

FURTHER MATHEMATICS

Units 3 & 4 – Written examination 1



2019 Trial Examination

SOLUTIONS

SECTION A: Multiple-choice questions (1 mark each)

Core: Data analysis

Question 1.

Answer: **B**

Explanation:

There are 4 scores above 85. $\frac{4}{30} \times 100 = \frac{40}{3} \approx 13.3333$

Question 2.

Answer: **C**

Explanation:

For this data min = 26, $Q_1 = 50$, $Q_2 = 61$, $Q_3 = 72$, max = 92

Question 3.

Answer: **D**

Explanation:

Using CAS:

Mean is 60.8 and standard deviation is $17.65 \approx 17.7$

	A	score	B	C
=				=OneVar(
1		26	Title	One-Va...
2		35	\bar{x}	60.8
3		37	Σx	1824.
4		38	Σx^2	119938.
5		40	$s_x := s_n - \dots$	17.6545...

Question 4.

Answer: D

Explanation:

The birth weight is 1.6 times the standard deviation above the mean

$$154 + 1.6 \times 27 = 197.2 \approx 197$$

Question 5.

Answer: C

Explanation:

100 grams is 2 standard deviations below the mean, therefore 2.5% of data lies below this.

Question 6.

Answer: D

Explanation:

127 grams is 1 standard deviation below the mean and 208 grams is 2 standard deviations above the mean, thus $34\% + 47.5\% = 81.5\%$ of goslings are expected to have birth weights between these values.

$$\frac{81.5}{100} \times 4809 = 3919.3 \approx 3920$$

Question 7.

Answer: D

Explanation:

Both variables are categorical. A segmented bar chart is the only appropriate one because all other representations listed require numerical data.

Question 8.

Answer: E

Explanation:

$$\text{predicted mortalities} = 296.317 - 45.2052 \times 4.48 = 93.80$$

$$\text{Residual} = \text{actual} - \text{predicted} = 54.11 - 93.80 = -39.69$$

Question 9.

Answer: D

Explanation:

$$r = -\sqrt{0.5921} = -0.76948 \approx -0.7695$$

r must be negative because the gradient of the regression line is negative.

Question 10.*Answer: D**Explanation:*

The shape of the data on the scatter plot is not linear, thus the regression equation shown will not give reliable predictions. The data will need to be transformed to linearise it before reliable predictions can be made.

Question 11.*Answer: C**Explanation:*

The best way to view a data set to determine if it is linear is to plot a residual plot. If there is no clear pattern in the residual plot then the assumption of linearity is confirmed. Correlation coefficient and coefficient of determination give an indication of the strength of a relationship, but do not give any indication of the shape of the scatter plot.

Question 12.*Answer: A**Explanation:*

Because this is the translated equation, interpretation of the coefficient of determination, slope and intercept are in terms of $\frac{1}{\text{obesity}}$ and *Year*, and not directly linking *obesity* to *Year*.

Applying the regression equation to calculate the obesity in 2004 on CAS:

$$\text{solve}\left(\frac{1}{\text{obesity}}=4.8224-0.002371 \cdot 2004, \text{obesity}\right) \\ \text{obesity}=14.1012$$

Question 13.*Answer: A**Explanation:*

Using CAS to determine the predicted obesity rate in 2016

$$\text{solve}\left(\frac{1}{\text{obesity}}=4.8224-0.002371 \cdot 2016, \text{obesity}\right) \\ \text{obesity}=23.5493594574$$

Question 14.*Answer: B**Explanation:*

The slope is calculated using the correlation coefficient, so can be used to determine the value of r

$$\text{slope} = r \times \frac{s_y}{s_x}$$

$$-0.0044 = r \times \frac{5.6}{803}$$

$$\therefore r = -0.6309 \approx -0.631$$

Question 15.*Answer: E**Explanation:*

Need to calculate the average across all months

$$\text{Year ave} = \frac{21.1 + 21.3 + 19.9 + 17.7 + 15.5 + 13.5 + 13.0 + 13.6 + 15.0 + 16.7 + 18.0 + 19.3}{12} = 17.05$$

$$\text{Feb SI} = \frac{21.3}{17.05} = 1.2492 \approx 1.25 \text{ rounded to two decimal places}$$

Question 16.*Answer: E**Explanation:*

To correct for seasonality, a figure must be divided by the seasonal index. Firstly the seasonal

$$\text{index for July} = \frac{13.0}{17.05}$$

$$\text{Deseasonal value} = \text{actual} \div \frac{13.0}{17.05} = \text{actual} \times \frac{17.05}{13.0} = 1.311... \times \text{actual} \text{ so average maximum temperatures for July will be increased by 31\%}$$

Core: Recursion and financial modelling

Question 17.

