

# 2022 VCE Further Mathematics Trial Examination 2



**Kilbaha Education**

Quality educational content

**Kilbaha Education  
PO Box 2227  
Kew Vic 3101  
Australia**

**Tel: (03) 9018 5376**

**[kilbaha@gmail.com](mailto:kilbaha@gmail.com)  
<https://kilbaha.com.au>**

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Quality educational content

**Kilbaha Education (Est. 1978) (ABN 47 065 111 373)**  
**PO Box 2227**  
**Kew Vic 3101**  
**Australia**

**Tel: +613 9018 5376**

**Email: [kilbaha@gmail.com](mailto:kilbaha@gmail.com)**

**Web: <https://kilbaha.com.au>**

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|                   |  |  |  |  |  |  |  |  |  |
|-------------------|--|--|--|--|--|--|--|--|--|
| STUDENT<br>NUMBER |  |  |  |  |  |  |  |  |  |
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# VICTORIAN CERTIFICATE OF EDUCATION 2022 FURTHER MATHEMATICS

## Trial Written Examination 2

Reading time: 15 minutes

Total writing time: 1 hour 30 minutes

### QUESTION AND ANSWER BOOK

#### Structure of book

| Section A - Core    | <i>Number of questions</i> | <i>Number of questions to be answered</i> | <i>Number of marks</i> |
|---------------------|----------------------------|-------------------------------------------|------------------------|
|                     | 7                          | 7                                         | 36                     |
| Section B - Modules | <i>Number of modules</i>   | <i>Number of modules to be answered</i>   | <i>Number of marks</i> |
|                     | 4                          | 2                                         | 24                     |
|                     |                            |                                           | Total 60               |

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

#### Materials supplied

- Question and answer book of 32 pages.
- Formula sheet
- Working space is provided throughout the book.

#### Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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**Section A – Core****Instructions for Section A**

Answer all questions in the spaces provided. Write using blue or black pen.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example,  $\pi$ , surds or fractions.

In ‘Recursion and financial modelling’, all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Data Analysis****Question 1 (5 marks)**

The table below shows the *name* and *height*, in centimetres, of 15 women in a study.

| <i>Name</i> | <i>Height (cm)</i> |
|-------------|--------------------|
| Angie       | 165                |
| Bella       | 152                |
| Cerise      | 165                |
| Donna       | 154                |
| Evelyn      | 164                |
| Freya       | 168                |
| Gabriella   | 172                |
| Hypatia     | 149                |
| Inez        | 150                |
| Jana        | 154                |
| Kari        | 161                |
| Lang        | 152                |
| Mahila      | 157                |
| Nazali      | 187                |
| Omari       | 160                |

- a. Which one of the two variables in this data set is a nominal variable?

1 mark

---

**Data Analysis****Question 1 (continued)**

- b. Determine the mean and standard deviation of the variable *height* for this sample of fifteen women.

Write your answers in the boxes provided below.

Round your answers to one decimal place.

1 mark

Mean =  cm      Standard deviation =  cm

The ordered stem plot below has been constructed from the data from this sample of fifteen women.

*height* (cm)

|    |  |             |
|----|--|-------------|
| 14 |  | 9           |
| 15 |  | 0 2 2 4 4 7 |
| 16 |  | 0 1 4 5 5 8 |
| 17 |  | 2           |
| 18 |  | 7           |

key: 14|9 = 149

- c. Write down the value of the third quartile ( $Q_3$ ) 1 mark

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- d. What percentage of these women is taller than 154 cm? 1 mark

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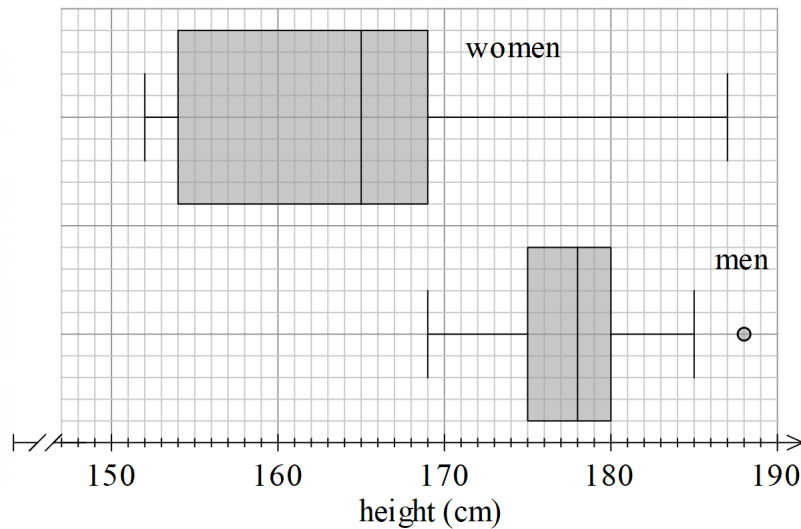
- e. The value of 187 cm is considered an outlier as it is above the upper fence.  
Show that the upper fence for this data is 184.5 cm. 1 mark

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**Data Analysis****Question 2 (4 marks)**

The parallel boxplots below show the *height*, in centimetres, for a sample of 40 men and 40 women.



Using the information in the parallel boxplots, complete the following statements by filling in the boxes.

- a. The number of women with a *height* less than or equal to the shortest man is .

1 mark

- b. The median value for the *height* of the men is  cm higher than the median value for the *height* of the women.

1 mark

- c.  % of the women has a *height* between 152 cm and 154 cm.

1 mark

- d. The interquartile range for the men is  less than the interquartile range for the women.

1 mark

**Data Analysis****Question 3 (3 marks)**

In The Netherlands, the height, in centimetres, of women is approximately normally distributed with a mean of 167 cm.

Queen Maxima of The Netherlands has a height of 178 cm.

Her standardized height is given by  $z = +2.9$

- a.** Show that the standard deviation for the height of women in The Netherlands is 3.8 rounded to one decimal place.

1 mark

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- b.** In a random sample of 2000 women in The Netherlands, how many are expected to have a height between 159.4 cm and 170.8 cm?

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The height, in centimetres, of men in The Netherlands is approximately normally distributed with a mean of 182.1 cm and a standard deviation of 0.9 cm.

King Willem-Alexander is 183 cm tall.

