

**Victorian Certificate of Education
2019**

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

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Letter

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FURTHER MATHEMATICS

Written examination 2

Monday 4 November 2019

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.45 am (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
	9	9	36
Section B – Modules	<i>Number of modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
	4	2	24
	Total 60		

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 37 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Core

Instructions for Section A

Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π , surds or fractions.

In ‘Recursion and financial modelling’, all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (4 marks)

Table 1 shows the *day number* and the *minimum temperature*, in degrees Celsius, for 15 consecutive days in May 2017.

Table 1

<i>Day number</i>	<i>Minimum temperature (°C)</i>
1	12.7
2	11.8
3	10.7
4	9.0
5	6.0
6	7.0
7	4.1
8	4.8
9	9.2
10	6.7
11	7.5
12	8.0
13	8.6
14	9.8
15	7.7

Data: Australian Government, Bureau of Meteorology,
<www.bom.gov.au/>

- a. Which of the two variables in this data set is an ordinal variable?1 mark
-

The incomplete ordered stem plot below has been constructed using the data values for days 1 to 10.

key: 4|1 = 4.1 $n = 15$

minimum temperature ($^{\circ}\text{C}$)

4		1	8
5			
6		0	7
7		0	
8			
9		0	2
10		7	
11		8	
12		7	

- b. Complete the **stem plot above** by adding the data values for days 11 to 15.

1 mark

(Answer on the stem plot above.)

- c. The ordered stem plot below shows the *maximum temperature*, in degrees Celsius, for the same 15 days.

key: 9|2 = 9.2 $n = 15$

maximum temperature ($^{\circ}\text{C}$)

9		2			
10					
11		5	6		
12		2	5		
13		5	5	7	
14		9	9		
15		0	2	5	6
16		0			

Data: Australian Government, Bureau of Meteorology,
<www.bom.gov.au/>

Use this stem plot to determine

- i. the value of the first quartile (Q_1)

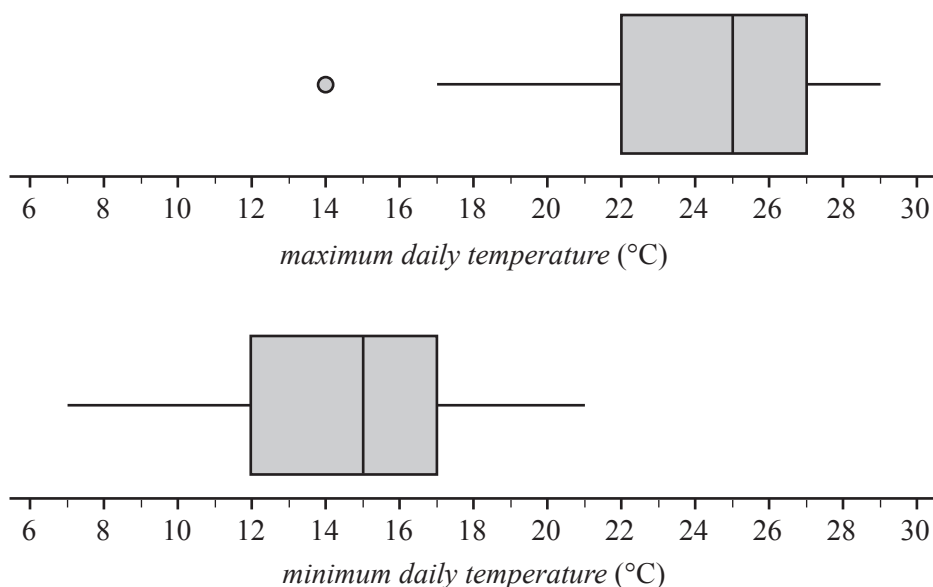
1 mark

- ii. the percentage of days with a maximum temperature higher than 15.3 $^{\circ}\text{C}$.

1 mark

Question 2 (4 marks)

The parallel boxplots below show the *maximum daily temperature* and *minimum daily temperature*, in degrees Celsius, for 30 days in November 2017.



Data: Australian Government, Bureau of Meteorology,
<www.bom.gov.au/>

- a. Use the information in the boxplots to complete the following sentences.

For November 2017

- i. the interquartile range for the *minimum daily temperature* was °C 1 mark

- ii. the median value for *maximum daily temperature* was °C higher than the median value for *minimum daily temperature* 1 mark

- iii. the number of days on which the *maximum daily temperature* was less than the median value for *minimum daily temperature* was . 1 mark

- b. The *temperature difference* between the *minimum daily temperature* and the *maximum daily temperature* in November 2017 at this location is approximately normally distributed with a mean of 9.4 °C and a standard deviation of 3.2 °C.

Determine the number of days in November 2017 for which this *temperature difference* is expected to be greater than 9.4 °C.

1 mark

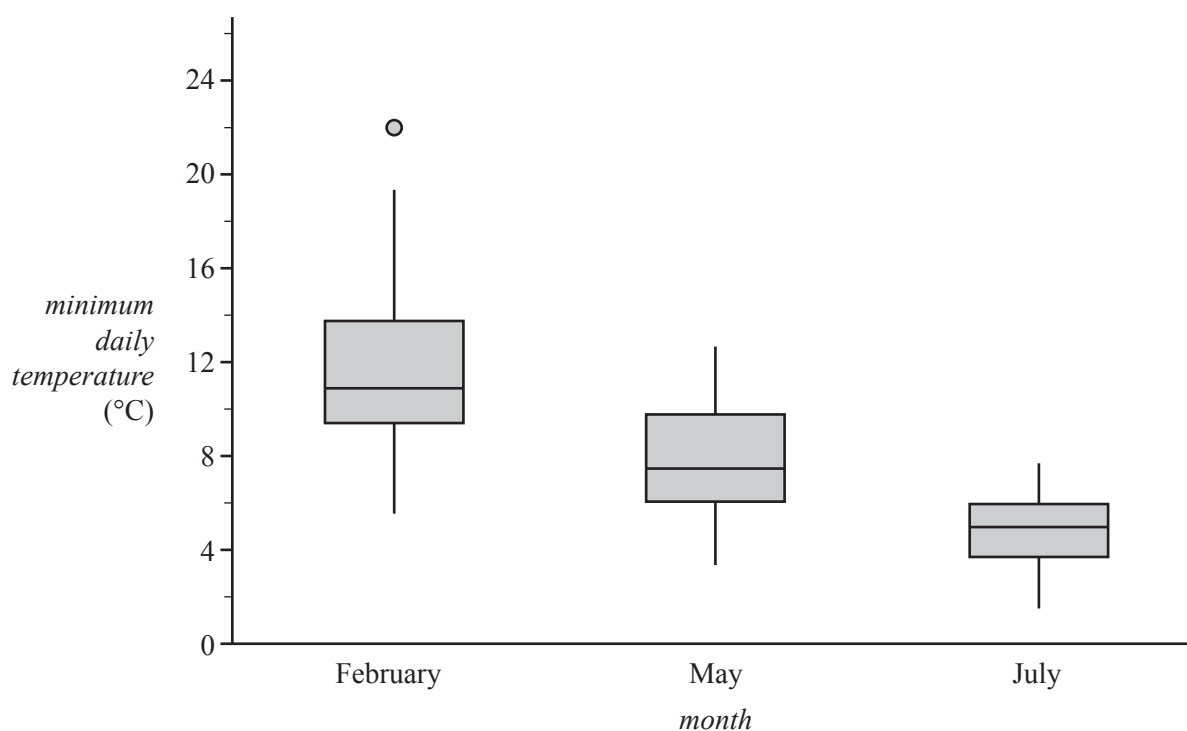
Question 3 (2 marks)

The five-number summary for the distribution of *minimum daily temperature* for the months of February, May and July in 2017 is shown in Table 2.