Answer: D

Explanation:

Using recursion on CAS:

15000	15000
$15000 \cdot 1.005 - 200$	14875.
$14875. \cdot 1.005 - 200$	14749.375
$14749.375 \cdot 1.005 - 200$	14623.121875
$14623.121875 \cdot 1.005 - 200$	14496.2374344
$14496.2374375 \cdot 1.005 - 200$	14368.7186718
$14368.718671797 \cdot 1.005 - 200$	14240.5622652
$14240.562265156 \cdot 1.005 - 200$	14111.7650765

Question 18.

Answer: D

Explanation:

Interest rate is 0.5% per month, so $12 \times 0.5\% = 6\%$ p.a.

Question 19.

Answer: D

Explanation:

Each of the recurrence relations can be used to generate a sequence to determine which is correct:

A: 4, 12, 20 so not A

B: 4, 12, 28 so not B

C: 4, 12, 36 so not C

D: 4, 12, 44, 172, 684 this is correct sequence

E: 4, 12, 52 so not E

Question 20.*Answer: E**Explanation:*

Because regular additions of \$250 are made each month, a total of \$750 is added to the investment in 3 months. This means that the amount of interest earned in 3 months is not \$1365.48, but $1365.48 - 750 = \$615.48$.

Question 21.*Answer: A**Explanation:*

Using finance solver as shown here to calculate the value of the investment after 12 months and 24 months

Finance Solver	
N:	12
I(%):	6.24
PV:	-39000
Pmt:	-250
FV:	44591.72635576
PpY:	12
CpY:	12
PmtAt:	END

Finance Solver	
N:	24
I(%):	6.24
PV:	-39000
Pmt:	-250
FV:	50542.53066849
PpY:	12
CpY:	12
PmtAt:	END

Thus the increase in the investment is $\$50542.53 - \$44591.73 = \$5950.80$

Question 22.*Answer: A**Explanation:*

We need to compare the effective interest rates for each of these options and then determine which is best for the BANK (not the borrower!)

eff(8.25,26)	8.58568369414
eff(8.24,52)	8.58192625278
eff(8.26,12)	8.57999714253
eff(8.32,4)	8.58320228254
eff(8.25,12)	8.56921386198

The bank will benefit most from the highest interest rate which is Option A

Question 23.*Answer: D**Explanation:*

Firstly need to calculate the interest rate so that we can use Finance Solver to determine the initial investment.

$$\text{Interest rate } \frac{178.34}{36273.08} \times 100 = 0.4916\% \text{ per month} = 5.8999 \approx 5.9\% \text{ per year}$$

Using finance solver on CAS:

Finance Solver	
N:	10
I(%):	5.9
PV:	-15067.311472178
Pmt:	-2000
FV:	36273.08
PpY:	12
CpY:	12
PmtAt:	END

So the initial investment is closest to \$15070

Question 24.*Answer: E**Explanation:*

Calculate the interest earned at the higher interest rate:

$$\$40640.47 \times \frac{6.1}{12 \times 100} = \$206.59$$

$$\text{Principal addition} = \text{Payment plus interest} = 2500 + 206.59 = \$2706.59$$

SECTION B: Modules Multiple-choice questions (1 mark each)

Module 1 - Matrices

Question 1.

Answer: B

Explanation:

Since the matrix is to be multiplied by a 3×1 matrix, the matrix represented by X must be $? \times 3$. Matrices C and E have 3 columns, so there are 2 matrices.

Question 2.

Answer: B

Explanation:

For an inverse matrix to exist, a matrix must be square and have a non-zero determinant. Matrices A , D and E are the only square matrices so we need to check the determinant of each of these:

$\det(A) = 0$ $\det(D) = 12$ $\det(E) = 8$, thus only 2 matrices will have a defined inverse.

