.1 6.2 6.2 6.5 6.5 6.9 6.9
7.2 7.4 7.5 7.6 7.6 7.7 7.8 7.9 8.2 8.5

- a.** What is the median of the data set? 1 mark

- b.** Calculate the interquartile range (IQR). 2 marks

- c.** After checking the data, the coach finds that one of the sprint times was recorded incorrectly and is an outlier.
Identify any outliers in the data set and justify your answer. 2 marks

Question 3 (5 marks)

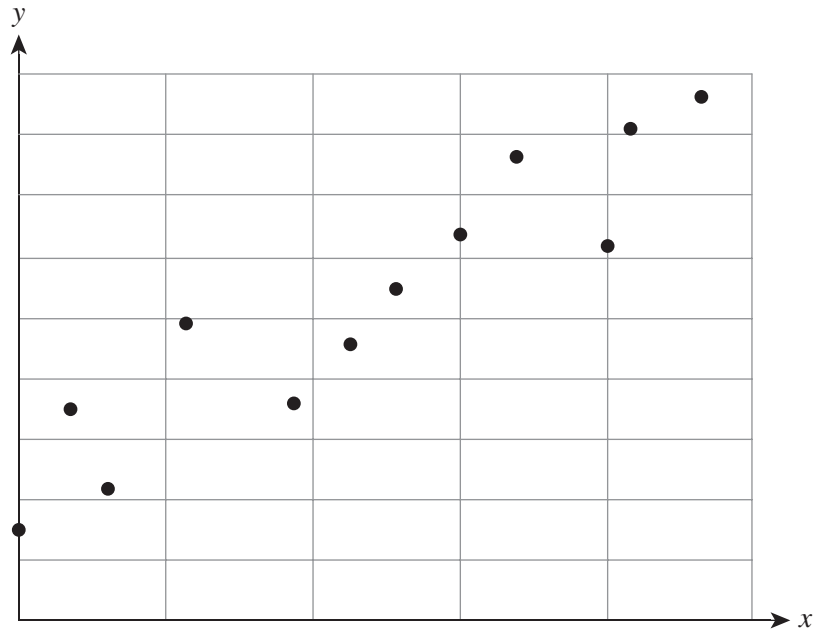
Ling recorded the number of ice creams sold by her business on various days throughout the year to determine if there is an association between a day's maximum temperature, in $^{\circ}\text{C}$, and the number of ice creams sold. The results are shown in the table below.

Maximum temperature ($^{\circ}\text{C}$)	Ice creams sold
0	15
2	35
3	22
6	49
9	36
12	46
13	55
15	64
17	77
20	62
21	81
23	87

- a.** Identify the explanatory variable and response variable of the data. 1 mark

- b.** Using the variables identified in **part a.**, write down the equation of the line of good fit. 1 mark

- c. Ling draws the following incomplete scatterplot of the data.



Using the scatterplot, describe the direction and strength of the association.

2 marks

- d. Ling has 100 ice creams in stock and the weather forecast indicates that the maximum temperature will be 32°C .

Determine whether Ling has enough stock for the day.

1 mark

AREA OF STUDY 2

- Algebra, number and structure
- Discrete mathematics

Question 4 (6 marks)

John decides to save up for a new car. When he starts saving, his bank balance is \$1500 and he will deposit \$500 per month into his account.

- a. i. Write down a recurrence relation that describes John's bank account balance, B_n , after n months. 1 mark

- ii. Find the value of John's account after two years. 1 mark

After two years of saving, John decides to take out a loan to cover the rest of the money he needs to purchase his new car. He borrows \$23 000 and the bank charges interest at 8% per annum, compounding monthly. The loan is scheduled to be fully paid off after five years.

- b. Find John's monthly repayment. Round your answer to the nearest cent. 1 mark

- c. Write down the recurrence relation that describes the balance of the loan, V_n , after n months. 2 marks

- d. What is the total amount of interest that John will pay? 1 mark

Question 5 (5 marks)

After purchasing his new car for \$46 000, John uses it for his business. John's accountant advises that there are two methods for depreciating the cost of the car.