The associated boxplots are shown below the table.

Table 2. Five-number summary for *minimum daily temperature*

<i>Month</i>	Minimum	Q_1	Median	Q_3	Maximum
February	5.9	9.5	10.9	13.9	22.2
May	3.3	6.0	7.5	9.8	12.7
July	1.6	3.7	5.0	5.9	7.7



Data: Australian Government, Bureau of Meteorology, <www.bom.gov.au/>

Explain why the information given above supports the contention that *minimum daily temperature* is associated with the *month*. Refer to the values of an appropriate statistic in your response.

Question 4 (3 marks)

The relative humidity (%) at 9 am and 3 pm on 14 days in November 2017 is shown in Table 3 below.

Table 3

Relative humidity (%)	
9 am	3 pm
100	87
99	75
95	67
63	57
81	57
94	74
96	71
81	62
73	53
53	54
57	36
77	39
51	30
41	32

Data: Australian Government,
Bureau of Meteorology,
<www.bom.gov.au/>

A least squares line is to be fitted to the data with the aim of predicting the relative humidity at 3 pm (*humidity 3 pm*) from the relative humidity at 9 am (*humidity 9 am*).

- a. Name the explanatory variable.

1 mark

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- b. Determine the values of the intercept and the slope of this least squares line.

Round both values to three significant figures and write them in the appropriate boxes provided.

1 mark

$$\text{humidity } 3 \text{ pm} = \boxed{} + \boxed{} \times \text{humidity } 9 \text{ am}$$

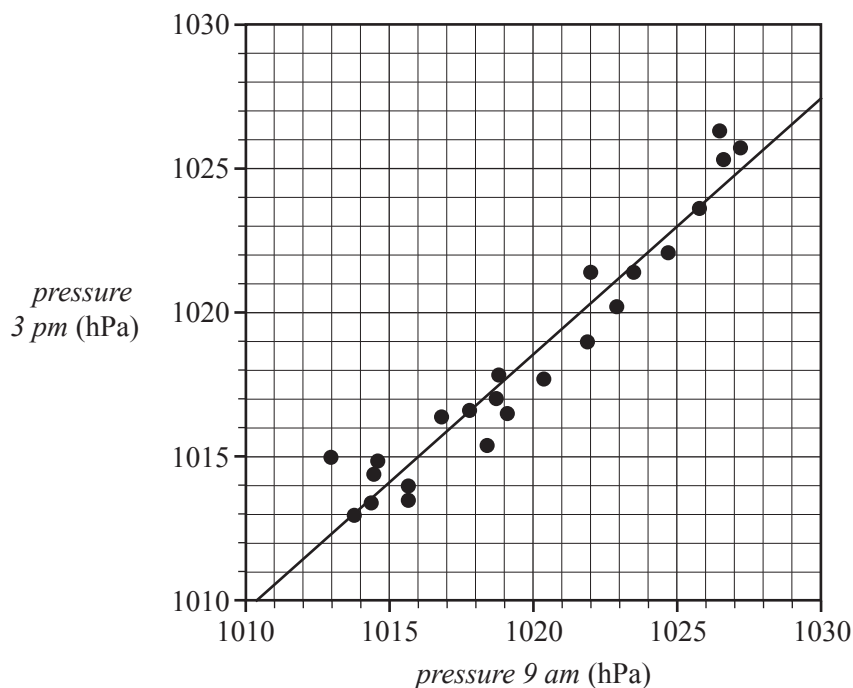
- c. Determine the value of the correlation coefficient for this data set.

Round your answer to three decimal places.

1 mark

Question 5 (8 marks)

The scatterplot below shows the atmospheric pressure, in hectopascals (hPa), at 3 pm (*pressure 3 pm*) plotted against the atmospheric pressure, in hectopascals, at 9 am (*pressure 9 am*) for 23 days in November 2017 at a particular weather station.



Data: Australian Government, Bureau of Meteorology, <www.bom.gov.au/>

A least squares line has been fitted to the scatterplot as shown.

The equation of this line is

$$\text{pressure 3 pm} = 111.4 + 0.8894 \times \text{pressure 9 am}$$

- a. Interpret the slope of this least squares line in terms of the atmospheric pressure at this weather station at 9 am and at 3 pm.

1 mark

- b. Use the equation of the least squares line to predict the atmospheric pressure at 3 pm when the atmospheric pressure at 9 am is 1025 hPa.

Round your answer to the nearest whole number.

1 mark

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- c. Is the prediction made in **part b.** an example of extrapolation or interpolation? 1 mark

- d. Determine the residual when the atmospheric pressure at 9 am is 1013 hPa.
Round your answer to the nearest whole number. 1 mark

- e. The mean and the standard deviation of *pressure 9 am* and *pressure 3 pm* for these 23 days are shown in Table 4 below.

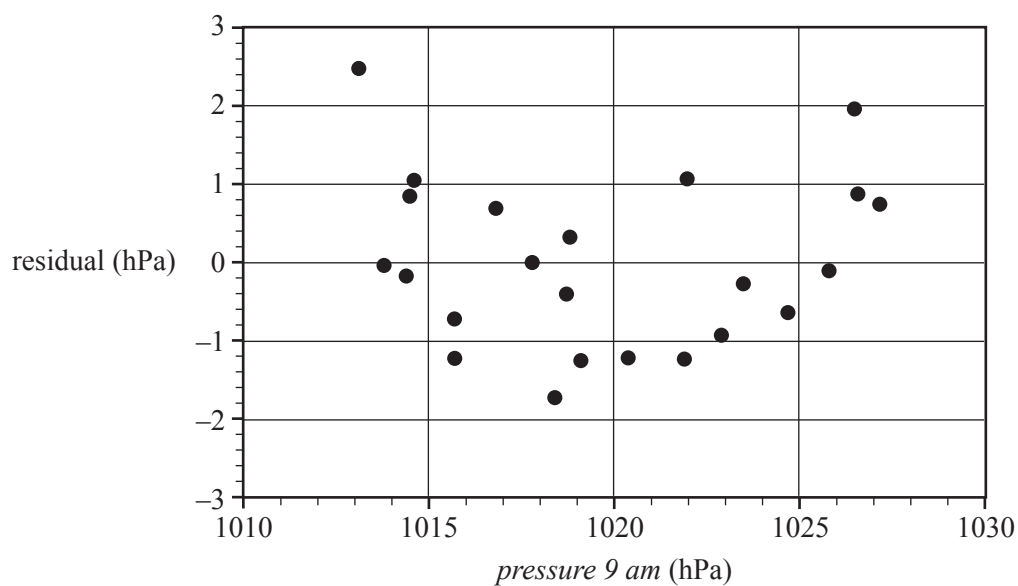
Table 4

	<i>Pressure 9 am</i>	<i>Pressure 3 pm</i>
Mean	1019.7	1018.3
Standard deviation	4.5477	4.1884

- i. Use the equation of the least squares line and the information in Table 4 to show that the correlation coefficient for this data, rounded to three decimal places, is $r = 0.966$ 1 mark

- ii. What percentage of the variation in *pressure 3 pm* is explained by the variation in *pressure 9 am*?
Round your answer to one decimal place. 1 mark

- f. The residual plot associated with the least squares line is shown below.