Question 3.

Answer: E

Explanation:

Using CAS to complete matrix multiplication:

$$\begin{bmatrix} -6 & -12 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ -3 & 6 \end{bmatrix} = \begin{bmatrix} 30 & -84 \\ -5 & 14 \end{bmatrix}$$

Question 4.*Answer: D**Explanation:*

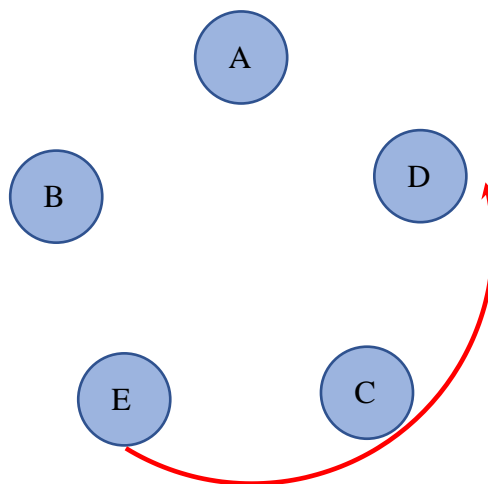
Communications matrix should be symmetric, so we need to determine which column is not the same as the corresponding row:

	A	B	C	D	E
A	0	1	0	1	0
B	1	0	0	0	1
C	0	0	0	1	1
D	1	0	0	0	0
E	0	1	1	0	0

The third row suggests that C can communicate with D, but the fourth row suggests that D cannot communicate with C. Since each student must be able to communicate directly with two other students then the fourth row must be incorrect.

Question 5.*Answer: D**Explanation:*

The communication matrix can be converted into a diagram showing which students sit next to each other:



Thus the shortest distance for a message to go from Eliza to Dennis is Eliza – Clara - Dennis

Question 6.

Answer: **E**

Explanation:

When multiplying a 4x4 matrix by the matrix in **E**:

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} b & a & d & c \\ f & e & h & g \\ j & i & l & k \\ n & m & p & o \end{bmatrix}$$

Question 7.

Answer: **A**

Explanation:

Matrix multiplication:

$$C = \begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 5 & 4 \end{bmatrix} \quad D = \begin{bmatrix} -1 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix}$$

$$D \times C = \begin{bmatrix} -1 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ -14 & -8 \end{bmatrix}$$

Question 8.

Answer: **C**

Explanation:

The columns in a transition matrix add to 1. Thus $X = 0.2$ and $Y = 0.2$

Use the fact that $S_1 = T \times S_0 + U$ so:

$$\begin{bmatrix} 296 \\ 405 \\ 352 \end{bmatrix} = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.1 & 0.6 & W \\ 0.2 & 0.2 & Z \end{bmatrix} \times \begin{bmatrix} 260 \\ 320 \\ 450 \end{bmatrix} + \begin{bmatrix} 5 \\ 7 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} 296 \\ 405 \\ 352 \end{bmatrix} = \begin{bmatrix} 0.7 \times 260 + 0.2 \times 320 + 0.1 \times 450 + 5 \\ 0.1 \times 260 + 0.6 \times 320 + W \times 450 + 7 \\ 0.2 \times 260 + 0.2 \times 320 + Z \times 450 + 11 \end{bmatrix}$$

$$\therefore 405 = 26 + 192 + 450W + 7 \text{ and } 352 = 52 + 64 + 450W + 11$$

$$W = 0.4 \text{ thus } Z = 0.5$$

from this we can see that $W > Z$ is the only untrue statement.

Module 2 – Networks and decision mathematics

Question 1.

Answer: D

Explanation:

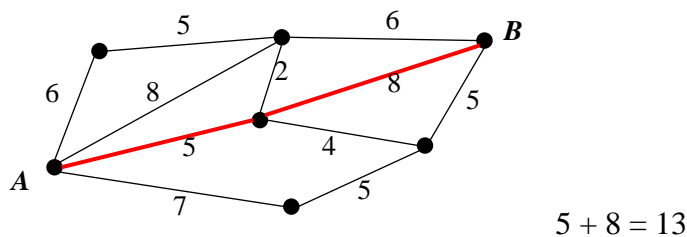
For an Euler circuit to exist the network all vertices must be even. All other networks contain odd vertices.