- Method 1 depreciates the value of the car by \$0.7 per kilometre driven.
- Method 2 depreciates the value of the car by 2.1% every month.

The vehicle is expected to be driven 1000 kilometres per month.

- a.** Using method 1, find the expected value of the vehicle after one month. 1 mark

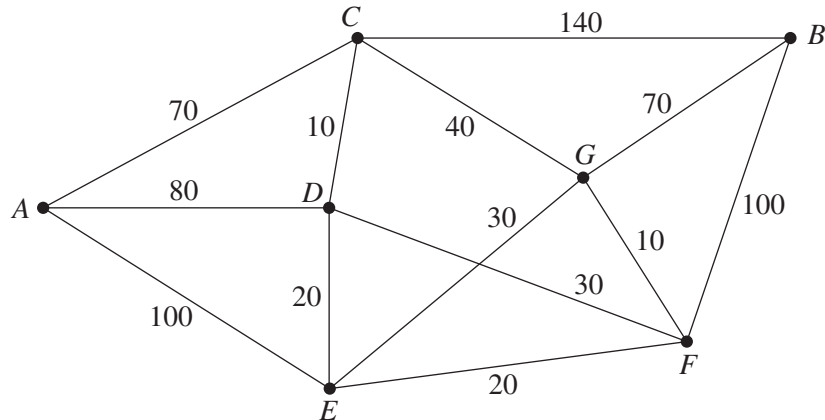
- b.** Using method 2, find the expected value of the vehicle after six months.
Round your answer to the nearest cent. 1 mark

- c.** Which method results in a larger expected value for the car after 20 months? 2 marks

- d.** After how many months will the car have the same value using both methods?
Round your answer to the nearest month. 1 mark

Question 6 (4 marks)

The diagram below shows a network of train lines joining seven regional towns in Victoria. The numbers indicate the length of the train lines, in kilometres.



- a. What is the shortest distance that a train can travel from town A to town B ? 1 mark

- b. In the space below, sketch the minimum spanning tree for the network. 1 mark

- c. John wants to travel from town A to town B and visit every town once along the way. Identify the path that will allow John to complete this journey in the shortest distance possible and calculate the length of this path. 2 marks

AREA OF STUDY 3

- Functions, relations and graphs

Question 7 (4 marks)

The following table shows a breakdown of the fares charged by a taxi company in Melbourne. The fare varies depending on the time of the journey.

Day rate (9:00 am–4:59 pm)			
Fixed cost	Distance rate	Booking fee	Waiting time
\$4.20	\$1.34/km	\$1.50	\$0.43/minute
Overnight rate (5:00 pm–8:59 am)			
Fixed cost	Distance rate	Booking fee	Waiting time
\$5.20	\$1.75/km	\$1.50	\$0.49/minute
Peak rate (10:00 pm–3:59 am on Friday and Saturday only)			
Fixed cost	Distance rate	Booking fee	Waiting time
\$6.20	\$1.85/km	\$1.50	\$0.56/minute

The total fare, F , in dollars, is calculated using the equation

$$F = \text{fixed cost} + (\text{distance} \times \text{rate}) + \text{booking fee} + (\text{waiting time} \times \text{rate})$$

- a. State the total fare, F , in terms of distance travelled, d km, for a journey that occurs at 11:00 am with a waiting time of 0 minutes. 1 mark

- b. At 8:00 pm, Josian takes a taxi from her workplace to her home, which is 15 km away. If the total waiting time is five minutes, what is Josian's total fare? 1 mark

- c. Josian plans to take a taxi when she visits her parents for Christmas. There are two routes that she can choose from. Route 1 has a total distance of 30 km and the traffic conditions on this route would result in a total waiting time of 10 minutes. Route 2 has a total distance of 24 km, but the traffic conditions would result in a total waiting time of 30 minutes.