- i. The residual plot above can be used to test one of the assumptions about the nature of the association between the atmospheric pressure at 3 pm and the atmospheric pressure at 9 am.

What is this assumption?

1 mark

- ii. The residual plot above does not support this assumption.

Explain why.

1 mark

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Question 6 (3 marks)

The total rainfall, in millimetres, for each of the four seasons in 2015 and 2016 is shown in Table 5 below.

Table 5

	Total rainfall (mm)			
Year	Summer	Autumn	Winter	Spring
2015	142	156	222	120
2016	135	153	216	96

- a. The seasonal index for winter is shown in Table 6 below.

Use the values in Table 5 to find the seasonal indices for summer, autumn and spring.

Write your answers in Table 6, rounded to two decimal places.

2 marks

Table 6

	Summer	Autumn	Winter	Spring
Seasonal index			1.41	

- b. The total rainfall for each of the four seasons in 2017 is shown in Table 7 below.

Table 7

	Total rainfall (mm)			
Year	Summer	Autumn	Winter	Spring
2017	141	156	262	120

Use the appropriate seasonal index from Table 6 to deseasonalise the total rainfall for winter in 2017.

Round your answer to the nearest whole number.

1 mark

Recursion and financial modelling

Question 7 (4 marks)

Phil is a builder who has purchased a large set of tools.

The value of Phil's tools is depreciated using the reducing balance method.

The value of the tools, in dollars, after n years, V_n , can be modelled by the recurrence relation shown below.

$$V_0 = 60\,000, \quad V_{n+1} = 0.9 V_n$$

- a. Use recursion to show that the value of the tools after two years, V_2 , is \$48 600. 1 mark

- b. What is the annual percentage rate of depreciation used by Phil? 1 mark

- c. Phil plans to replace these tools when their value first falls below \$20 000.
After how many years will Phil replace these tools? 1 mark

- d. Phil has another option for depreciation. He depreciates the value of the tools by a flat rate of 8% of the purchase price per annum.
Let V_n be the value of the tools after n years, in dollars.
Write down a recurrence relation, in terms of V_0 , V_{n+1} and V_n , that could be used to model the value of the tools using this flat rate depreciation. 1 mark

Question 8 (4 marks)

Phil invests \$200 000 in an annuity from which he receives a regular monthly payment.

The balance of the annuity, in dollars, after n months, A_n , can be modelled by the recurrence relation

$$A_0 = 200\,000, \quad A_{n+1} = 1.0035 A_n - 3700$$

- a. What monthly payment does Phil receive? 1 mark

- b. Show that the annual percentage compound interest rate for this annuity is 4.2%. 1 mark

At some point in the future, the annuity will have a balance that is lower than the monthly payment amount.

- c. What is the balance of the annuity when it first falls below the monthly payment amount?
Round your answer to the nearest cent. 1 mark

- d. If the payment received each month by Phil had been a different amount, the investment would act as a simple perpetuity.
What monthly payment could Phil have received from this perpetuity? 1 mark

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Question 9 (4 marks)

Phil would like to purchase a block of land.

He will borrow \$350 000 to make this purchase.

Interest on this loan will be charged at the rate of 4.9% per annum, compounding fortnightly.

After three years of equal fortnightly repayments, the balance of Phil's loan will be \$262 332.33

- a. What is the value of each fortnightly repayment Phil will make?

Round your answer to the nearest cent.

1 mark

- b. What is the total interest Phil will have paid after three years?

Round your answer to the nearest cent.

1 mark

- c. Over the next four years of his loan, Phil will make monthly repayments of \$3517.28 and will be charged interest at the rate of 4.8% per annum, compounding monthly.

Let B_n be the balance of the loan n months after these changes apply.

Write down a recurrence relation, in terms of B_0 , B_{n+1} and B_n , that could be used to model the balance of the loan over these four years.

2 marks

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END OF SECTION A

SECTION B – Modules**Instructions for Section B**

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π , surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

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Module 1 – Matrices

Question 1 (5 marks)

The car park at a theme park has three areas, A , B and C .

The number of empty (E) and full (F) parking spaces in each of the three areas at 1 pm on Friday are shown in matrix Q below.

$$Q = \begin{array}{cc} & \begin{array}{cc} E & F \end{array} \\ \begin{array}{c} \left[\begin{array}{cc} 70 & 50 \\ 30 & 20 \\ 40 & 40 \end{array} \right] \end{array} & \begin{array}{c} A \\ B \\ C \end{array} \end{array} \text{ area}$$

- a. What is the order of matrix Q ?

1 mark

- b. Write down a calculation to show that 110 parking spaces are full at 1 pm.

1 mark

Drivers must pay a parking fee for each hour of parking.

Matrix P , below, shows the hourly fee, in dollars, for a car parked in each of the three areas.

$$P = \begin{array}{cc} & \begin{array}{ccc} & \text{area} \\ A & B & C \end{array} \\ \left[\begin{array}{ccc} 1.30 & 3.50 & 1.80 \end{array} \right] \end{array}$$

- c. The total parking fee, in dollars, collected from these 110 parked cars if they were parked for one hour is calculated as follows.

$$P \times L = [207.00]$$

where matrix L is a 3×1 matrix.

Write down matrix L .

1 mark

$$L =$$

The number of whole hours that each of the 110 cars had been parked was recorded at 1 pm. Matrix R , below, shows the number of cars parked for one, two, three or four hours in each of the areas A , B and C .

$$R = \begin{array}{c} \begin{array}{ccc} & \text{area} & \\ A & B & C \end{array} \\ \begin{bmatrix} 3 & 1 & 1 \\ 6 & 10 & 3 \\ 22 & 7 & 10 \\ 19 & 2 & 26 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \end{array} \text{ hours}$$

- d. Matrix R^T is the transpose of matrix R .

Complete the matrix R^T below.

1 mark

$$R^T = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

- e. Explain what the element in row 3, column 2 of matrix R^T represents.

1 mark

Question 2 (4 marks)

The theme park has four locations, Air World (A), Food World (F), Ground World (G) and Water World (W).

The number of visitors at each of the four locations is counted every hour.

By 10 am on Saturday the park had reached its capacity of 2000 visitors and could take no more visitors.

The park stayed at capacity until the end of the day.