- c.** What percentage of men in The Netherlands is expected to be taller than King Willem-Alexander?

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**Data Analysis****Question 4 (10 marks)**

The *height at age 10*, and the *height at age 25*, both in centimetres, for a sample of 11 people are shown in the table below.

| <i>height at age 10 (cm)</i> | <i>height at age 25 (cm)</i> |
|------------------------------|------------------------------|
| 145                          | 182                          |
| 143                          | 182                          |
| 148                          | 183                          |
| 141                          | 182                          |
| 136                          | 179                          |
| 147                          | 181                          |
| 134                          | 178                          |
| 137                          | 180                          |
| 150                          | 182                          |
| 132                          | 174                          |
| 129                          | 170                          |

A least-squares line is to be fitted to the data to see if height at age 25 can be predicted from height at age 10.

The equation of the least-squares line is

$$\text{height at age 25} = 109.99558 + 0.494843 \times \text{height at age 10}$$

- a. i. Rewrite this least-squares equation expressing the slope and intercept correct to 3 significant figures

1 mark

$$\text{height at age 25} = \boxed{\phantom{000}} + \boxed{\phantom{000}} \times \text{height at age 10}$$

- ii. Name the response variable

1 mark

---

- iii. Interpret the slope in terms of the variables *height at age 25* and *height at age 10*.

1 mark

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**Data Analysis****Question 4 (continued)**

The value of the correlation coefficient,  $r$ , is 0.8624

- b.** What percentage of the variation in *height at age 25* can be explained by the variation in *height at age 10*?  
Give your answer correct to the nearest whole percentage. 1 mark

---

- c.** Use the equation of the least-squares line given in **a. i.** to predict the *height at age 25* of a person who had a *height at age 10* of 139 cm. Write your answer in centimetres correct to one decimal place. 1 mark

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- d.** Is the prediction made in **part c.** an example of interpolation or extrapolation? 1 mark

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- e.** Determine the residual for the person who had a height of 141 cm at age 10.  
Round your answer to the nearest whole number.

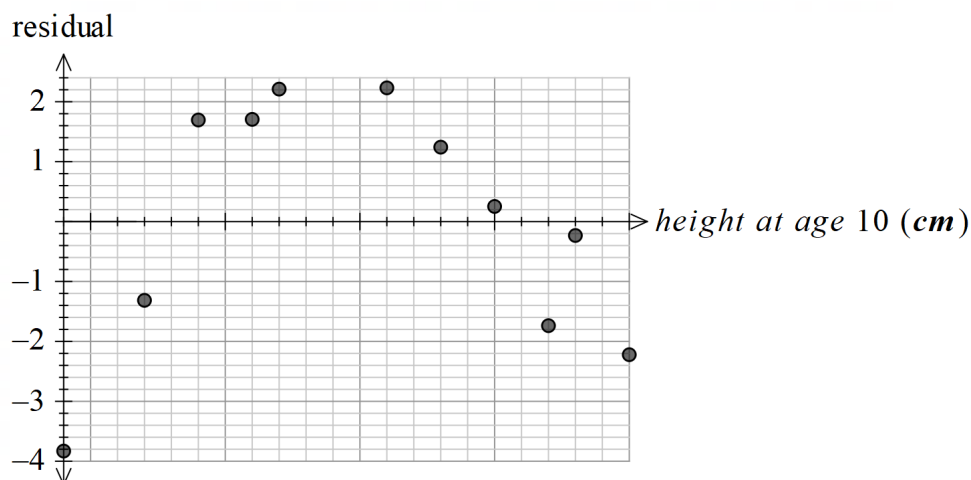
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1 mark

**Data Analysis****Question 4 (continued)**

- f. The residual plot associated with the least squares line is shown below.



The residual plot indicates that the association between *height at age 25* and *height at age 10* is non-linear.

Explain why.

1 mark

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- g. Apply a  $\log_{10}$  transformation to the variable *height at age 10* to linearise the data. Write down the equation of the least-squares line for the transformed data. Round the values of the intercept and slope to one decimal place.

2 marks

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**Data Analysis****Question 5 (2 marks)**

The table below shows the average height, in centimetres of nineteenth century German men by year of birth.

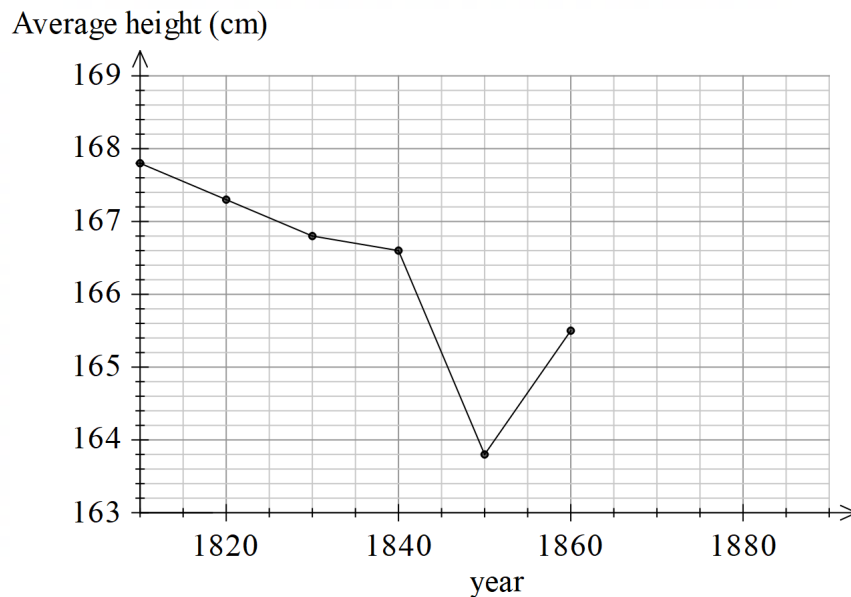
| Year                | 1810  | 1820  | 1830  | 1840  | 1850  | 1860  | 1870  | 1880  | 1890  |
|---------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Average height (cm) | 167.8 | 167.3 | 166.8 | 166.6 | 163.8 | 165.5 | 166.8 | 167.8 | 168.6 |

Data: OurWorldInData.org/human-height

- a. Use the data to complete the time series plot below.

(Answer on the time series plot below.)

1 mark



- b. A five-point moving median is applied to smooth the data.  
What is the smoothed value for the year 1860?