Question 2.

Answer: D

Explanation:

Shortest time path is shown on the diagram below:



Question 3.

Answer: D

Explanation:

Using the formula $v + f = e + 2$ and given that $e = 13$, then $v + f = 15$. The only combination of vertices and faces that does not add up to 15 is answer D

Question 4.

Answer: C

Explanation:

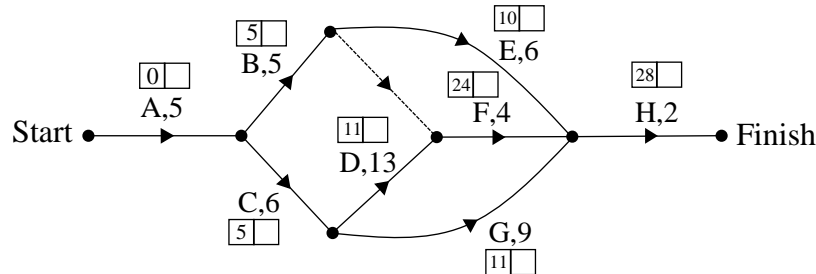
The adjacency matrix needs to be created:

	A	B	C	D	E
A	0	0	0	2	1
B	0	1	2	1	0
C	0	2	0	1	1
D	2	1	1	0	0
E	1	0	1	0	1

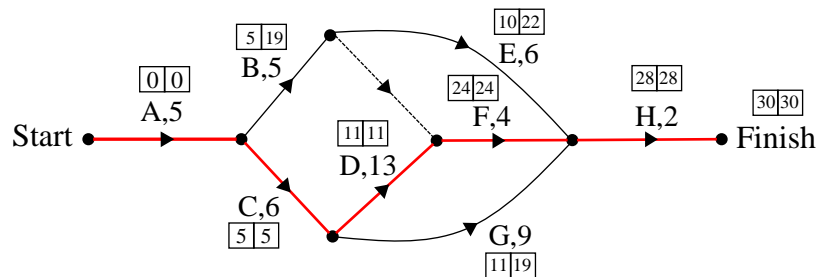
From this we can see that there are four 2's in the adjacency matrix.

Question 5.**Answer: E***Explanation:*

By completing a forward scan we find the earliest start time for activity H is 28 days.

**Question 6.****Answer: B***Explanation:*

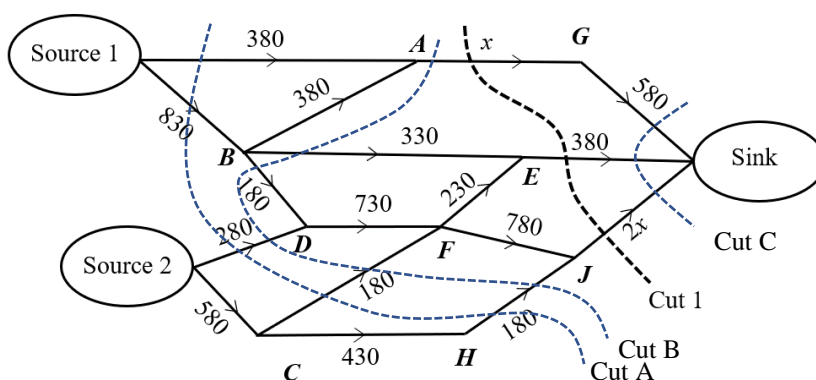
This question requires a backward scan to be completed to determine the critical path.



There are 5 activities along the critical path. For an activity to be reduced by 3 days and not reduced by more than half, then the activity needs to have a duration of 6 days or more. There are just two activities along the critical path that are 6 days or more in duration.

Question 7.**Answer: D***Explanation:*

Analysis of possible cuts is required for this question.



$$\text{Cut A} = 1850$$

$$\text{Cut B} = x + 1150$$

$$\text{Cut C} = 2x + 960$$

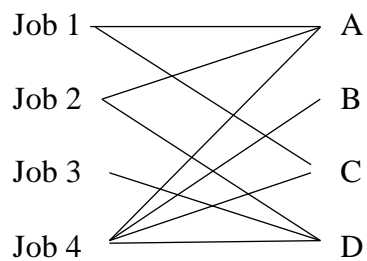
$$\text{Cut 1} = 3x + 380$$

Compare which is smallest by substituting values of x from the proposed answers. Thus answer D is correct. If $x = 385$ then Cut B is equal to Cut 1.