The state matrix, S_0 , below, shows the number of visitors at each location at 10 am on Saturday.

$$S_0 = \begin{bmatrix} 600 \\ 600 \\ 400 \\ 400 \end{bmatrix} \begin{matrix} A \\ F \\ G \\ W \end{matrix}$$

- a. What percentage of the park's visitors were at Water World (W) at 10 am on Saturday?

1 mark

Let S_n be the state matrix that shows the number of visitors expected at each location n hours after 10 am on Saturday.

The number of visitors expected at each location n hours after 10 am on Saturday can be determined by the matrix recurrence relation below.

$$S_0 = \begin{bmatrix} 600 \\ 600 \\ 400 \\ 400 \end{bmatrix}, \quad S_{n+1} = T \times S_n \quad \text{where} \quad T = \begin{matrix} & \begin{matrix} \text{this hour} \\ A & F & G & W \end{matrix} \\ \begin{matrix} A \\ F \\ G \\ W \end{matrix} & \begin{bmatrix} 0.1 & 0.2 & 0.1 & 0.2 \\ 0.3 & 0.4 & 0.6 & 0.3 \\ 0.1 & 0.2 & 0.2 & 0.1 \\ 0.5 & 0.2 & 0.1 & 0.4 \end{bmatrix} \end{matrix} \begin{matrix} A \\ F \\ G \\ W \end{matrix} \begin{matrix} \\ \\ \text{next hour} \\ \end{matrix}$$

- b. Complete the state matrix, S_1 , below to show the number of visitors expected at each location at 11 am on Saturday.

1 mark

$$S_1 = \begin{bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \\ 300 \\ \underline{\hspace{2cm}} \end{bmatrix} \begin{matrix} A \\ F \\ G \\ W \end{matrix}$$

- c. Of the 300 visitors expected at Ground World (G) at 11 am, what percentage was at either Air World (A) or Food World (F) at 10 am?

1 mark

- d. The proportion of visitors moving from one location to another each hour on Sunday is different from Saturday.

Matrix V , below, shows the proportion of visitors moving from one location to another each hour after 10 am on Sunday.

$$V = \begin{array}{c} \begin{array}{cccc} & \text{this hour} & & \\ & A & F & G & W \\ \begin{bmatrix} 0.3 & 0.4 & 0.6 & 0.3 \\ 0.1 & 0.2 & 0.1 & 0.2 \\ 0.1 & 0.2 & 0.2 & 0.1 \\ 0.5 & 0.2 & 0.1 & 0.4 \end{bmatrix} & \begin{array}{l} A \\ F \\ G \\ W \end{array} \end{array} & \begin{array}{l} \\ \\ \text{next hour} \\ \end{array} \end{array}$$

Matrix V is similar to matrix T but has the first two rows of matrix T interchanged.

The matrix product that will generate matrix V from matrix T is

$$V = M \times T$$

where matrix M is a binary matrix.

Write down matrix M .

1 mark

$$M = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

Question 3 (3 marks)

On Sunday, matrix V is used when calculating the expected number of visitors at each location every hour after 10 am. It is assumed that the park will be at its capacity of 2000 visitors for all of Sunday.

Let L_0 be the state matrix that shows the number of visitors at each location at 10 am on Sunday.

The number of visitors expected at each location at 11 am on Sunday can be determined by the matrix product

$$V \times L_0 \quad \text{where} \quad L_0 = \begin{bmatrix} 500 \\ 600 \\ 500 \\ 400 \end{bmatrix} \begin{matrix} A \\ F \\ G \\ W \end{matrix} \quad \text{and} \quad V = \begin{matrix} \begin{matrix} & \text{this hour} \\ & A & F & G & W \end{matrix} \\ \begin{matrix} A \\ F \\ G \\ W \end{matrix} \begin{bmatrix} 0.3 & 0.4 & 0.6 & 0.3 \\ 0.1 & 0.2 & 0.1 & 0.2 \\ 0.1 & 0.2 & 0.2 & 0.1 \\ 0.5 & 0.2 & 0.1 & 0.4 \end{bmatrix} \begin{matrix} A \\ F \\ G \\ W \end{matrix} \end{matrix} \begin{matrix} \\ \\ \\ \text{next hour} \end{matrix}$$

- a. Safety restrictions require that all four locations have a maximum of 600 visitors.

Which location is expected to have more than 600 visitors at 11 am on Sunday?

1 mark

- b. Whenever more than 600 visitors are expected to be at a location on Sunday, the first 600 visitors can stay at that location and all others will be moved directly to Ground World (G).

State matrix R_n contains the number of visitors at each location n hours after 10 am on Sunday, after the safety restrictions have been enforced.

Matrix R_1 can be determined from the matrix recurrence relation

$$R_0 = \begin{bmatrix} 500 \\ 600 \\ 500 \\ 400 \end{bmatrix} \begin{matrix} A \\ F \\ G \\ W \end{matrix}, \quad R_1 = V \times R_0 + B_1$$

where matrix B_1 shows the required movement of visitors at 11 am.

- i. Determine the matrix B_1 .

1 mark

$$B_1 =$$

- ii. State matrix R_2 can be determined from the new matrix rule

$$R_2 = VR_1 + B_2$$

where matrix B_2 shows the required movement of visitors at 12 noon.

Determine the state matrix R_2 .

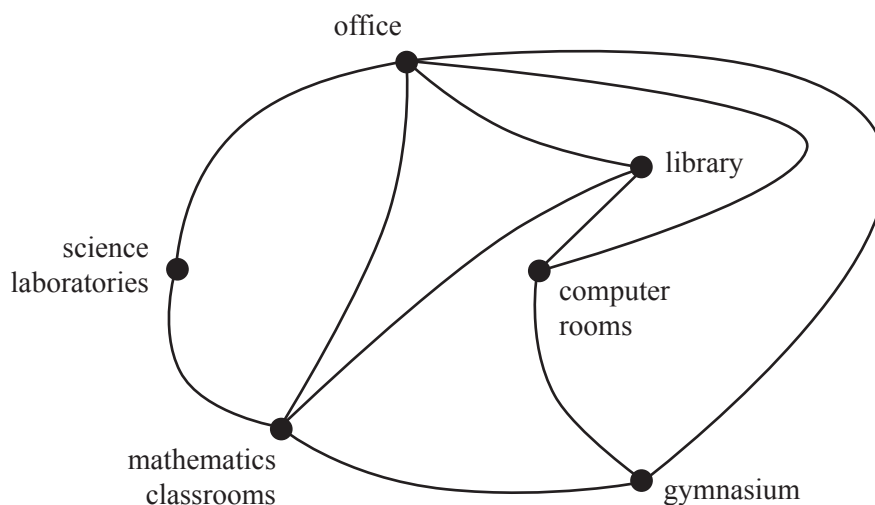
1 mark

$$R_2 =$$

Module 2 – Networks and decision mathematics

Question 1 (3 marks)

Fencedale High School has six buildings. The network below shows these buildings represented by vertices. The edges of the network represent the paths between the buildings.

