Neap

Trial Examination 2022

VCE Further Mathematics Units 3&4

Written Examination 2

Suggested Solutions

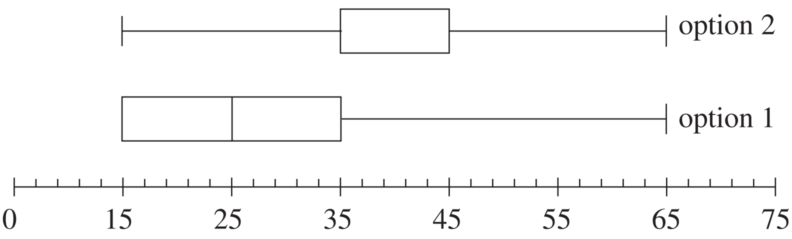
SECTION A – CORE

Data analysis

Question 1 (11 marks)

a. Tourists are the main targets, so conducting the survey at different venues and times will randomise the people being asked while still maximising the number of tourists asked. A1

b.



correct five-number summary M1

A1

c. Option 1 is positively skewed, A1

and option 2 is symmetrical. A1

Note: Consequential on answer to ***Question 1b.***

d. IQR = 10

35 – 1.5(10) = 20, so outliers < 20

45 + 1.5(10) = 60, so outliers > 60 A1

e. **i.** 3.05 A1

ii. 1.58 A1

iii. 3 A1

f. Using the fact that 95% of the data lies within two standard deviations:

$\mu \pm 2\sigma$

3.05 ± 1.58 M1

limits –0.11 to 6.21 hours, or 0 to 6 hours (to the nearest hour)

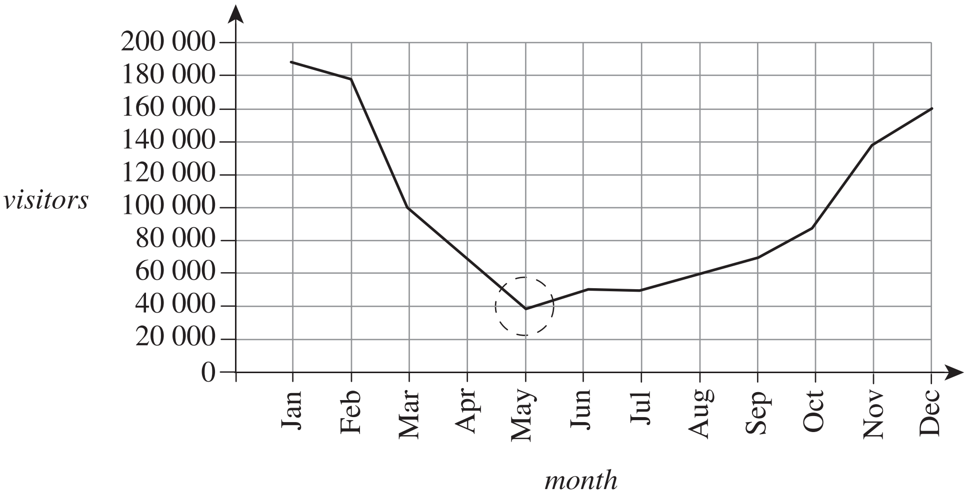
The cost is $50 per hour, so the income limits are $0 to $300. A1

Question 2 (4 marks)

a. time series graph A1

b. seasonal data with an increasing trend line A1

c. The point (May, 40 000) becomes (May, 50 000). A1



d. $\dfrac{190\,000 + 180\,000 + 100\,000}{3} = 156\,667$

$= 157\,000$ (to 3 significant figures) A1

Question 3 (9 marks)

a. attendance = 141 × temperature – 868 A1

b. A temperature of 45° is outside the range of the experimental data. So, any estimation will be an extrapolation A1
and hence unreliable. A1

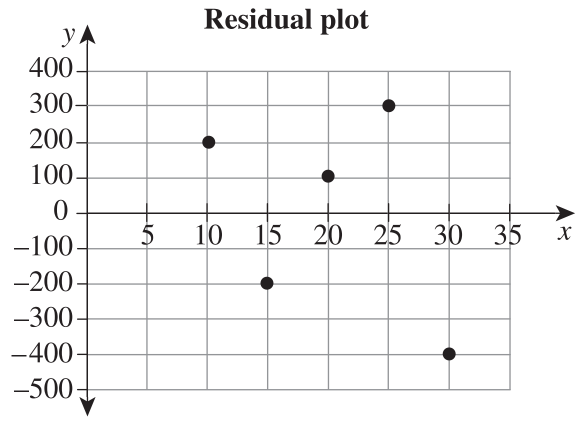
c.