Question 8.**Answer: D***Explanation:*

Noting that all values of x given as options are 6 or less, start by completing a row reduction and then column reduction followed by an adjustment so as to determine what the bipartite graph will look like.

$$\begin{bmatrix} 5 & 8 & 5 & 7 \\ 5 & 7 & 6 & 4 \\ 7 & 7 & 9 & x \\ 4 & 4 & 4 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 3 & 0 & 2 \\ 1 & 3 & 2 & 0 \\ 7-x & 7-x & 9-x & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 2 & 0 & 2 \\ 1 & 2 & 2 & 0 \\ 7-x & 6-x & 9-x & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 3 & 0 & 3 \\ 0 & 2 & 1 & 0 \\ 6-x & 6-x & 8-x & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



The allocation will have Job 3 allocated to D provided $6-x \neq 0$. For this reason if x is less than or equal to 5, the optimum allocation will be that given in the question.

Module 3 – Geometry and measurement**Question 1.***Answer: D**Explanation:*

360° is divided into 12 equal parts $\frac{360}{12} = 30^\circ$

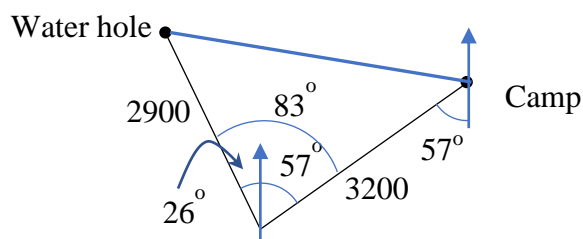
Question 2.*Answer: A**Explanation:*

The related sides on the triangles are NO is similar to QR and MO is similar to PR thus:

$$\frac{QR}{PR} = \frac{NO}{MO} \Rightarrow QR = \frac{NO}{MO} \times PR = \frac{133.2}{92.5} \times 66.6 = 95.904 \approx 95.9$$

Question 3.*Answer: D**Explanation:*

The sun rises in the east and sinks in the west. Places located furthest East will see sun rise first. Those furthest to the West will see the sun rise last. Thus Point A (116° East) followed by Point B (97° East) and then Point C (62° West)

Question 4.*Answer: B**Explanation:*

Using the cosine rule

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$

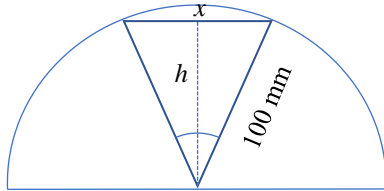
$$c^2 = (3200)^2 + (2900)^2 - 2 \times 3200 \times 2900 \cos(83^\circ)$$

$$c^2 = 16388104.98...$$

$$c = 4048.22.. \approx 4050 \text{ m}$$

Question 5.**Answer: C***Explanation:*

Use the answers given and test to see which is correct. If the area covered is a total of 1812 mm² then the area of one segment must be approximately 906 mm². The segment is the area of the sector minus the area of the triangle as shown below:



$$h = \sqrt{100^2 - \left(\frac{x}{2}\right)^2} \quad \text{and}$$

$$\frac{\theta}{2} = \sin^{-1}\left(\frac{\frac{x}{2}}{100}\right) \therefore \theta = 2 \sin^{-1}\left(\frac{x}{200}\right)$$

$$\text{Area of Triangle} = \frac{1}{2} xh$$

$$\text{Area of sector} = \frac{\theta}{360} \times \pi \times 100^2$$

$$\text{Area of segment} = \frac{\theta}{360} \times \pi \times 100^2 - \frac{1}{2} xh$$

$$\text{Try } x = 80 \quad h = \sqrt{100^2 - 40^2} \approx 91.65... \quad \theta = 2 \sin^{-1}\left(\frac{80}{200}\right) \approx 47.156...$$

$$\text{Segment area} = \frac{47.156...}{360} \times \pi \times 100^2 - \frac{1}{2} \times 80 \times 91.65... \approx 449.1 \text{ mm}^2 \quad \text{Too small!}$$

$$\text{Try } x = 90 \quad h = \sqrt{100^2 - 45^2} \approx 89.30... \quad \theta = 2 \sin^{-1}\left(\frac{90}{200}\right) \approx 53.487...$$