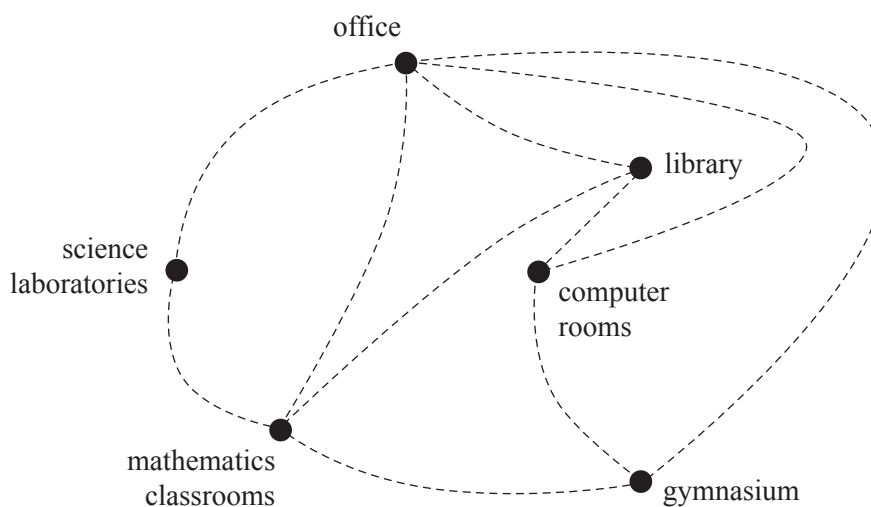


- a. Which building in the school can be reached directly from all other buildings? 1 mark

- b. A school tour is to start and finish at the office, visiting each building only once.

- i. What is the mathematical term for this route? 1 mark

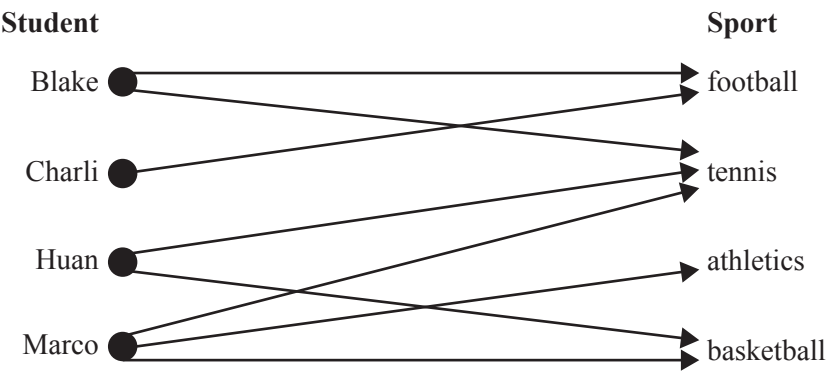
- ii. Draw in a possible route for this school tour on the diagram below. 1 mark



Question 2 (3 marks)

Fencedale High School offers students a choice of four sports, football, tennis, athletics and basketball.

The bipartite graph below illustrates the sports that each student can play.



Each student will be allocated to only one sport.

- a. Complete the table below by allocating the appropriate sport to each student.
- 1 mark

Student	Sport
Blake	
Charli	
Huan	
Marco	

- b. The school medley relay team consists of four students, Anita, Imani, Jordan and Lola.

The medley relay race is a combination of four different sprinting distances: 100 m, 200 m, 300 m and 400 m, run in that order.

The following table shows the best time, in seconds, for each student for each sprinting distance.

Student	Best time for each sprinting distance (seconds)			
	100 m	200 m	300 m	400 m
Anita	13.3	29.6	61.8	87.1
Imani	14.5	29.6	63.5	88.9
Jordan	13.3	29.3	63.6	89.1
Lola	15.2	29.2	61.6	87.9

The school will allocate each student to one sprinting distance in order to minimise the total time taken to complete the race.

To which distance should each student be allocated?

Write your answers in the table below.

2 marks

Student	Sprinting distance (m)
Anita	
Imani	
Jordan	
Lola	

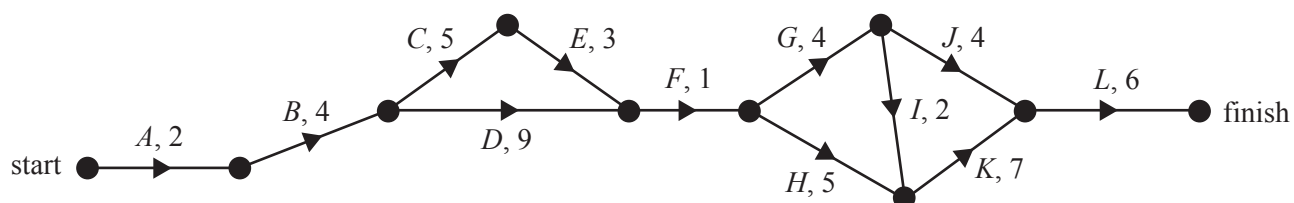
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Question 3 (6 marks)

Fencedale High School is planning to renovate its gymnasium.

This project involves 12 activities, *A* to *L*.

The directed network below shows these activities and their completion times, in weeks.



The minimum completion time for the project is 35 weeks.

- a. How many activities are on the critical path? 1 mark

- b. Determine the latest start time of activity *E*. 1 mark

- c. Which activity has the longest float time? 1 mark

It is possible to reduce the completion time for activities *C*, *D*, *G*, *H* and *K* by employing more workers.

- d. The completion time for each of these five activities can be reduced by a maximum of two weeks.
What is the minimum time, in weeks, that the renovation project could take? 1 mark

- e. The reduction in completion time for each of these five activities will incur an additional cost to the school.

The table below shows the five activities that can have their completion times reduced and the associated weekly cost, in dollars.

Activity	Weekly cost (\$)
<i>C</i>	3000
<i>D</i>	2000
<i>G</i>	2500
<i>H</i>	1000
<i>K</i>	4000

The completion time for each of these five activities can be reduced by a maximum of two weeks. Fencedale High School requires the overall completion time for the renovation project to be reduced by four weeks at minimum cost.

Complete the table below, showing the reductions in individual activity completion times that would achieve this.

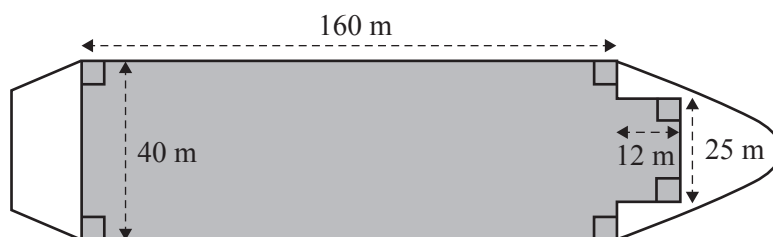
2 marks

Activity	Reduction in completion time (0, 1 or 2 weeks)
<i>C</i>	
<i>D</i>	
<i>G</i>	
<i>H</i>	
<i>K</i>	

Module 3 – Geometry and measurement

Question 1 (4 marks)

The following diagram shows a cargo ship viewed from above.



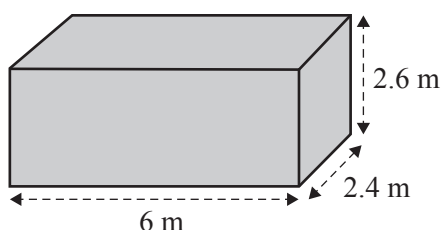
The shaded region illustrates the part of the deck on which shipping containers are stored.