1 mark

**Recursion and financial modelling****Question 6 (3 marks)**

Sue has a business printing promotional brochures for local events and is saving for a new printing machine.

She opens a savings account.

The amount of money, in dollars, in the account after  $n$  months,  $A_n$ , is modelled by the recurrence relation

$$A_0 = 45000, \quad A_{n+1} = 1.0045A_n$$

- a.** How much money did Sue initially deposit into this savings account? 1 mark

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- b.** What is the annual percentage compounding interest rate for this savings account? 1 mark

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- c.** The amount of money in the account after  $n$  months,  $A_n$ , can also be determined using a rule.

Complete the rule by filling in the boxes below.

1 mark

$$A_n = \boxed{\phantom{000000}}^n \times \boxed{\phantom{000000}}$$

**Recursion and financial modeling****Question 7 (6 marks)**

Sue buys a new printing machine valued at \$95000.

Sue's printing machine will be depreciated using the unit cost method.

After printing 2500 brochures, the value of the machine will have decreased by \$125.

- a. What will be the value of the machine after printing 2500 brochures? 1 mark

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- b. Show that the unit cost depreciation of Sue's printing machine is \$0.05 per brochure.

1 mark

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- c. Sue finds that the machine depreciates by \$1600 per year. 1 mark  
How many brochures does she print in a year?

The value of the machine, in dollars, after  $n$  years,  $V_n$ , can be modelled by the recurrence relation

$$V_0 = 95000, \quad V_{n+1} = V_n - 1600$$

- d. Use recursive calculations to show that the value of the machine after 3 years is \$90200.

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**Recursion and financial modeling****Question 7 (continued)**

- e. The value of the machine, in dollars after  $n$  years,  $V_n$ , could also be determined using a rule of the form  $V_n = c + dn$

Write down the values of  $c$  and  $d$ .

1 mark

$c =$                        $d =$

- f. After 11 years, Sue changes to the reducing balance method of depreciation for the printing machine, using a reducing balance rate of 12% per annum.

What will be the value of the machine, to the nearest whole dollar, after a total of 19 years?

1 mark

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**Recursion and financial modelling****Question 8 (3 marks)**

When Sue's business expands, she needs to buy new premises.

She borrows \$265000 at an interest rate of 3.5% per annum.

Interest on the unpaid balance of the loan is charged to the loan account monthly.

The loan is to be fully repaid in 15 years by making equal monthly instalments of \$1894.44

- a.** Calculate the principal amount that will have been paid off at the end of the first year.  
Round your answer to the nearest dollar.

1 mark

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- b.** At the end of the seventh year, Sue makes an additional one-off payment of \$25000.  
No other changes are made to the conditions of the loan.  
Determine how many months, to the nearest month, Sue saves in repaying the loan.

2 marks

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**END OF SECTION A**

**Section B – Modules****Instructions for Section B**

Select two modules and answer all questions within the selected modules. Write using blue or black pen. You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example,  $\pi$ , surds or fractions. Unless otherwise indicated, the diagrams in this book are not drawn to scale.

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**Module 1 - Matrices****Question 1 (4 marks)**

Nazeem is running an indoor games competition for his friends on a rainy day. The games are Snap (S), Ludo (L), Darts (D) and Checkers (C).

The points awarded for winning each type of game are shown in the table below.

| Game     | Points |
|----------|--------|
| Snap     | 2      |
| Ludo     | 4      |
| Darts    | 6      |
| Checkers | 8      |

Matrix P lists the number of points scored for winning each type of game.

$$P = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \end{bmatrix}$$

- a. Write down the order of matrix P. 1 mark

On the day, Mahila won 3 games of Snap and 2 games of Darts, but did not win at either Ludo or Checkers

- b. Row matrix, R, shows the number of wins she had for each type of game.  
Complete the matrix by writing in its elements.

$$R = \begin{bmatrix} & \text{S} & \text{L} & \text{D} & \text{C} \\ & & & & \end{bmatrix}$$

1 mark

**Module 1 - Matrices****Question 1 (continued)**

- c. Matrix A is found by evaluating the product of matrix R and matrix P so that  $A = R \times P$

Evaluate matrix A.

1 mark

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- d. What information does matrix A provide in this context?

1 mark

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**Module 1 – Matrices****Question 2 (2 marks)**

Mahila (M), Nazeem (N), Oliver (O), Paula (P) and Questo (Q) compete in a darts round robin where each player competes against every other player exactly once.

The one-step dominance matrix below shows the winner of each game with a “1” and the loser or no match played with a “0”.

For example, row 3 indicates that Oliver loses to Questo but defeats Mahila, Nazeem and Paula.

|        |   | loser |   |   |   |   |
|--------|---|-------|---|---|---|---|
|        |   | M     | N | O | P | Q |
| winner | M | 0     | 1 | 0 | 0 | 1 |
|        | N | 0     | 0 | 0 | 0 | 1 |
|        | O | 1     | 1 | 0 | 1 | 0 |
|        | P | 1     | 1 | 0 | 0 | 1 |
|        | Q | 0     | 0 | 1 | 0 | 0 |

- a. How many two-step dominances does Paula have?

1 mark

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- b. When the players are ranked from first to last, which two players come equal third?

1 mark

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**Module 1 - Matrices****Question 3 (6 marks)**

Nazeem decides to run a game convention over four days.

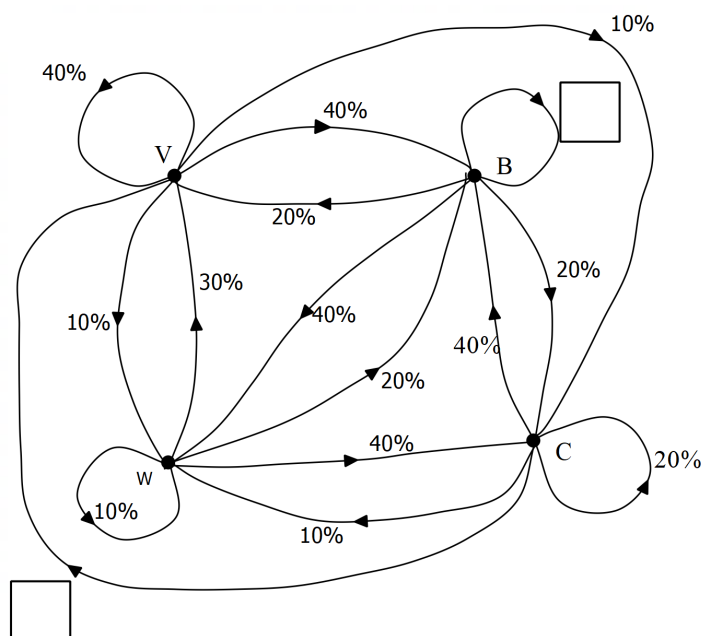
Each day, participants can choose to play one type of game.