Max temperature	10	15	20	25	30
Number of visitors	1670	2020	3070	4020	4070
Predicted number of visitors	1470	2220	2970	**3720**	**4470**
Residual	200	–200	100	**300**	**–400**

A2

Note: Award one mark for at least two correct answers.

d.



correct plot A1

e. If the points in the residual plot are randomly scattered about the zero residual line and do not show a pattern, the relationship is likely to be linear. A1

f. Applying a $\log_x$ transformation and a linear regression on a CAS calculator gives the following information:

```
LinReg
 y=ax+b
 a=5672.589552
 b=-4236.464837
 r²=.9298226078
 r=.9642730982
```

Therefore, the new correlation coefficient is $r = 0.96$. A1

g. The value of $r = 0.96$ for the $\log_x$ transformation is higher than the given value of $r = 0.85$ for the linear transformation. Therefore, the $\log_x$ transformation fits the data better than the linear one. A1

Recursion and financial modelling

Question 4 (4 marks)

a. $V_n = V_0 \times 1.04^n$, $V_0 = 2\ 200\ 000$ A1

b. Using the solve function on a CAS calculator gives:

$2\ 500\ 000 = 2\ 200\ 00 \times 1.04^n$

OR

$V_3 = 2\ 200\ 000 \times 1.04^3$

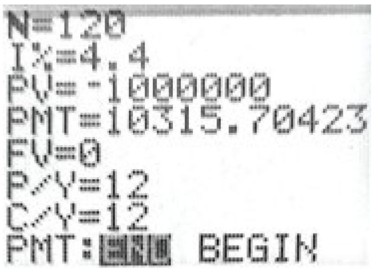
$= 2\ 474\ 700.80$

$V_4 = 2\ 200\ 000 \times 1.04^4$

$= 2\ 573\ 688$

After 4 years, the value has passed \$2 500 000. A1

c. Using the financial solver on a CAS calculator gives:



The monthly payment is \$10 315.70. A1

d. $t_3 = 1.075 \times 250\,000$

$= 268\,750$ (and so on)

Year	1	2	3	4	5
Profit (\$)	0	250 000	**268 750**	**288 906.25**	**310 574.22**

A1

Question 5 (5 marks)

a. depreciation over 10 million tickets = \$46 000 M1

$\frac{46\,000}{10\,000\,000} = \$0.0046$ or 0.46 cents per ticket A1

b. scrap value $= 50\,000 \times 0.8^5$

$= \$16\,384$ A1

c. *Any one of:*

- Graph A shows a steady drop of 20% each year. It is a smooth curve and shows the reducing balance method.
- Graph B reflects the changing ticket sales each year. The depreciation increases with more sales.

identifies the graph A1
explains the reasoning mathematically A1

Question 6 (3 marks)

a. Using the financial solver on a CAS calculator with the following settings gives:

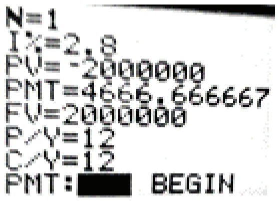
$N = 1$

$I = 2.8$

$PV = -2\,000\,000$

$FV = 2\,000\,000$

$P/Y = 12$



OR

$$A = 2\,000\,000 \times \left(1 + \frac{\frac{2.8}{12}}{100}\right)$$

$$= 2\,004\,666.67$$

$$I = \$4666.67$$

Therefore, the monthly payment is \$4666.67. A1

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b. Using the financial solver on a CAS calculator with the following settings to find the monthly repayment for the $6 million loan gives:

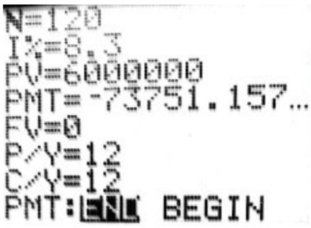
$N = 120$

$I = 8.3$

$PV = –6\ 000\ 000$

$FV = 0$

$P/Y = 12$



If $6 million were borrowed, the repayment would be $73 751.16 per month.

