



Trial Examination 2019

# **VCE Further Mathematics Units 3&4**

Written Examination 2

**Suggested Solutions**

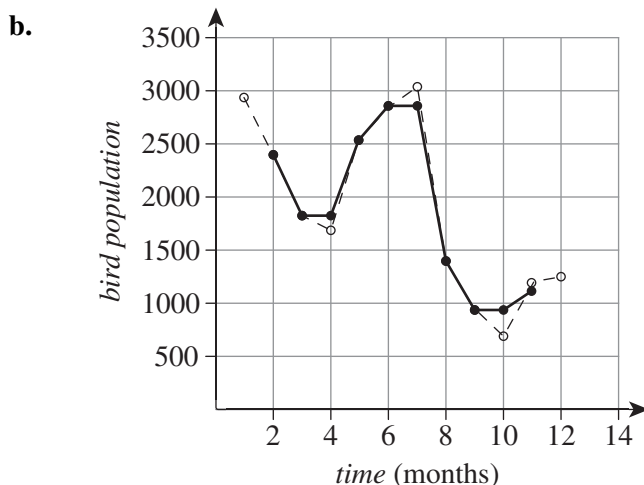
**SECTION A – CORE****Data analysis****Question 1** (8 marks)

- a. Each airline has 30 days recorded. A1
- b. No. The TG Airlines data is positively skewed. A1
- c. TG Airlines have eight days with zero passengers, indicating that there were no flights on those days or that Supreme Airlines' data is negatively skewed. A1
- d. There are 30 data points, so the median is between the 15<sup>th</sup> and 16<sup>th</sup> –  $Q_1$  the median of the first 15 is the 8<sup>th</sup> data point 26, and  $Q_3$  the median of the second 15 is the 23<sup>rd</sup> data point 48. The minimum value is 15 and the maximum 50.  
Therefore the correct values are 15, 36, 43.5, 48 and 50. M1 A1
- e. The IQR for Supreme Airlines is 18 ( $48 - 36 = 12$ ,  $1.5 \times 12 = 18$ ). The fences for outliers are 18 and 66 ( $36 - 18 = 18$  and  $48 + 18 = 66$ ). The figure of 15 is lower than the lower boundary and so is an outlier. A1
- f. i. 25% A1
- ii.  $\frac{53}{2} = 26.5\%$  A1

**Question 2** (6 marks)

- a. It is a decreasing trend. A1

*Note: There is also a seasonal element to the data, but the decreasing trend must be noted to achieve the mark.*



- c.  $\frac{2400 + 1800 + 1700}{3} = 1967$  A1

$$\text{d. } \frac{\frac{15\,000}{14\,250} + \frac{15\,500}{13\,750} + \frac{14\,000}{13\,875}}{3} = 1.06$$

M1 A1

$$\text{e. } \frac{13\,000}{0.85} = 15\,294$$

A1

**Question 3** (10 marks)

$$\text{a. } \text{passengers} = 229 \times \text{flights} - 10\,067$$

A1

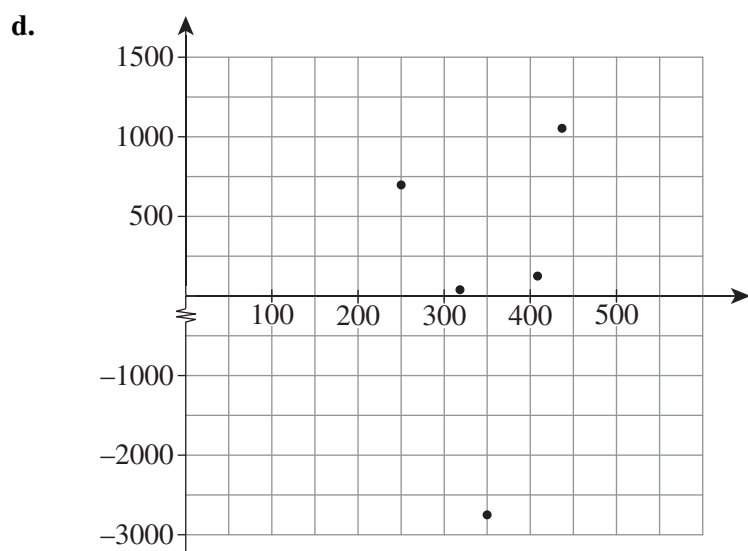
$$\begin{aligned} \text{b. } \text{flight} &= \frac{75\,000 + 10\,067}{229} \\ &= 371.5 \end{aligned}$$

Therefore 372 flights is the minimum.