The types of games on offer are board games (B), card games (C), video games (V) and word games (W).

The transition matrix T shows the way in which the participants are expected to change their choice of game from day to day.

$$T = \begin{array}{c} \begin{array}{ccccc} & \text{today} & & & \\ & \text{B} & \text{C} & \text{V} & \text{W} \\ \begin{bmatrix} 0.2 & 0.4 & 0.4 & 0.2 \\ 0.2 & 0.2 & 0.1 & 0.4 \\ 0.2 & 0.3 & 0.4 & 0.3 \\ 0.4 & 0.1 & 0.1 & 0.1 \end{bmatrix} & \text{B} \\ & \text{C} \\ & \text{V} \\ & \text{W} \end{array} & \text{next day} \end{array}$$

- a. The information in the transition matrix T is shown by the transition diagram below, with two pieces of information missing. Complete the transition diagram by filling in the boxes with the appropriate percentages.



1 mark

**Module 1 - Matrices****Question 2 (continued)**

Let  $G_n$  be the state matrix for the number of participants expected to choose each game type in week  $n$ .

For the given matrix  $G_1$ , a matrix rule that can be used to predict the number of participants choosing each game type on days 2, 3 and 4 is

$$G_1 = \begin{bmatrix} 300 \\ 200 \\ 400 \\ 100 \end{bmatrix} \begin{matrix} \text{B} \\ \text{C} \\ \text{V} \\ \text{W} \end{matrix}, \quad G_{n+1} = TG_n$$

- b. Complete the state matrix,  $G_2$ , for day 2 below.

1 mark

$$G_2 = \begin{bmatrix} \phantom{000} \\ \phantom{000} \\ \phantom{000} \\ \phantom{000} \end{bmatrix} \begin{matrix} \text{B} \\ \text{C} \\ \text{V} \\ \text{W} \end{matrix}$$

- c. How many participants change their choice of game type from day 1 to day 2?

1 mark

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- d. Of the participants expected to choose Video games (V) on day 3, what percentage chose card games (C) on day 2? Give your answer to the nearest whole percentage.

2 marks

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- e. What is the maximum number of participants expected to choose word games (W) on any one of the four days?

1 mark

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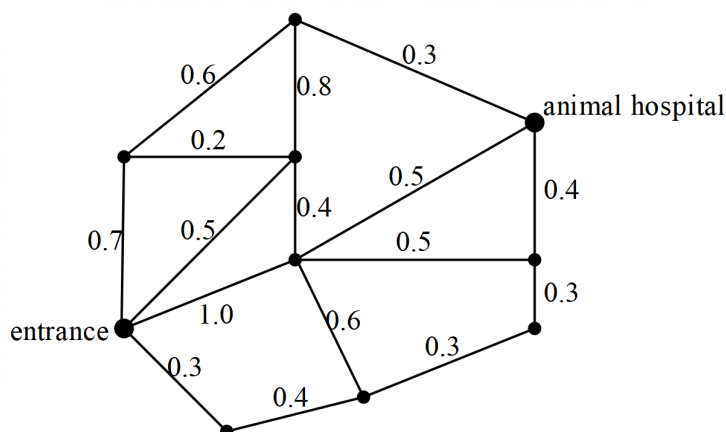
**End of Module 1: Matrices**

**Module 2: Networks and decision mathematics****Question 1 (4 marks)**

Jasper is visiting the Willburton Native Animal Sanctuary.

The network diagram below shows the tracks (as edges) between the entrance, the animal enclosures, and the animal hospital (all represented as vertices).

The numbers on the edges indicate the distance, in kilometres, of each track.



- a. What is the shortest distance, in kilometres, from the entrance to the animal hospital?

1 mark

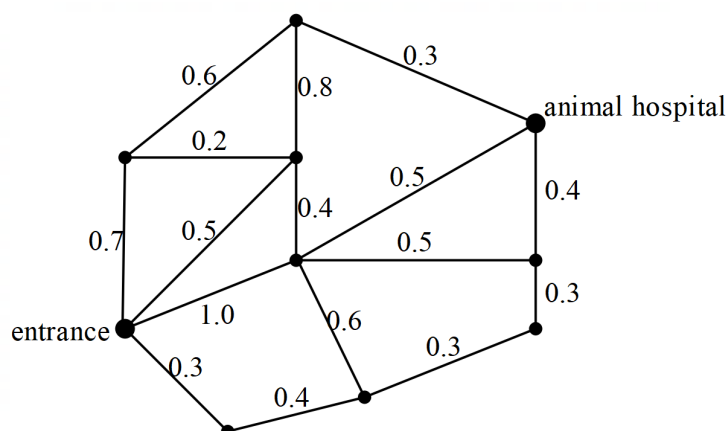
- b. Jasper decides to follow a route from the entrance to the animal hospital that visits every animal enclosure once only.

- i. What mathematical term is used to describe such a route?

1 mark

- ii. On the network diagram below, draw in the route that Jasper takes from the entrance to the animal hospital that has her visiting each animal enclosure once only.

1 mark



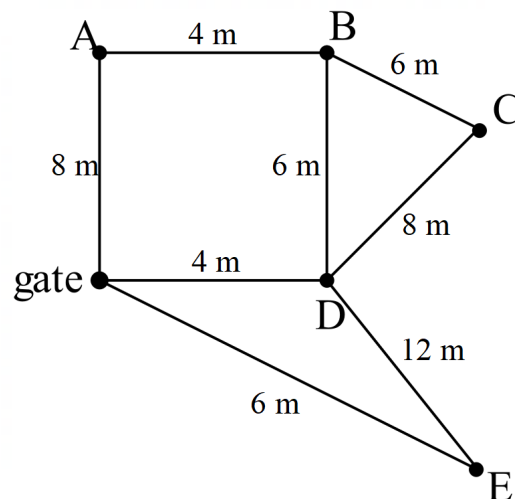
**Module 2: Networks and decision mathematics****Question 1 (continued)**

- c. In one very wet season, all the gravel was washed off the tracks.  
What is the minimum length of track that needs to be resurfaced with gravel to connect the entrance, café and animal enclosures with graveled tracks?