Which route will result in a lower fare if Josian intends to leave her home at 10:00 am? 2 marks

Question 8 (7 marks)

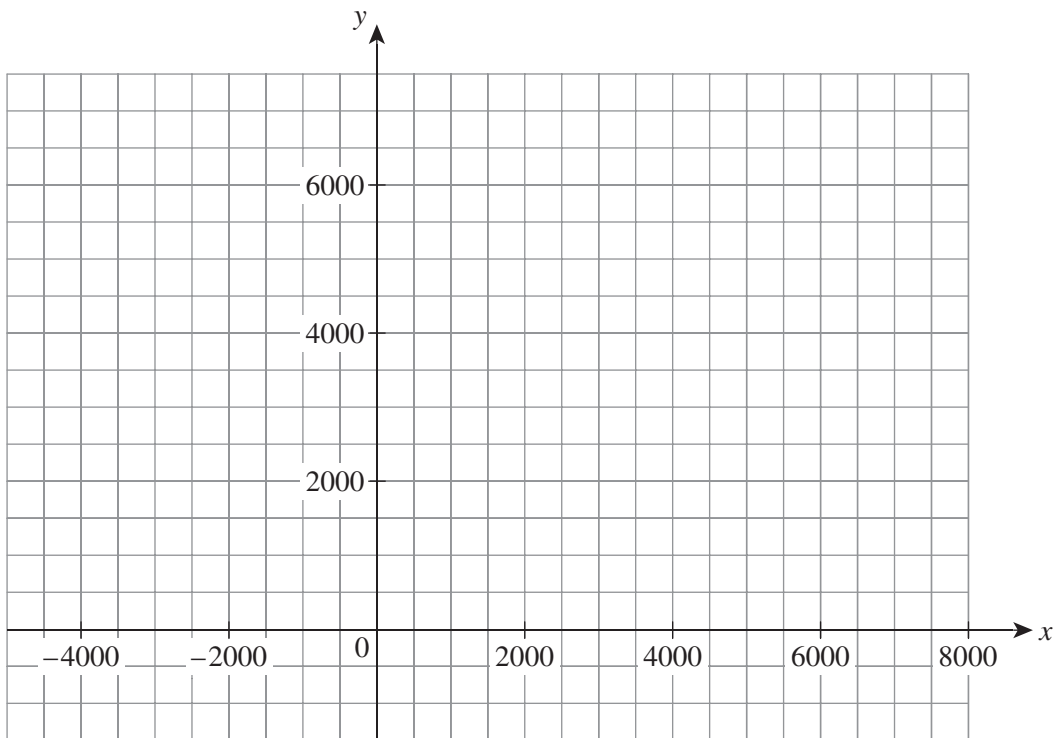
Rio owns a doughnut stall at a market. The monthly cost of production, C , in dollars, consists of a fixed amount of \$1500 and an additional 45 cents for each doughnut made.

- a. Write an equation that models the monthly cost of production, C , for Rio's stall if he makes x doughnuts per month. 1 mark

Rio sells his doughnuts for \$1.50 each.

- b. Write an equation that models the monthly revenue, R , in dollars, for Rio's stall if he sells x doughnuts per month. 1 mark

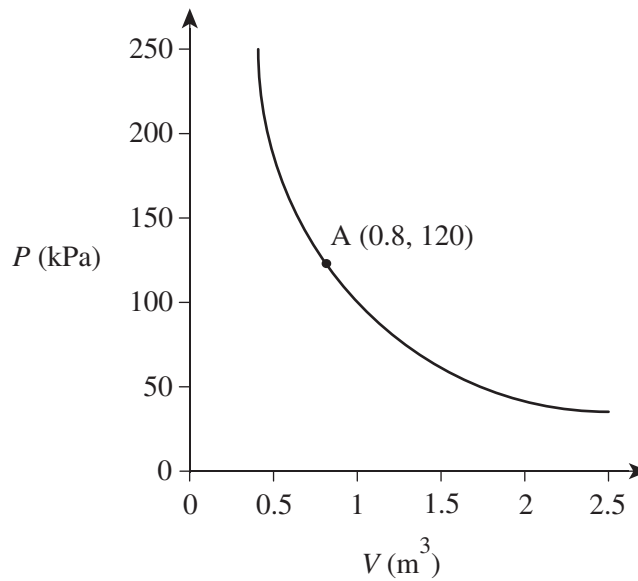
- c. On the axes below, sketch the graphs of C and R . Clearly label all y-axis intercepts and the point of intersection. 3 marks