A1

c.

| Number of flights                | 250    | 320    | 350    | 410           | 430           |
|----------------------------------|--------|--------|--------|---------------|---------------|
| Number of passengers (actual)    | 49 230 | 62 180 | 65 250 | 79 830        | 84 630        |
| Number of passengers (predicted) | 48 500 | 62 150 | 68 000 | <b>79 700</b> | <b>83 600</b> |
| Residual                         | 730    | 30     | -2750  | <b>130</b>    | <b>1030</b>   |

*missing passenger numbers* A1*residuals* A1

A1

e. There is a pattern which does not support the idea of a linear relationship.

A1

- f. Enter the data into your technology and apply an  $x^2$  transformation to the  $x$  values before doing the regression calculation.

i.  $y = 0.29x^2 + 31\,740$  A1

ii. 0.997 or 1.00 A1

- g. Both the linear and  $x^2$  models have strong correlation, 0.99 and 1.00 respectively. A1

The residual plot for the linear does not support the assumption of linearity. A residual plot for the  $x^2$  transformation would need to be done to see if that was the preferred model. A1

*Note: The actual details do not need to match the answer supplied but must be accurate.*

### Recursion and financial modelling

#### Question 4 (2 marks)

$$t_1 = 6$$

$$t_2 = 7$$

$$t_3 = 6 + 7 = 13$$

$$t_4 = 7 + 13 = 20$$

$$t_5 = 13 + 20 = 33$$

$$6 + 7 + 20 = 33$$

M1 A1

#### Question 5 (5 marks)

- a. Enter the data into the financial application of your technology.

The monthly repayment is \$4087.94.

A1

b.  $P_n = 80000(1.05)^{(n-2)}$  A1

c.

| Year   | 1 | 2      | 3      | 4      | 5      | 6         | 7          |
|--------|---|--------|--------|--------|--------|-----------|------------|
| Profit | 0 | 80 000 | 84 000 | 88 200 | 92 610 | 97 240.50 | 102 102.53 |

M1 A1

- d. Firstly find the value after 5 years using your technology.

```

N=60
I% = 4.2
PV = 400000
PMT = -4087.93501
FV = -220886.8999
P/Y = 12
C/Y = 12
PMT: [ ] [ ] [ ] BEGIN

```

After 60 months (5 years) the loan is now \$120 886.90.

Substitute the value of 120 886.8999 into the present value, leave all other values the same and recalculate  $N$ .

```

N = 31.27110302
I% = 4.2
PV = 120886.9
PMT = -4087.93501
FV = 0
P/Y = 12
C/Y = 12
PMT: [ ] [ ] [ ] BEGIN

```

$N$  is now 31.27 months (32 months). It was 60 months, so the time saved is 28 months.

A1

#### Question 6 (2 marks)

$$\text{Method 1} = 60\,000 \times (0.8)^3$$

$$= \$30\,720$$

A1

$$\text{Method 2} = 60\,000 - 3 \times 10\,000$$

$$= \$30\,000$$

A1

#### Question 7 (3 marks)

$$\text{a.} \quad 800\,000 \left( 1 + \frac{\frac{3.1}{12}}{100} \right) = \$802\,066.67$$

Therefore the monthly repayment is \$2066.67.

A1

b.

| Date      | Deposit                           | Withdrawal | Balance                     |
|-----------|-----------------------------------|------------|-----------------------------|
| 1/1/2019  | <b>2066.67</b>                    |            | \$34 560                    |
| 1/2/2019  | <b>2066.67</b>                    |            | <b>\$36 626.67</b>          |
| 15/2/2019 |                                   | \$2000     | <i>A</i> <b>\$34 626.34</b> |
| 1/3/2019  | <b>2066.67</b>                    |            | <b>\$36 693.67</b>          |
| 31/3/2019 | interest <i>B</i> <b>\$158.82</b> |            |                             |

*A* is calculated by adding the income for two months and subtracting the withdrawal, which equals \$34 626.67.