1 mark

**Question 2 (5marks)**

Jasper and Laura go into the nocturnal animal house. The house consists of five enclosures, shown as vertices A, B, C, D and E in the network diagram below. Tunnels between the enclosures are represented by edges, and the numbers on the edges give the length, in metres, of each of the tunnels.



- a. How many vertices have an even degree?

1 mark

- b. Starting at the gate, Jasper wants to walk through every tunnel exactly once.

- i. At which enclosure will she finish?

1 mark

- ii. What mathematical term is used to describe such a route?

1 mark

**Module 2: Networks and decision mathematics****Question 2 (continued)**

While Laura is at enclosure E, she realizes she has dropped her wallet. Starting from E, she wants to go through every tunnel at least once and return to enclosure E.

- c.      i. What is the least distance, in metres, she will need to walk? 1 mark

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- ii. In walking the least distance, which enclosures, apart from E and the gate, will she visit more than once?

1 mark

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**Module 2: Networks and decision mathematics****Question 3 (3 marks)**

A new Platypus enclosure is to be constructed at the sanctuary.

The construction project involves activities A to I.

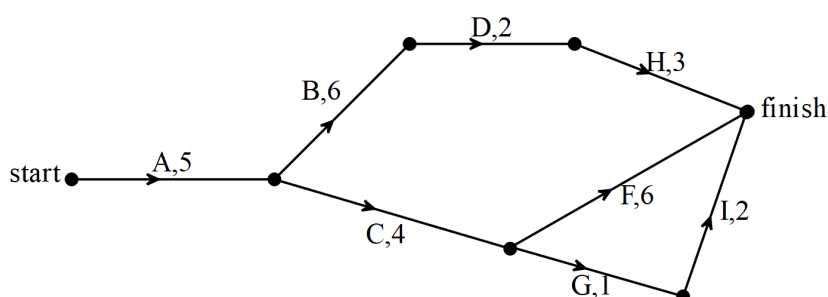
The table below shows the earliest starting time (EST), latest starting time (LST) and duration, in weeks, for each activity. The immediate predecessor is also shown.

The latest start time for activity G is missing.

| Activity | EST<br>(weeks) | LST<br>(weeks) | Duration<br>(weeks) | Immediate<br>predecessor |
|----------|----------------|----------------|---------------------|--------------------------|
| A        | 0              | 0              | 5                   | -                        |
| B        | 5              | 5              | 6                   | A                        |
| C        | 5              | 6              | 4                   | A                        |
| D        | 11             | 11             | 2                   | B                        |
| E        | 5              | 9              | 7                   | A                        |
| F        | 9              | 10             | 6                   | C                        |
| G        | 9              |                | 1                   | C                        |
| H        | 13             | 13             | 3                   | D                        |
| I        | 10             | 14             | 2                   | G                        |

- a. The directed network below represents this construction project, but the edge for activity E is missing.

Complete the network by including activity E, along with its duration time. 1 mark



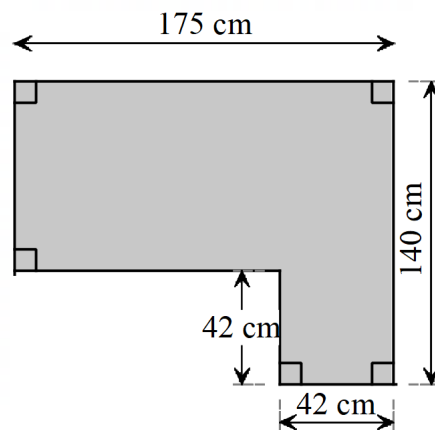
- b. What is the shortest time, in weeks, in which this construction project can be completed? 1 mark

- c. What is the latest start time for activity G if the project is not to be delayed? 1 mark

**End of Module 2: Networks and decision mathematics**

**Module 3: Geometry and measurement****Question 1 (4 marks)**

Carolyn has a studio where she makes wearable art.  
The following diagram shows the top surface of her work bench.



- a. What is the area, in square centimetres, of the top surface of Carolyn's work bench?

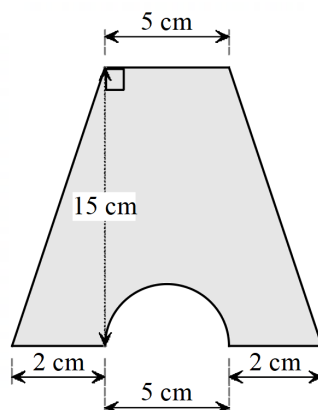
1 mark

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Carolyn has a flat wooden bench pin for working on small jewellery pieces.  
The bench pin is in the shape of a trapezium with a semicircle cut out as shown.



- b. Calculate the area, in square centimetres, of the bench pin, correct to one decimal place.

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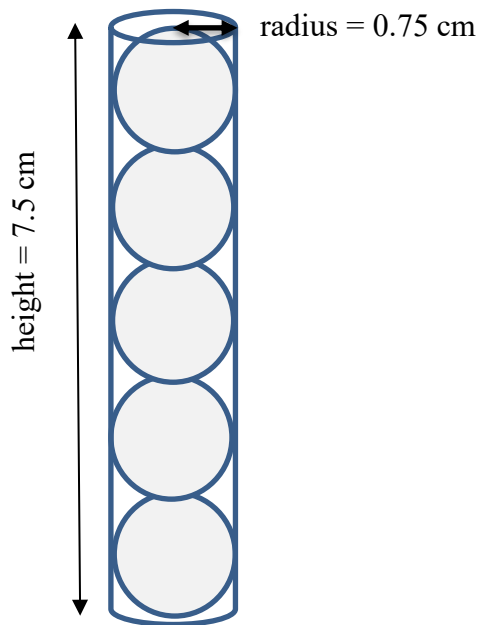
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1 mark

**Module 3: Geometry and measurement****Question 1 (continued)**

Carolyn uses precious metal beads in her work.