- d. Determine the minimum number of doughnuts that Rio needs to sell each month to make a profit. Justify your answer. 2 marks

Question 9 (4 marks)

A group of Physics students are investigating the relationship between the air pressure inside a balloon, P , in kilopascals (kPa), and the volume of the balloon, V , in cubic metres. It is known that air pressure varies inversely to the volume of the balloon. The graph below shows the relationship between the two variables.



- a. Write an equation that describes the relationship between P and V . 1 mark

- b. What is the air pressure in the balloon when its volume is 1.5 m^3 ? 1 mark

- c.** When the air pressure in the balloon is greater than 144 kPa, the balloon will explode.

What is the maximum volume that the balloon can reach without exploding?

2 marks

AREA OF STUDY 4

- Discrete mathematics
- Space and measurement

Question 10 (5 marks)

Leanne (L), Marco (M), Natalia (N), Oscar (O) and Penelope (P) are friends from primary school. Some of them have lost contact with others over years. The communication matrix is shown below.

		From				
		L	M	N	O	P
$C =$ To	L	0	1	0	1	0
	M	1	0	1	0	1
	N	1	1	0	1	0
	O	1	0	1	0	0
	P	0	1	1	0	0

A '1' indicates that two people are still in contact and a '0' indicates that two people are no longer in contact.

- a. How many people is Oscar still in contact with? 1 mark

- b. Find the two-step communication matrix, C^2 . 1 mark

- c. How many ways can Natalia ask someone else to pass on a message to Leanne? List all the possible ways. 2 marks

- d.** By adding matrix C and matrix C^2 , find which friend has the most contacts. 1 mark

Question 11 (6 marks)

Sera has started a personal clothing brand. She buys shirts at a cost of $\$a$ per shirt, and resells each shirt for $\$b$. On average, Sera sells 15 shirts every day and makes a profit of \$22 500 in 30 days.

As a marketing strategy, Sera decides to offer a 20% discount for the next 30 days. As a result, she sells 25 shirts every day, but still makes a profit of \$22 500.

- a.** Express the profit, p , from the sale of each shirt in terms of a and b . 1 mark

- b.** Write down two simultaneous equations that could be used to find the value of a and b . 2 marks

- c.** Express the simultaneous equations found in **part b.** as a matrix equation. 2 marks

- d.** Find the values of a and b . 1 mark

Question 12 (4 marks)

Theo purchases two cylindrical water tanks, A and B. It is known that the ratio of tank A's base area to tank B's base area is 5 : 3. Tank A initially contains water with a height of 6 m and tank B initially contains water with a height of 4 m.

- a. Find the ratio between the volume of water in tank A and the volume of water in tank B. 1 mark

- b. After the same amount of water is poured into tanks A and B, both tanks have the same height of water.

If the height of the water in tank A is increased by x m, find the value of x . 3 marks

END OF QUESTION AND ANSWER BOOKLET

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VCE General Mathematics Units 1&2

Written Examinations 1 & 2

Formula Sheet

Instructions

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GENERAL MATHEMATICS FORMULAS**Data analysis**

lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
line of good fit	$y = a + bx$, where $b = r \frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{\text{effective}} = \left[\left(1 + \frac{r}{100n} \right)^n - 1 \right] \times 100\%$

Matrices

determinant of a 2×2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix	$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \quad \text{where } \det A \neq 0$

Networks and decision mathematics

Euler's formula	$v + f = e + 2$
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Geometry and measurement

area of a triangle	$A = \frac{1}{2}bc \sin(\theta^\circ)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc \cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta^\circ$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta^\circ}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base $\times$ height
volume of a pyramid	$\frac{1}{3} \times$ area of base $\times$ height

Graphs and relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$

END OF FORMULA SHEET