A1

Interest (*B*) is  $\frac{1.8}{12}\%$  of the lowest balance each month. January's lowest balance is \$34 560, February's is \$34 626.67 and March's is \$36 693.34.

$$\begin{aligned}\text{Therefore, } B &= \left( \frac{\frac{1.8}{12}}{100} \right) \times (34\,560 + 34\,626.67 + 36\,693.34) \\ &= \$158.82\end{aligned}$$

A1

**SECTION B – MODULES****Module 1 – Matrices****Question 1** (4 marks)

a. 275 A1

b. i. 
$$\begin{bmatrix} 312 & 612 & 315 \\ 405 & 513 & 275 \\ 370 & 570 & 401 \end{bmatrix} \begin{bmatrix} 15 \\ 20 \\ 30 \end{bmatrix} = \begin{bmatrix} 26\,370 \\ 24\,585 \\ 28\,980 \end{bmatrix}$$
 A1

ii.  $R_{21}$  represents the total ticket revenue for week 2. A1

c. 
$$\begin{bmatrix} 36 & 42 \\ 81 & 37 \end{bmatrix}^{-1} \begin{bmatrix} 462.60 \\ 713.10 \end{bmatrix} = \begin{bmatrix} 6.2 \\ 5.7 \end{bmatrix}$$
  
 \$6.20 A1

**Question 2** (2 marks)

a. 
$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$
 A1

b. 
$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \rightarrow d$$

$$d + d^2 = \begin{bmatrix} 0 & 2 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 2 & 0 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$
 A1

|                 |        |
|-----------------|--------|
| 1 <sup>st</sup> | Gold   |
| 2 <sup>nd</sup> | Eagles |
| 3 <sup>rd</sup> | Fires  |
| 4 <sup>th</sup> | Heroes |

*Note: For full marks the order must be completely correct.*

**Question 3** (6 marks)

a.  $S_0 = T^{-1} \times S_1$

$$T^{-1} \times \begin{bmatrix} 2952 \\ 1800 \\ 2050 \\ 3200 \end{bmatrix} = \begin{bmatrix} 4000 \\ 2000 \\ 1000 \\ 3000 \end{bmatrix} \quad \text{A1}$$

**OR**

Use solve function.

$$\text{solve} \left( t, \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2950. \\ 1800. \\ 2050. \\ 3200. \end{bmatrix}, a, b, c, d \right)$$

4000. and  $b=2000.$  and  $c=1000.$  and  $d=3000$

b.  $2950 \times 0.1 + 1800 \times 0.5 + 2050 \times 0.1 + 3200 \times 0.1 = 1720$  A1

c. 2950 walked in June, and 20% of them will drive in July.

$$0.2 \times 2950 = 590 \quad \text{M1}$$

$$S_2 = \begin{bmatrix} 2372.5 \\ 1720.0 \\ 2627.5 \\ 3280.0 \end{bmatrix}$$

$$\frac{590}{3280} \times 100 = 17.987804878$$

$$18\% \quad \text{A1}$$

d. The marketing campaign was successful, as the long-term steady state shows a clear increase in the numbers of people walking and cycling to work. A1

cycling – 1000 to 3333

walking – 3000 to 3333

*Note: Figures must be used to show that the campaign was successful.*

e.  $t \times \begin{bmatrix} 2000 \\ 500 \\ 500 \\ 1000 \end{bmatrix} + \begin{bmatrix} 100 \\ 25 \\ 25 \\ 50 \end{bmatrix} = \begin{bmatrix} 1475 \\ 625 \\ 850 \\ 1250 \end{bmatrix}$

$$t \times \begin{bmatrix} 1475 \\ 625 \\ 850 \\ 1250 \end{bmatrix} + \begin{bmatrix} 100 \\ 25 \\ 25 \\ 50 \end{bmatrix} = \begin{bmatrix} 1215 \\ 695 \\ 110 \\ 1390 \end{bmatrix}$$

$$695 \quad \text{A1}$$

**Module 2 – Networks and decision mathematics****Question 1** (3 marks)

a. 50 minutes A1

b. For example:

$A-B-E-D-F-C-A$  A1

*Note: Any Hamiltonian cycle that starts and ends at A is a viable answer.*

c.  $v = 6$

$$f = 6$$

$$e = 10$$

$$\therefore 6 + 6 = 10 + 2$$

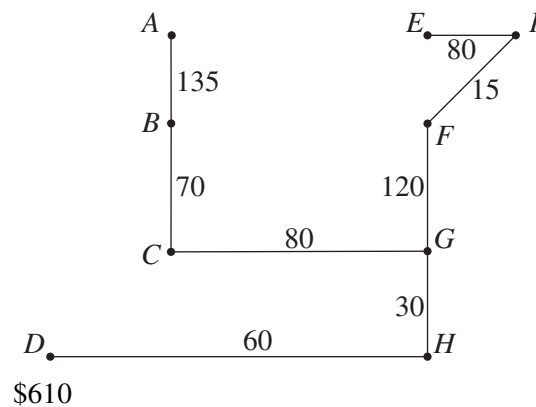
A1

**Question 2** (2 marks)

a. minimum spanning tree

A1

b.



A1

**Question 3** (2 marks)

a.  $15 + 10 + 6 + 12 = 43$  L

A1

b.  $20 + 6 + 5 = 31$  L

A1

**Question 4** (5 marks)

a. G: C and F

K: B, E, G and H

A1

*Note: All preceding activities must be correct for full marks.*

b. 19 days

A1

c.  $A-B-I-J-M$

A1

d. D and F

A1

**e.** Reduce  $I$  by 2 days:  $800 \times 2 = \$1600$

Reduce  $K$  by 1 day: \$600

$$1600 + 600 = \$2200$$

A1

**Module 3 – Geometry and measurement****Question 1** (4 marks)

a. area of rectangle + area of semicircles =  $100 \times 50 + 2(0.5 \times \pi \times 25^2)$   
 $= 6963.49540849$

$$\therefore 6963 \text{ m}^2$$

A1

b. Using Pythagoras:  $\sqrt{150^2 + 40^2} = 155.241746963$   
 $\therefore 155.2 \text{ m}$

A1

c. Using trigonometry to find angle:  $\tan^{-1}\left(\frac{40}{150}\right) = 14.9314171781$

Add angle to 90 to get bearing.

$$\therefore 105^\circ$$

A1

d. surface area = curved area of cylinder + curved surfaces of hemisphere  
 $= 2\pi \times 21 \times 8 + \frac{1}{2} \times 4\pi \times 21^2$   
 $= 3826.45985207$

$$\therefore 3826 \text{ m}^2$$

A1

**Question 2** (4 marks)

a. Using the formula:  $\frac{1}{3} \times \pi \times 20^2 \times 80 = 33\,510.3216383$

$$\therefore 33\,510 \text{ m}^3$$

A1

b.  $1 : 80 = x : 20$

$$\text{solve}\left(\frac{1}{80} = \frac{x}{20}, x\right)$$

$$x = 0.25$$

**OR**

$$\frac{20}{80} = 0.25$$

$$\therefore 0.25 \text{ m}$$

A1

- c. arc length = 8825

Find latitude using distance along a meridian.

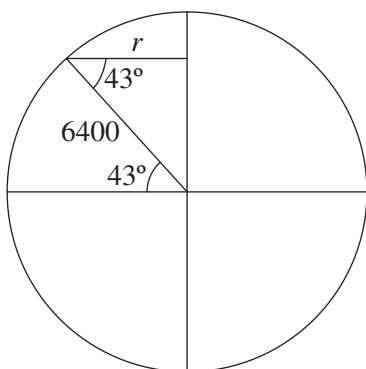
$$\text{solve}\left(\frac{\pi \times 6400 \times x}{180} = 8825, x\right)$$

$$x = 79.0055084692$$

$$\text{latitude} = 79.0055084692 - 36$$

$$\approx 43$$

M1



$$r = 6400 \times \cos(43)$$

$$= 4680.24403383$$

$$\therefore 4700 \text{ km}$$

A1

### Question 3 (4 marks)

- a. major sector =  $290^\circ$

$$\frac{290}{360} \times \pi \times 8^2 = 161.966554585$$

$$\therefore 162 \text{ m}^2$$

A1

- b. Use  $\frac{1}{2}ab \sin(c)$ .

$$\frac{1}{2} \times 40 \times 50 \times \sin(63) = 891.006524188$$

$$\therefore 891 \text{ m}^2$$

A1

- c. Use cosine formula with correct substitution.