The beads come in cylindrical containers holding five beads as shown in the diagram below.



The container has a radius of 0.75 cm and a height of 7.5 cm.  
Each bead is spherical with a diameter of 1.5 cm.

- c. Write a calculation to show that the volume, rounded to two decimal places, of one bead is  $1.77 \text{ cm}^3$ .

1 mark

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- d. To protect the beads, the empty space in the container is filled with oil.  
What volume of oil, in cubic centimetres, is needed to fill the space?  
Give your answer correct to one decimal place.

1 mark

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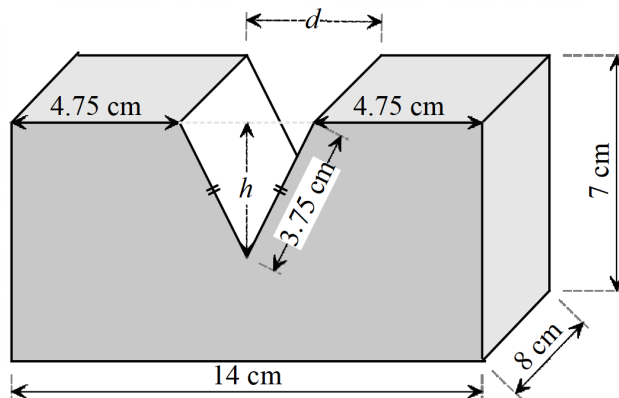
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**Module 3: Geometry and measurement****Question 2 (4 marks)**

Carolyn has a jump-ring vice for making small components.

The vice is made by cutting out a triangular prism from a wooden cuboid as shown.



- a. **Show** that the length  $d$ , is 4.5 cm. 1 mark

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- b. The vertical height of the triangular face of the cut-out prism is  $h$ .  
**Show** that  $h$  is equal to 3 cm. 1 mark

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- c. What is the volume, in cubic centimetres, of the triangular prism that has been cut out?

1 mark

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- d. What is the volume, in cubic centimetres, of the jump-ring vice?

1 mark

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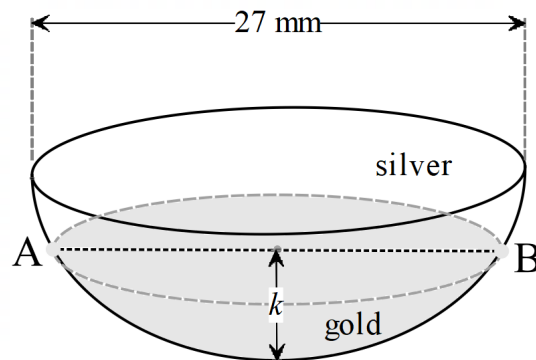


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**Module 3: Geometry and measurement****Question 3 (2 marks)**

Carolyn is making a small silver and gold bead.

It is in the shape of a hemisphere. The bottom part of the hemispherical bead is made of gold as shown in the diagram below.



The area of the top of the gold section is one ninth of the area of the top of the silver section.

- a. Show that the diameter, AB, of the top of the gold section is 9 mm.

1 mark

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- b. Calculate the height,  $k$ , in millimetres, of the gold section of the bead.  
Round your answer correct to two decimal places.

1 mark

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**Question 4 (2 marks)**

Carolyn is going to an artist's convention in Bangkok, Thailand.  
She departs Melbourne ( $38^\circ \text{ S}$ ,  $145^\circ \text{ E}$ ) at 8:25 pm on Sunday, 8 May 2022.  
She arrives in Bangkok ( $14^\circ \text{ N}$ ,  $100^\circ \text{ E}$ ) at 2:50 am on Monday, 9 May 2022.  
The time difference between Melbourne and Bangkok is three hours.

- a. How long, in hours and minutes, does it take Carolyn to travel between Melbourne and Bangkok?

1 mark

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Carolyn will travel on to Chennai ( $14^\circ \text{ N}$ ,  $80^\circ \text{ E}$ ) after the convention.  
Assume that the radius of Earth is 6400 km.

- b. Find the shortest small circle distance between Bangkok and Chennai.  
Round your answer to the nearest kilometre.

1 mark

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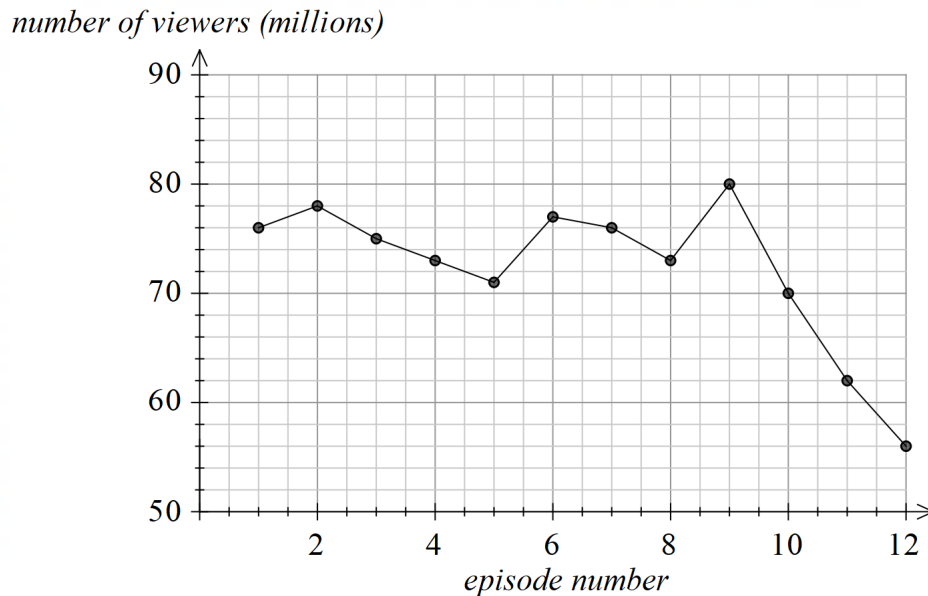
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**End of Module 3: Geometry and measurement**

**Module 4: Graphs and relations****Question 1 (4 marks)**

The graph below shows the *number of viewers*, in millions, for the first twelve episodes of a television drama.