- a. What is the area, in square metres, of the shaded region?

1 mark

Each shipping container is in the shape of a rectangular prism.

Each shipping container has a height of 2.6 m, a width of 2.4 m and a length of 6 m, as shown in the diagram below.



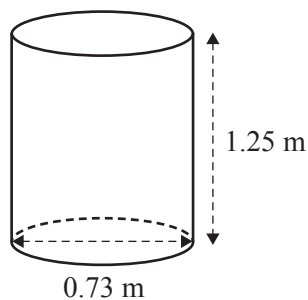
- b. What is the volume, in cubic metres, of one shipping container?

1 mark

- c. What is the total surface area, in square metres, of the outside of one shipping container?

1 mark

- d. One shipping container is used to carry barrels. Each barrel is in the shape of a cylinder. Each barrel is 1.25 m high and has a diameter of 0.73 m, as shown in the diagram below. Each barrel must remain upright in the shipping container.



What is the maximum number of barrels that can fit in one shipping container?

1 mark

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Question 2 (3 marks)

A cargo ship travels from Magadan (60° N, 151° E) to Sydney (34° S, 151° E).

- a. Explain, with reference to the information provided, how we know that Sydney is closer to the equator than Magadan.

1 mark

- b. Assume that the radius of Earth is 6400 km.

Find the shortest great circle distance between Magadan and Sydney.

Round your answer to the nearest kilometre.

1 mark

- c. The cargo ship left Sydney (34° S, 151° E) at 6 am on 1 June and arrived in Perth (32° S, 116° E) at 10 am on 11 June.

There is a two-hour time difference between Sydney and Perth at that time of year.

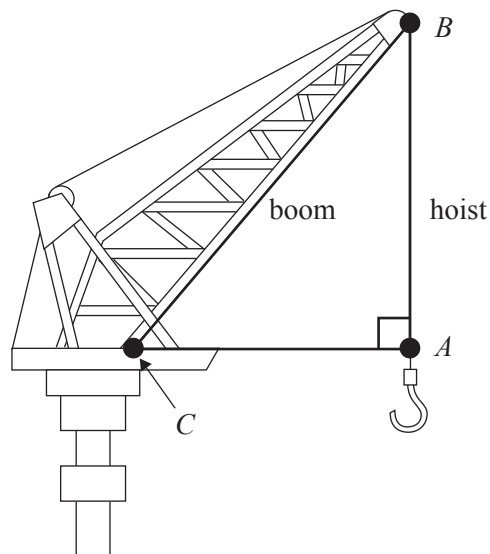
How many hours did it take the cargo ship to travel from Sydney to Perth?

1 mark

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Question 3 (5 marks)

The following diagram shows a crane that is used to transfer shipping containers between the port and the cargo ship.



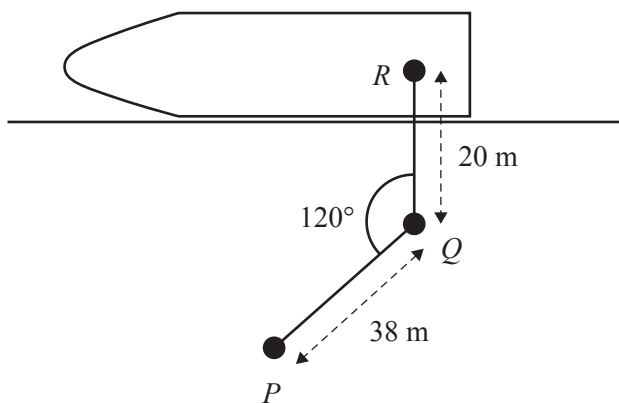
The length of the boom, BC , is 25 m. The length of the hoist, AB , is 15 m.

- a. i. Write a calculation to show that the distance AC is 20 m. 1 mark

- ii. Find the angle ACB . 1 mark
Round your answer to the nearest degree.

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- b. The diagram below shows a cargo ship next to a port. The base of a crane is shown at point Q .



The base of the crane (Q) is 20 m from a shipping container at point R . The shipping container will be moved to point P , 38 m from Q . The crane rotates 120° as it moves the shipping container anticlockwise from R to P .

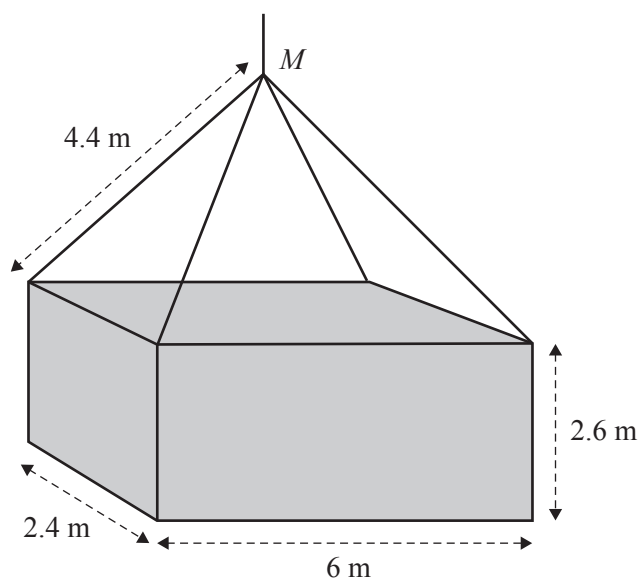
What is the distance RP , in metres?

Round your answer to the nearest metre.

1 mark

- c. A shipping container is a rectangular prism.

Four chains connect the shipping container to a hoist at point M , as shown in the diagram below.



The shipping container has a height of 2.6 m , a width of 2.4 m and a length of 6 m .

Each chain on the hoist is 4.4 m in length.

What is the vertical distance, in metres, between point M and the top of the shipping container?

Round your answer to the nearest metre.