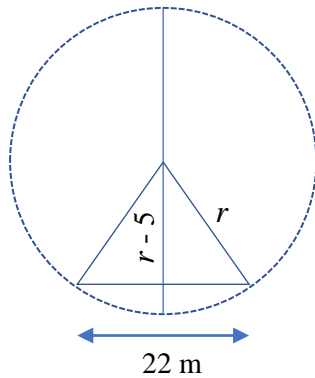
$$\text{Segment area} = \frac{53.487...}{360} \times \pi \times 100^2 - \frac{1}{2} \times 90 \times 89.30... \approx 649.0 \text{ mm}^2 \quad \text{still too small}$$

$$\text{Try } x = 100 \quad h = \sqrt{100^2 - 50^2} \approx 86.60... \quad \theta = 2 \sin^{-1}\left(\frac{100}{200}\right) = 60$$

$$\text{Segment area} = \frac{60}{360} \times \pi \times 100^2 - \frac{1}{2} \times 100 \times 86.60... \approx 905.86 \text{ mm}^2 \quad \text{This is closest}$$

Question 6.**Answer: D***Explanation:*

Create a right angle triangle using the measurements we have:



Applying Pythagoras theorem we get:

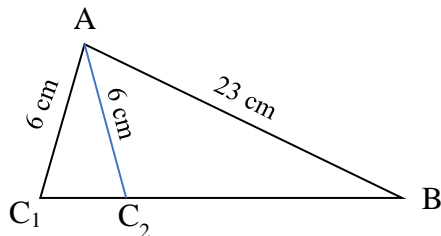
$$11^2 + (r-5)^2 = r^2$$

This can then be solved using CAS:

$$\text{solve}(11^2 + (r-5)^2 = r^2, r) \quad r = 14.6$$

Question 7.**Answer: D***Explanation:*

We need to determine possible side lengths and angles for this triangle.



Using the sine rule:

$$\frac{\sin C_1}{23} = \frac{\sin 14^\circ}{6}$$

$$\Rightarrow \sin C_1 = \frac{23 \sin 14^\circ}{6} = 0.92736...$$

$$\Rightarrow C_1 \approx 68^\circ \text{ or } C_2 = 180 - 68 = 112^\circ$$

If $C_1 = 68^\circ$ then $A_1 = 98^\circ$ or if $C_2 = 112^\circ$ then $A_2 = 54^\circ$ From these results we see that answer A is incorrect because it is not angle A equal to 68° , but angle C. Answer C is also incorrect as is answer E.

Now we need to determine the possible lengths of side CB. Using the cosine rule:

$$C_1B = \sqrt{23^2 + 6^2 - 2 \times 23 \times 6 \cos 98^\circ} \approx 24.6 \text{ cm} \text{ so answer D is correct.}$$

$$C_2B = \sqrt{23^2 + 6^2 - 2 \times 23 \times 6 \cos 54^\circ} \approx 20.0 \text{ cm}$$

Question 8.

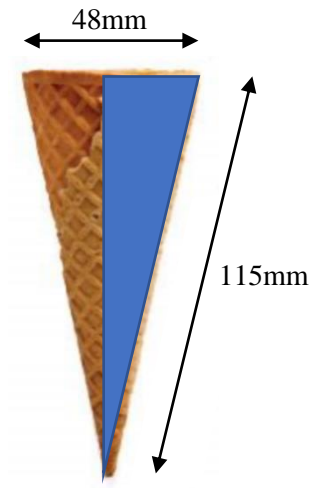
Answer: A

Explanation:

$$\text{Volume} = \text{hemisphere} + \text{Cone} = \frac{1}{2} \left(\frac{4}{3} \pi r^3 \right) + \frac{1}{3} \pi r^2 h$$

$$\text{Calculate height of cone: } h = \sqrt{11.5^2 - 2.4^2} \approx 11.247$$

$$\text{Volume} = \frac{1}{2} \left(\frac{4}{3} \pi (2.4)^3 \right) + \frac{1}{3} \pi (2.4^2 \times 11.247) \approx 96.793... \text{ cm}^3$$



Module 4 – Graphs and relations

Question 1.