- a. How many viewers watched the first episode? 1 mark

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- b. Over this period, the show had its highest number of viewers for an episode in which a long-running mystery was to be solved.  
What number episode was this? 1 mark

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- c. Show that the average rate of change of *number of viewers* from episode 7 to episode 12 is -4 million per episode.

1 mark

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**Module 4: Graphs and relations****Question 1 (continued)**

- d. Between which two consecutive episodes was the change in the *number of viewers* the least?

1 mark

**Question 2 (4 marks)**

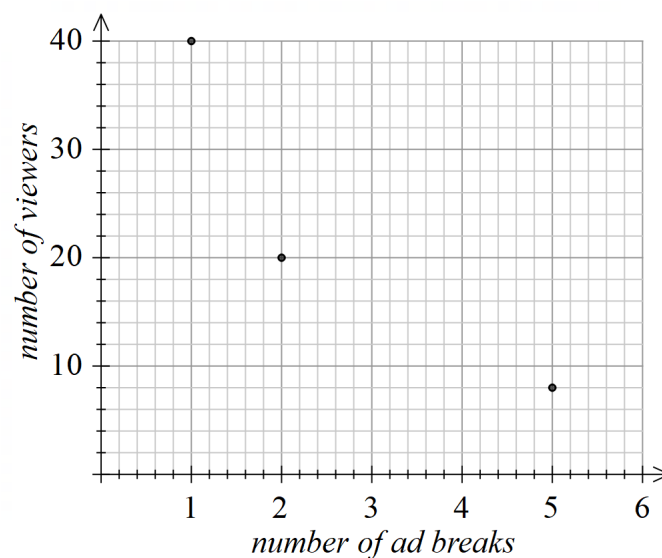
The producers of the television drama want to see if there is an association between the *number of viewers* and the *number of ad breaks* during the episode. They use a small focus group of people.

The following table shows the different *number of ad breaks* tried and the associated *number of viewers*.

|                     |    |    |    |   |
|---------------------|----|----|----|---|
| number of ad breaks | 1  | 2  | 4  | 5 |
| number of viewers   | 40 | 20 | 10 | 8 |

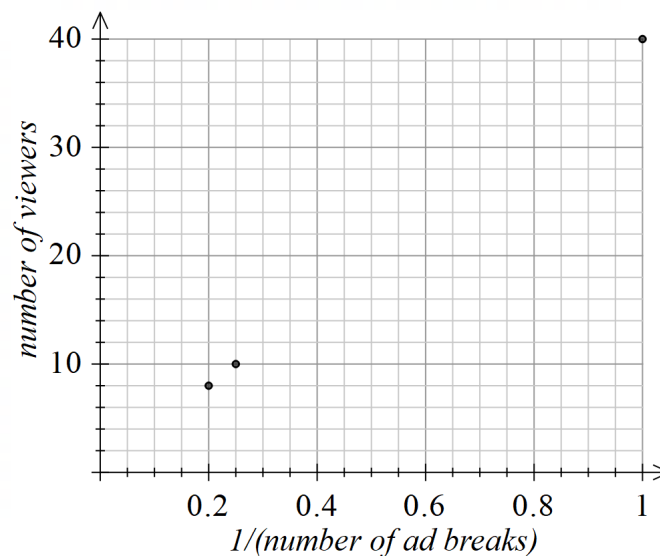
- a. Complete the graph below by plotting the number of viewers when the number of ad breaks is 4 by marking this value with a cross (x).

1 mark



**Module 4: Graphs and relations****Question 2 (continued)**

- b. A graph of number of viewers against the reciprocal of number of ad breaks is made. Fill in the missing value for number of viewers when the reciprocal of number of ad breaks is 0.5. Mark this point with a cross (x) on the graph below. 1 mark



- c. The graph in part b suggests that the relationship between *number of viewers* and *number of ad breaks* is of the form  

$$\text{number of viewers} = k/(\text{number of ad breaks})$$
  
 Find the value of  $k$ .

1 mark

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- d. What would be the expected number of viewers from this focus group if the number of ad breaks was 20?

1 mark

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**Module 4: Graphs and relations****Question 3 (4 marks)**

The canteen at the television studio must be set up with tables and benches each day for the production crew.

Let  $x$  be the number of tables set up on any one day

Let  $y$  be the number benches set up on any one day

It takes 10 minutes to set up a table and 8 minutes to set up a bench.

There is a total of 120 minutes available each day to set up the tables and benches.

This information can be written as inequalities 1 to 3.