The total to be repaid is 73 751.16 × 120 = $8 850 139.20.

Therefore, the interest paid is 8 850 139.20 – 6 000 000 = $2 850 139.20. M1

Using the financial solver on a CAS calculator with the following settings to find the monthly repayment for the $3.2 million loan gives:

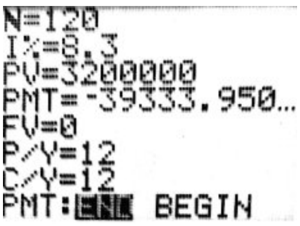
$N = 120$

$I = 8.3$

$PV = –3\ 200\ 000$

$FV = 0$

$P/Y = 12$



If $3.2 million is borrowed, the repayment is $39 333.95 per month.

The total to be repaid is 39 333.95 × 120 = $4 720 074.

Therefore, the interest paid is 4 720 074 – 3 200 000 = $1 520 074.

The interest saved by borrowing $3.2 million instead of $6 million is:

2 850 139.20 – 1 520 074 = $1 330 065.20. A1

SECTION B – MODULES

Module 1 – Matrices

Question 1 (2 marks)

a. 3×5 A1

b. No, it is not possible because in period 2 there are two classes on at once. A1

Question 2 (6 marks)

a. $5a + 5b = 58.50$

$7a + 6b = 96.75$ A1

b. $\begin{bmatrix} 5 & 3 \\ 7 & 6 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 58.5 \\ 96.75 \end{bmatrix}$ A1

c. $A = \begin{bmatrix} 5 & 3 \\ 7 & 6 \end{bmatrix}, B = \begin{bmatrix} 58.5 \\ 96.75 \end{bmatrix}$

Solve the equation $AX = B$:

$A^{-1}B = \begin{bmatrix} 6.75 \\ 8.25 \end{bmatrix}$ M1

The type A lunch costs \$6.75 and the type B lunch costs \$8.25. A1

d. $\begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix}$ A1

e. $\begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix}^{10}\begin{bmatrix} 240 \\ 160 \end{bmatrix} = \begin{bmatrix} 136.35 \\ 263.65 \end{bmatrix}$

After 10 days, 136 type A lunches and 264 type B lunches will be ordered. A1

Question 3 (4 marks)

a.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{matrix} A & B & C & D \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

A1

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b.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 1+1=2 \\ 1 \\ 1+1=2 \\ 1 \end{array}$$

Both Alexander and Cavendish won two matches. Therefore, based on one-step dominance, both houses are equally ranked the highest. A1

c. To find one- and two-step dominance, calculate the square of the initial matrix.

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \begin{array}{r} \\ 1+1=2 \\ =1 \\ 1+2=3 \\ 1+1=2 \end{array}$$

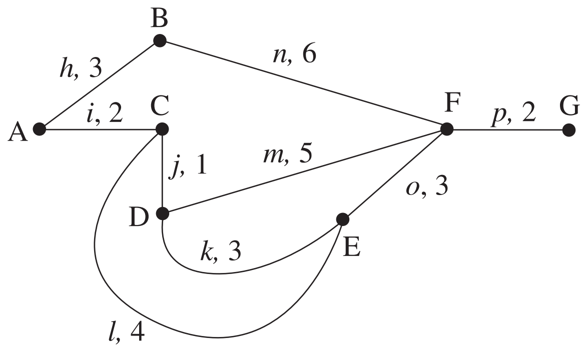
M1

Cavendish has two-step dominance and, since both Alexander and Cavendish had the same figure for one-step dominance, Cavendish is ranked number 1. A1

Module 2 – Networks and decision mathematics

Question 1 (3 marks)

a. *For example:*



M1 and A1

Note: Responses may differ, but the connections must be the same.

b. A–B–F–G = 11, A–C–E–F–G = 11, A–C–D–E–F–G = 11

Therefore, the minimum time needed to complete the project is 11 days. A1

Question 2 (2 marks)

a. Euler circuit A1

b. Since all the vertices are even, an Euler circuit exists. A1

Question 3 (2 marks)

a. Task *B* is not on the critical path, so the only effect of employing the additional labourer would be increasing the slack time for task *B*. A1

b. Since task *I* is on the critical path, it does not have a float time. A1

Question 4 (5 marks)

a.