2 marks

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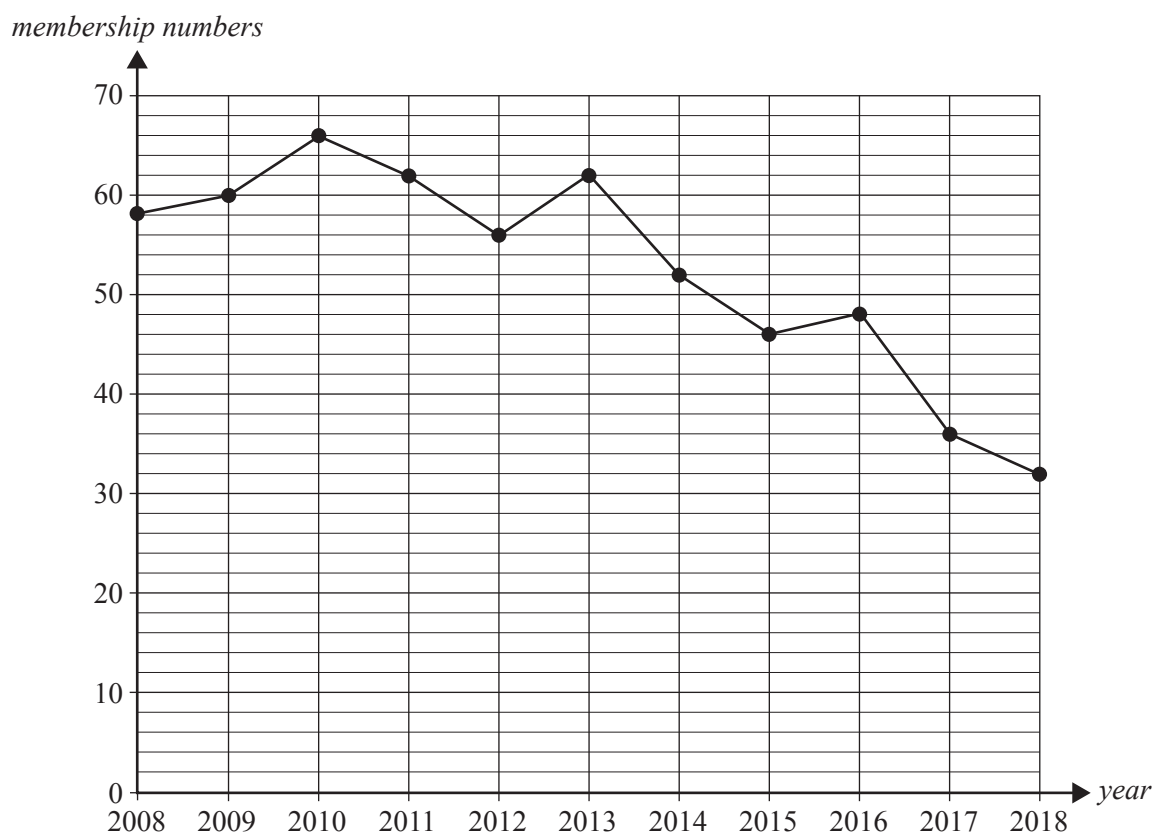
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Module 4 – Graphs and relations

Question 1 (3 marks)

The graph below shows the *membership numbers* of the Wombatong Rural Women's Association each year for the years 2008–2018.



- a. How many members were there in 2009? 1 mark

- b. i. Show that the average rate of change of *membership numbers* from 2013 to 2018 was -6 members per year. 1 mark

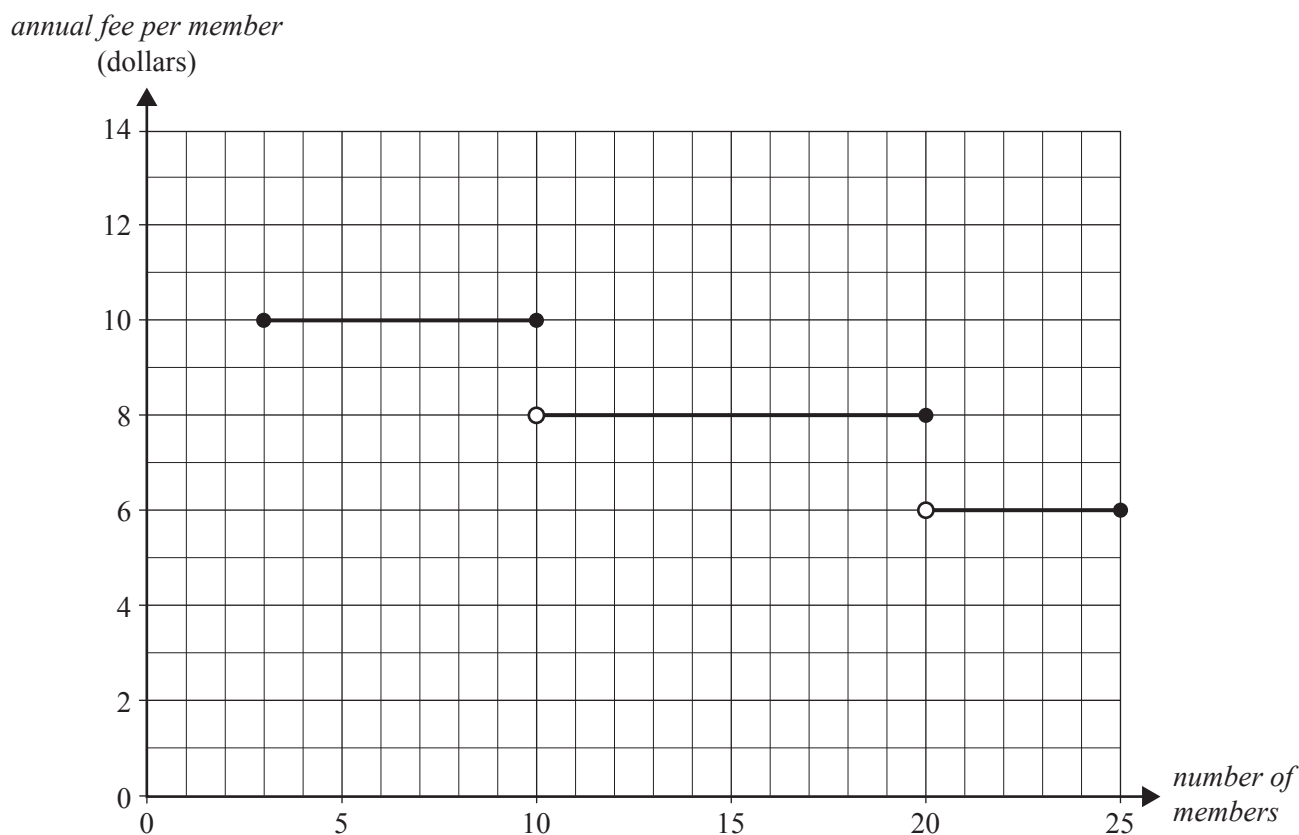
- ii. If the change in *membership numbers* continues at this rate, how many members will there be in 2021? 1 mark

Question 2 (4 marks)

Each branch within the association pays an annual fee based on the number of members it has.

To encourage each branch to find new members, two new annual fee systems have been proposed.

Proposal 1 is shown in the graph below, where the proposed *annual fee per member*, in dollars, is displayed for branches with up to 25 members.



- a. What is the smallest number of members that a branch may have?

1 mark

- b. The incomplete inequality below shows the number of members required for an annual fee per member of \$10.

Complete the inequality by writing the appropriate symbol and number in the box provided.