Inequality 1

$$x \geq 0$$

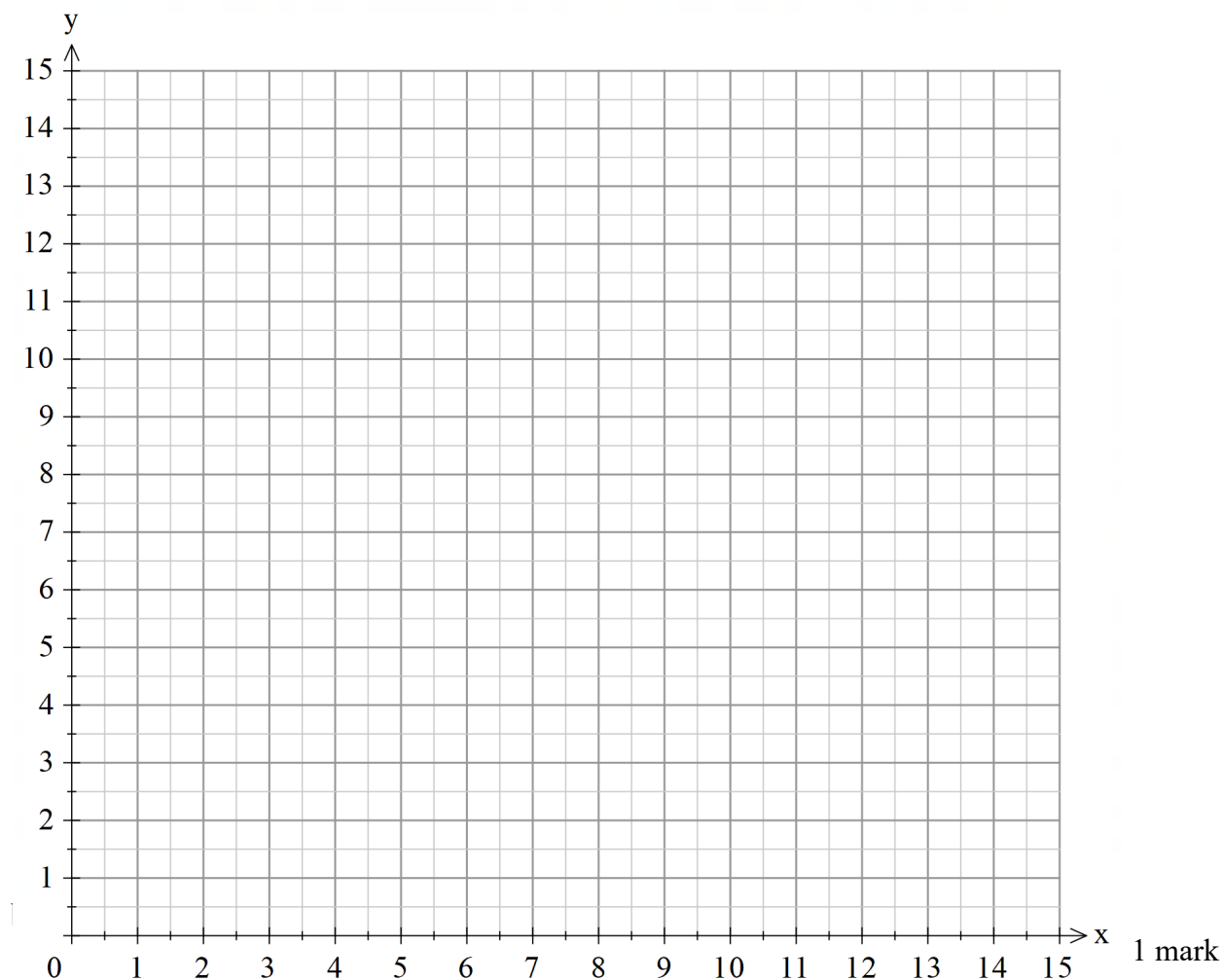
Inequality 2

$$y \geq 0$$

Inequality 3

$$10x + 8y \leq 120$$

- a. Draw the line that represents  $10x + 8y = 120$  on the graph below.



**Module 4: Graphs and relations****Question 3 (continued)**

On any one day, the number of benches is at least three times the number of tables.

- b. Write an inequality to describe this information in terms of  $x$  and  $y$ .

|               |  |
|---------------|--|
| Inequality 4: |  |
|---------------|--|

1 mark

- c. On the graph in part a. draw and clearly indicate the boundaries of the region represented by inequalities 1 to 4.

1 mark

Each table can seat 8 people and each bench can seat 6 people.

The total number of people that can be seated in the canteen each day is given by

$$N = 8x + 6y$$

- d. Determine the maximum number of people that can be seated in the canteen on any one day.

1 mark

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**End of Module 4: Graphs and relations**

**End of 2022 VCE Further Mathematics Written Examination 2**  
**Question and Answer Book**

**Kilbaha Education**  
**PO Box 2227**  
**Kew Vic 3101**  
**Australia**

**Tel: (03) 9018 5376**

[kilbaha@gmail.com](mailto:kilbaha@gmail.com)  
<https://kilbaha.com.au>

# **FURTHER MATHEMATICS**

## **Written examinations 1 and 2**

### **FORMULA SHEET**

#### **Directions to students**

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

## Further Mathematics Formulas

### Core: Data analysis

|                                    |                                                                              |
|------------------------------------|------------------------------------------------------------------------------|
| standardised score:                | $z = \frac{x - \bar{x}}{s_x}$                                                |
| lower and upper fence in a boxplot | lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$                    |
| least squares line:                | $y = a + bx$ where $b = r \frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$      |
| residual value:                    | residual value = actual value – predicted value                              |
| seasonal index:                    | seasonal index = $\frac{\text{actual figure}}{\text{deseasonalised figure}}$ |

### Core: Recursion and financial modelling

|                                                                       |                                                                                              |
|-----------------------------------------------------------------------|----------------------------------------------------------------------------------------------|
| first-order linear recurrence relation                                | $u_0 = a, \quad u_{n+1} = bu_n + c$                                                          |
| effective rate of interest for a compound interest loan or investment | $r_{\text{effective}} = \left[ \left( 1 + \frac{r}{100n} \right)^n - 1 \right] \times 100\%$ |

### Module 1: Matrices

|                                       |                                                                                                                     |
|---------------------------------------|---------------------------------------------------------------------------------------------------------------------|
| determinant of a $2 \times 2$ matrix: | $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}; \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ |
| inverse of a $2 \times 2$ matrix:     | $A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$                    |
| recurrence relation:                  | $S_0 = \text{initial state}, \quad S_{n+1} = TS_n + B$                                                              |

### Module 2: Networks and decision mathematics

|                  |                 |
|------------------|-----------------|
| Euler's formula: | $v + f = e + 2$ |
|------------------|-----------------|

**Module 3: Geometry and measurement**

|                            |                                                              |
|----------------------------|--------------------------------------------------------------|
| area of a triangle:        | $A = \frac{1}{2}bc \sin(\theta^0)$                           |
| Heron's formula:           | $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$ |
| sine rule:                 | $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$  |
| cosine rule:               | $a^2 = b^2 + c^2 - 2bc \cos(A)$                              |
| circumference of a circle: | $2\pi r$                                                     |
| length of an arc:          | $r \times \frac{\pi}{180} \times \theta^0$                   |
| area of a circle:          | $\pi r^2$                                                    |
| area of sector             | $\pi r^2 \times \frac{\theta^0}{360}$                        |
| volume of a sphere:        | $\frac{4}{3}\pi r^3$                                         |
| surface area of a sphere:  | $4\pi r^2$                                                   |
| volume of a cone:          | $\frac{1}{3}\pi r^2 h$                                       |
| volume of a prism:         | area of base $\times$ height                                 |
| volume of a pyramid:       | $\frac{1}{3} \times$ area of base $\times$ height            |

**Module 4: Graphs and relations**

|                                      |                                   |
|--------------------------------------|-----------------------------------|
| gradient (slope) of a straight line: | $m = \frac{y_2 - y_1}{x_2 - x_1}$ |
| equation of a straight line:         | $y = mx + c$                      |

**END OF FORMULA SHEET**