Company	**Building material allocated**
A	concrete
B	drainpipes
C	wood
D	brick

M1 and A1

b.

Labourer	**Painting**	**Plastering**	**Concreting**	**Plumbing**
Archie	27	30	34	40
Belinda	32	36	32	38
Charli	31	33	35	37
Depak	28	32	31	34

Find the smallest element in each row and subtract it from every element in the row.

0	3	7	13
0	4	0	6
0	2	4	6
0	4	3	6

Find the smallest element in each column and subtract it from every element in the column.

0	1	7	7
0	2	0	0
0	0	4	0
0	2	3	0

M1

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Cover the zeroes with the minimum number of straight lines.

0	1	7	7
0	2	0	0
0	0	4	0
0	2	3	0

M1

Since the number of straight lines equals the size of the matrix, the optimum solution has been found.

Since there is only one 0 in column 2, allocate Charli the plastering task.

Since there is only one 0 in column 3, allocate Belinda the concreting task.

Since there is only one 0 in row 1, allocate Archie the painting task.

Therefore, Depak must be allocated the plumbing task. A1

Module 3 – Geometry and measurement

Question 1 (3 marks)

a. Djoser is 31° N, which means it is 31° above the equator. Darwin is only 12° S, which means it is below the equator but is the closer of the two. A1

b. The height of each step is $\frac{60}{6} = 10$ m.

The bottom step measures 120 × 105.

The second step measures 100 × 85.

The third step measures 80 × 65.

The surface areas of the lowest three steps are:

$$L_1 = 2 \times 10 \times (120 + 105)$$
$$= 4500 \text{ m}^2$$
$$L_2 = 2 \times 10 \times (100 + 85)$$
$$= 3700 \text{ m}^2$$
$$L_3 = 2 \times 10 \times (80 + 65)$$
$$= 2900 \text{ m}^2$$

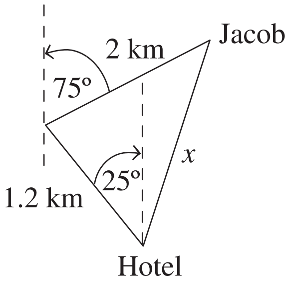
M1

$$\text{total surface area of lowest three steps} = 4500 + 3700 + 2900$$
$$= 11100 \text{ m}^2$$

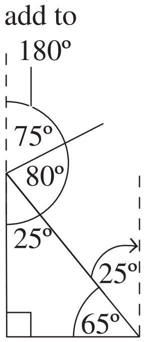
A1

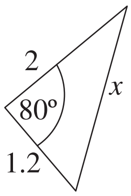
Question 2 (5 marks)

a. The diagrams below illustrate the angles and distances needed to solve the problem.



A1





Note: Responses do not require diagrams to obtain full marks.

$$x^2 = b^2 + c^2 - 2bc\cos(A)$$
$$= 1.2^2 + 2^2 - 2 \times 2 \times 1.2\cos(80)$$
$$= 4.61$$

M1

$$x = 2.15$$

Jacob is 2.15 km from the hotel. A1

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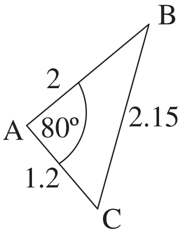
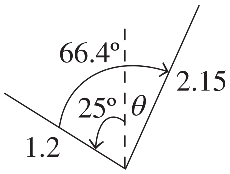
b. Using the sine rule to find angle *C*:



$$\frac{2}{\sin C} = \frac{2.15}{\sin 80}$$
$$\sin C = \frac{2\sin 80}{2.15}$$
$$C = 66.4°$$ M1

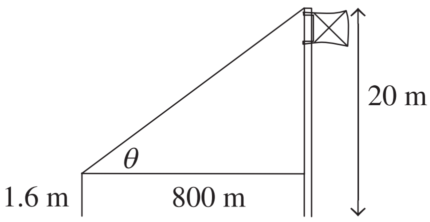
The size of the angle from the north is 66.4 – 25 = 41.4°.



The bearing of Jacob from the hotel is N 41.4° E. A1

*Note: Consquential on answer to **Question 2a.***

c. The following diagram illustrates the scenario in the question.