1 mark

$3 \leq$ number of members

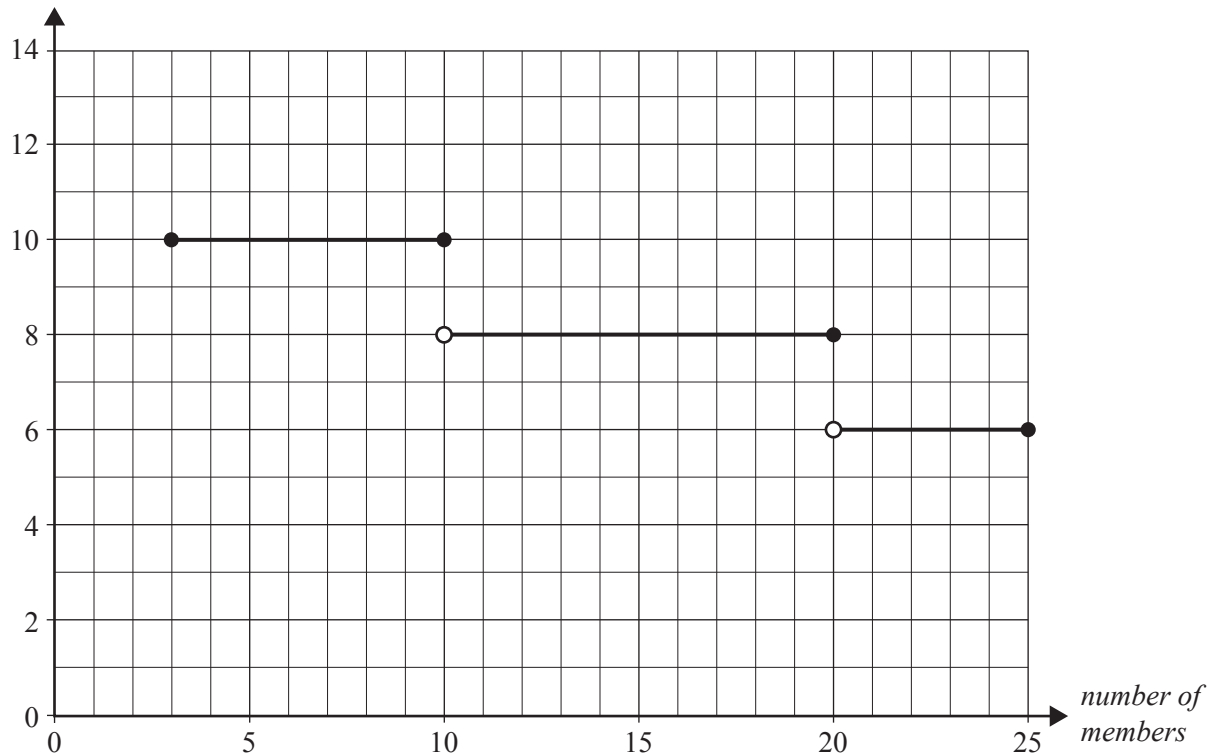
Proposal 2 is modelled by the following equation.

$$\text{annual fee per member} = -0.25 \times \text{number of members} + 12.25$$

- c. Sketch this equation on the graph for Proposal 1, shown below.

1 mark

annual fee per member
(dollars)



- d. Proposal 1 and Proposal 2 have the same *annual fee per member* for some values of the *number of members*.

Write down all values of the *number of members* for which this is the case.

1 mark

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Question 3 (5 marks)

Members of the association will travel to a conference in cars and minibuses:

- Let x be the number of cars used for travel.
- Let y be the number of minibuses used for travel.
- A maximum of eight cars and minibuses in total can be used.
- At least three cars must be used.
- At least two minibuses must be used.

The constraints above can be represented by the following three inequalities.

Inequality 1 $x + y \leq 8$

Inequality 2 $x \geq 3$

Inequality 3 $y \geq 2$

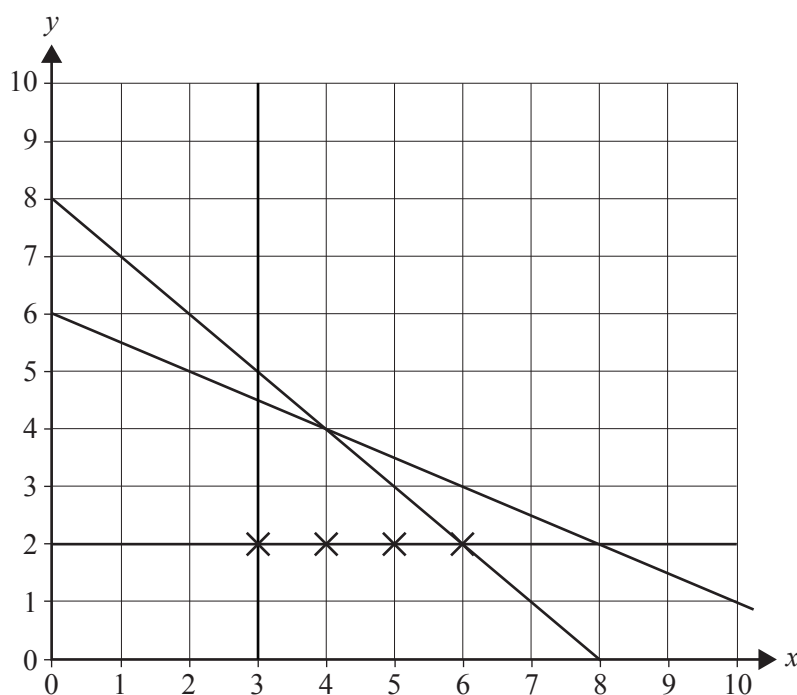
- a. Each car can carry a total of five people and each minibus can carry a total of 10 people.
A maximum of 60 people can attend the conference.

Use this information to write Inequality 4.

1 mark

The graph below shows the four lines representing Inequalities 1 to 4.

Also shown on this graph are four of the integer points that satisfy Inequalities 1 to 4. Each of these integer points is marked with a cross (X).



- b. On the **graph above**, mark clearly, with a circle ($\circ$), the remaining integer points that satisfy Inequalities 1 to 4.

1 mark

(Answer on the graph above.)

Each car will cost \$70 to hire and each minibus will cost \$100 to hire.

- c. What is the cost for 60 members to travel to the conference? 1 mark

- d. What is the minimum cost for 55 members to travel to the conference? 1 mark

- e. Just before the cars were booked, the cost of hiring each car increased.

The cost of hiring each minibus remained \$100.

All original constraints apply.

If the increase in the cost of hiring each car is **more than** k dollars, then the maximum cost of transporting members to this conference can only occur when using six cars and two minibuses.

Determine the value of k .

1 mark

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**Victorian Certificate of Education
2019**

FURTHER MATHEMATICS

Written examination 2

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Further Mathematics formulas

Core – Data analysis

standardised score	$z = \frac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r \frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	seasonal index = $\frac{\text{actual figure}}{\text{deseasonalised figure}}$

Core – Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{\text{effective}} = \left[\left(1 + \frac{r}{100n} \right)^n - 1 \right] \times 100\%$

Module 1 – Matrices

determinant of a 2×2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix	$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \quad \text{where} \quad \det A \neq 0$
recurrence relation	$S_0 = \text{initial state}, \quad S_{n+1} = TS_n + B$

Module 2 – Networks and decision mathematics

Euler's formula	$v + f = e + 2$
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Module 3 – Geometry and measurement

area of a triangle	$A = \frac{1}{2}bc \sin(\theta^\circ)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}, \quad \text{where} \quad s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc \cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta^\circ$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta^\circ}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base $\times$ height
volume of a pyramid	$\frac{1}{3} \times \text{area of base} \times \text{height}$

Module 4 – Graphs and relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$