Answer: A

Explanation:

Calculate gradient: $m = \frac{\frac{3}{13}}{\frac{6}{13}} = \frac{3}{6} = \frac{1}{2}$ $c = 3$

Equation could be written as: $y = \frac{6}{13}x + 3$ since this form is not here we need to rearrange.

$$y = \frac{6}{13}x + 3 \quad (\times 13)$$

$$13y = 6x + 39 \quad (-13y)$$

$$6x - 13y + 39 = 0$$

Question 2.

Answer: C

Explanation:

The profit from A will be $P_A = 20n - 110$

At event B, if 10% of sales are paid for the site then the revenue per unit will be \$18 per unit.

The profit from B will be $P_B = 18n - 20$

The profit will be the SAME for each event when:

$$20n - 110 = 18n - 20$$

$$2n - 110 = -20$$

$$2n = 90$$

$$n = 45$$

The profit at event A will be higher when at least 46 units are sold.

Question 3.*Answer: A**Explanation:*

By substituting the coordinates into the equation: $80 = \frac{k}{25} \Rightarrow k = 80 \times 25 = 2000$

Question 4.*Answer: A**Explanation:*

Calculate the gradient. Bailee commences her swim when the time is 90 sec and distance is 100m and completes her swim when the time is 135 sec and the distance is 150m.

$$\text{Gradient } m = \frac{150-100}{135-90} = \frac{50}{45} = \frac{10}{9}$$

Equation $d = \frac{10}{9}t + c$ substitute in $t = 90$ and $d = 100$ to find c

$$100 = \frac{10}{9} \times 90 + c \Rightarrow c = 0 \text{ So the equation is } d = \frac{10}{9}t$$

Question 5.*Answer: E**Explanation:*

The line goes through the points $(0,9)$ and $(5,0) \therefore m = \frac{-9}{5}$

$$\therefore y \leq \frac{-9}{5}x + 9 \quad (\times 5)$$

$$5y \leq -9x + 45 \quad (+9x)$$

$$9x + 5y \leq 45$$

Question 6.*Answer: C**Explanation:*

The corner points of the feasible region are $(0,0)$ $(0,7)$ $(5,0)$ and $(2.5,4.5)$ Substitute these values into the objective function $C = 10x + 12y$

$$(0,0) \quad C = 0$$

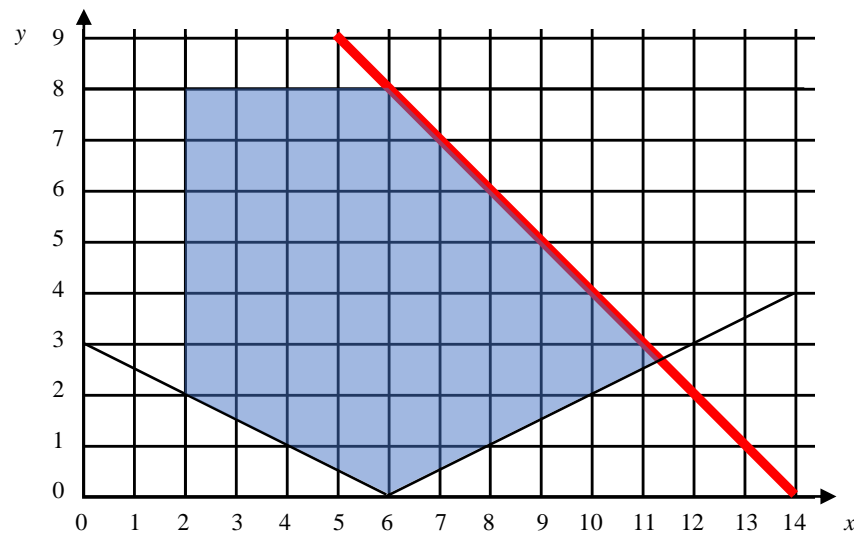
$$(0,7) \quad C = 0 + 12 \times 7 = 84$$

$$(5,0) \quad C = 10 \times 5 + 0 = 50$$

$$(2.5,4.5) \quad C = 10 \times 2.5 + 12 \times 4.5 = 79$$

Question 7.**Answer: C***Explanation:*

The slope of the objective function must be more negative than the slope of the line shown below:



The gradient of the highlighted line is -1 , so we require an objective function with a gradient more negative, that is, $m < -1$.

$$\text{A. } P = 18x + 24y \Rightarrow 24y = P - 18x \therefore y = \frac{P}{24} - \frac{18}{24}x \quad m = \frac{-18}{24} = -\frac{3}{4} \Rightarrow m > -1$$

$$\text{B. } P = 12x + 16y \Rightarrow 16y = P - 12x \therefore y = \frac{P}{16} - \frac{12}{16}x \quad m = \frac{-12}{16} = -\frac{3}{4} \Rightarrow m > -1$$

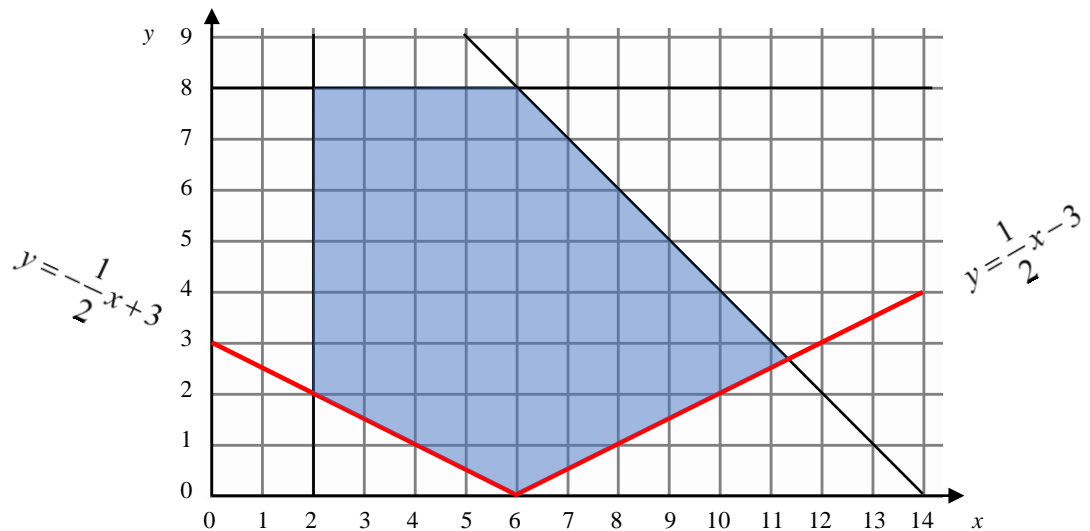
$$\text{C. } P = 50x + 45y \Rightarrow 45y = P - 50x \therefore y = \frac{P}{45} - \frac{50}{45}x \quad m = \frac{-50}{45} = -\frac{10}{9} \Rightarrow m < -1$$

$$\text{D. } P = 20x - 10y \Rightarrow 10y = 20x - P \therefore y = \frac{20}{10}x - \frac{P}{10} \quad m = \frac{20}{10} = 2 \Rightarrow m > -1$$

$$\text{E. } P = 3x + 3y \Rightarrow 3y = P - 3x \therefore y = \frac{P}{3} - \frac{3}{3}x \quad m = \frac{-3}{3} = -1$$

Question 8.*Answer: E**Explanation:*

The slope of the objective function must be “between” the slopes of the two constraints which meet at the point $(6,0)$.



The gradient of the objective function is:

$$P = mx + 12y \quad (-mx)$$

$$12y = P - mx \quad (\div 12)$$

$$y = \frac{P}{12} - \frac{m}{12}x \quad \text{Gradient of objective function is } -\frac{m}{12}$$

$$\text{This means that } -\frac{1}{2} < -\frac{m}{12} < \frac{1}{2} \quad (\times 12)$$

$$-6 < -m < 6 \quad (\times -1)$$

$$6 > m > -6 \text{ or } -6 < m < 6$$