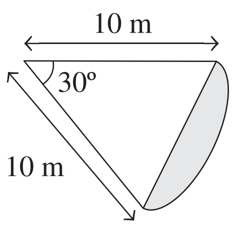
Note: Responses do not require a diagram to obtain full marks.

$$\tan\theta = \frac{18.4}{800}$$
$$\theta = 1.32°$$

The angle of elevation is 1.3°. A1

Question 3 (4 marks)

a.



area of the segment = area of the sector – area of the triangle

$$A = \frac{30}{360}\pi(10)^2 - \frac{1}{2}\times 10\times 10\sin 30$$ M1

$$= 1.8 \text{ m}^2$$ A1

b. area of the pool = $\frac{1}{2} \times 10 \times 10 \sin 30 = 25$ M1

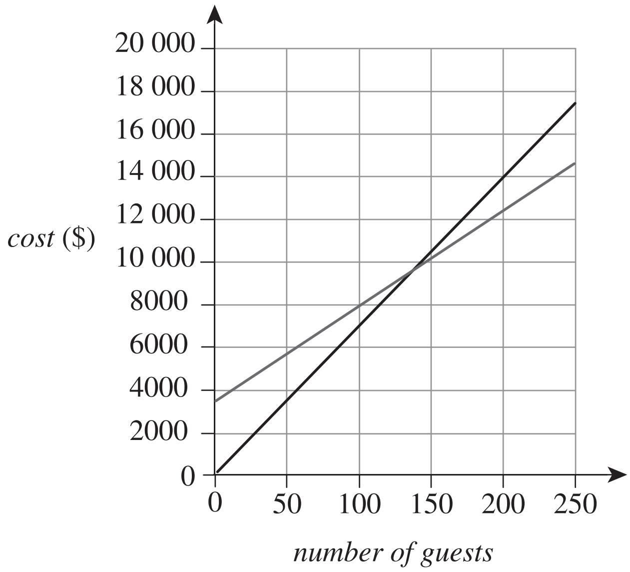
volume = 25 × 1.2 = 30 m^3 A1

Module 4 – Graphs and relations

Question 1 (4 marks)

a. $c = 45n + 3500$ A1

b.



correct shape A1
correct scale A1

c. the breakeven point A1

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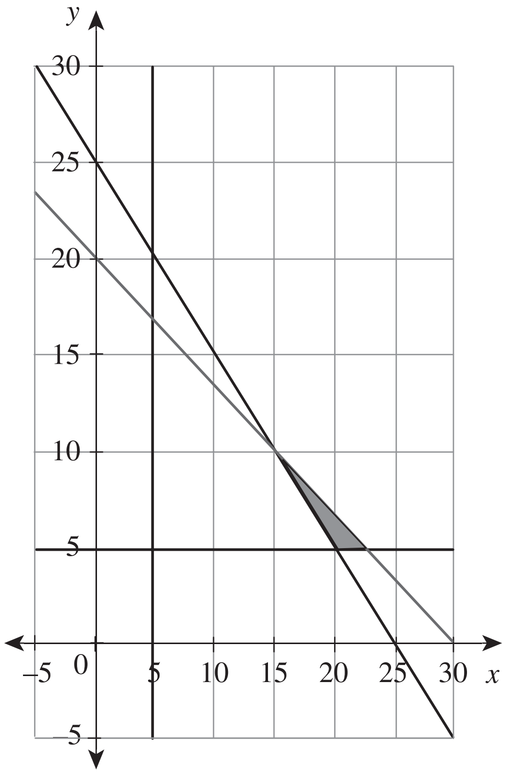
Question 2 (5 marks)

a. $2x + 3y \le 60$ A1

b.



A1

c. $z = 4x + 5y$ A1

d. Substituting (15, 10), (20, 5) and (22.5, 5) into $z = 4x + 5y$:

Vertex	$z = 4x + 5y$
(15, 10)	$z = 4(15) + 5(10) = \$110$
(20, 5)	$z = 4(20) + 5(5) = \$105$
(22.5, 5)	$z = 4(22.5) + 5(5) = \$115$

M1

Since 0.5 of a box cannot be made, the maximum profit of \$113 is found by making 22 square boxes and 5 triangular boxes. A1

Question 3 (3 marks)

a. Substitute the point (10, 3200) into $y = kx^2$:

$$3200 = k(10)^2$$
$$k = 32$$

A1

b. the cost per square metre A1

c. $\text{cost} = 75x^2 + 250$ A1