M1

$$\sqrt{60^2 + 40^2 - 2 \times 60 \times 40 \times \cos(63)} = 54.9622197628$$

$$\therefore 55 \text{ m}$$

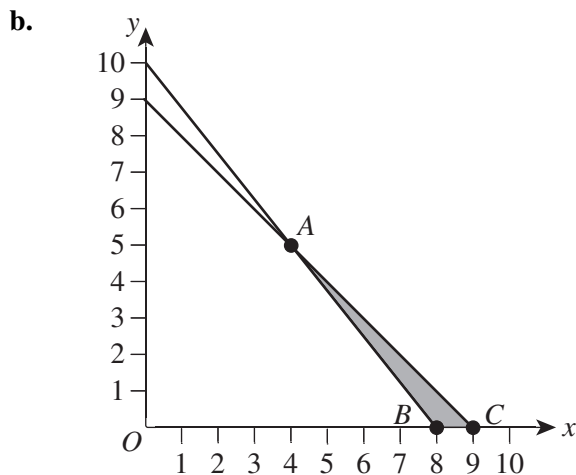
A1

**Module 4 – Graphs and relations****Question 1** (2 marks)

- a. 6 pm is the first time the clearance is over 4.5 m. A1
- b. Between 3 pm and 6 pm the clearance is less than 4.5 m, so the time is 3 hours. A1

**Question 2** (6 marks)

- a. There are  $x$  number of buses with 25 seats and  $y$  buses with 20 seats. The total number of seats must be at least 200. A1



*labels three correct points* A1  
*shades correct area* A1

- c.  $Z = 600x + 500y$  A1
- d. The important points are (4, 5), (8, 0) and (9, 0). A1

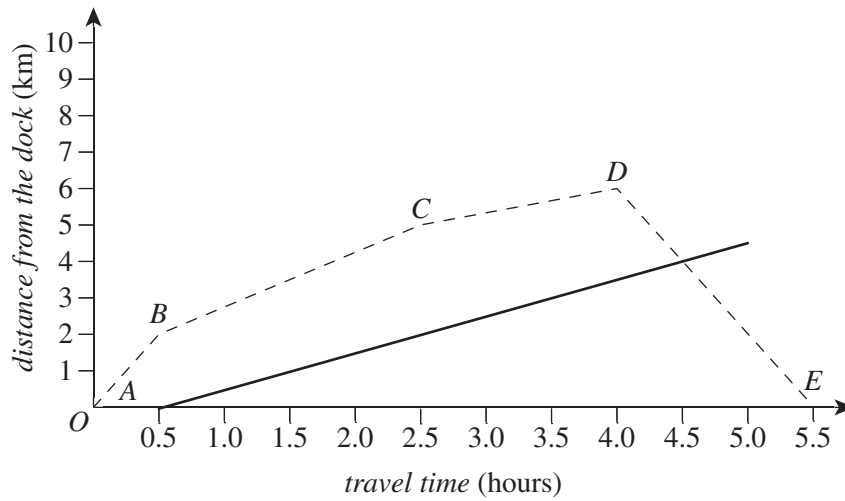
| Point    | Cost                                   |
|----------|----------------------------------------|
| A (4, 5) | $600 \times 4 + 500 \times 5 = \$4900$ |
| B (8, 0) | $600 \times 8 + 500 \times 0 = \$4800$ |
| C (9, 0) | $600 \times 9 + 500 \times 0 = \$5400$ |

The cheapest option is to use eight 25-seater buses. A1

**Question 3** (4 marks)

- a. At point D Gazza's boat begins getting closer to the starting point. A1
- b. In 2 hours a distance of 3 km was travelled. The average speed is  $\frac{3}{2} = 1.5 \text{ km h}^{-1}$ . A1

c.



A1

- d. 4 hours. The boats will meet 4.5 hours after Gazza sailed, which is 4 hours after the second boat sails.

A1