

**Victorian Certificate of Education
2018****FURTHER MATHEMATICS
Written examination 1****Wednesday 30 May 2018****Reading time: 2.00 pm to 2.15 pm (15 minutes)****Writing time: 2.15 pm to 3.45 pm (1 hour 30 minutes)****MULTIPLE-CHOICE QUESTION BOOK****Structure of book**

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
A – Core	24	24			24
B – Modules	32	16	4	2	16
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question book of 36 pages
- Formula sheet
- Answer sheet for multiple-choice questions
- Working space is provided throughout the book.

Instructions

- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination

- You may keep this question book and the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A – Core**Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Use the following information to answer Questions 1 and 2.

The stem plot below displays the distribution of *beak length*, in millimetres, of a sample of 33 female birds of the same species.

key: 23 | 9 = 23.9 $n = 33$

22							
22	8	9					
23	0	1	2	3	4	4	
23	5	6	6	7	8	9	9
24	0	3	4	4			
24	6	7	8	9			
25	0	1	2				
25	5	7					
26	3	4					
26	6						
27	3	4					
27							

Question 1

For these 33 female birds, the median *beak length* is

- A. 24.0 mm
- B. 24.1 mm
- C. 24.3 mm
- D. 24.6 mm
- E. 25.0 mm

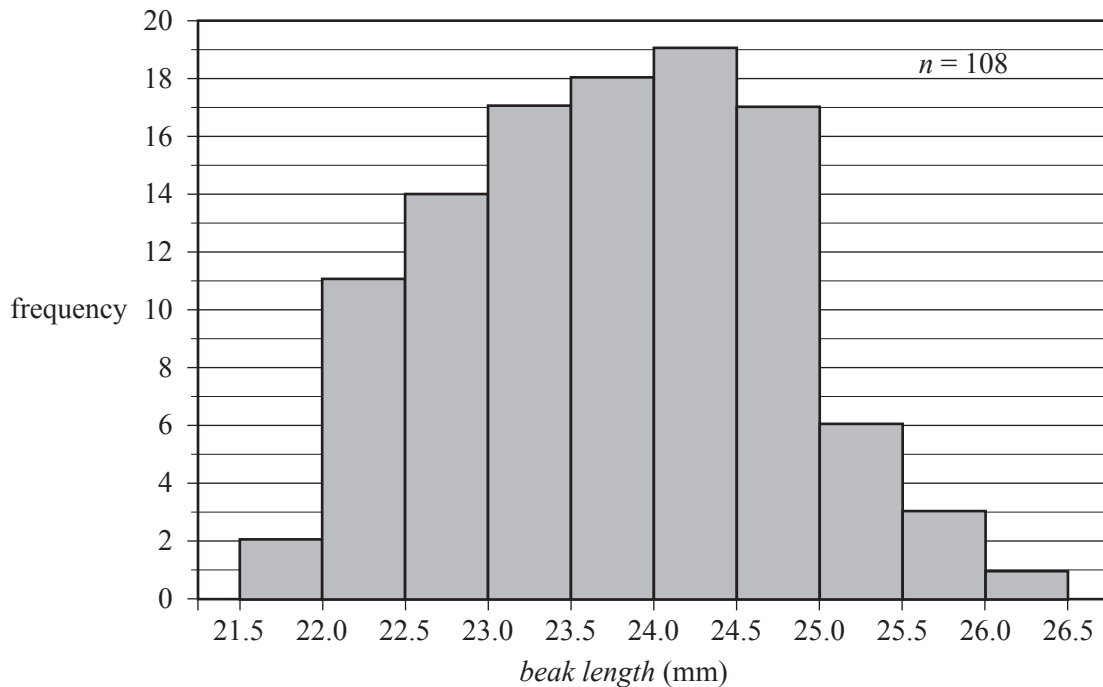
Question 2

The percentage of these 33 female birds with a beak length of less than 25.4 mm is closest to

- A. 21.2%
- B. 25.4%
- C. 27.0%
- D. 75.8%
- E. 78.8%

Use the following information to answer Questions 3 and 4.

The histogram below shows the distribution of *beak length*, in millimetres, of a sample of 108 birds of the same species. Both male and female birds are included in this sample.



Question 3

The *beak length* for this sample of 108 birds is most frequently

- A. greater than or equal to 22.5 mm and less than 23.0 mm.
- B. greater than or equal to 23.0 mm and less than 23.5 mm.
- C. greater than or equal to 23.5 mm and less than 24.0 mm.
- D. greater than or equal to 24.0 mm and less than 24.5 mm.
- E. greater than or equal to 24.5 mm and less than 25.0 mm.

Question 4

The interquartile range (IQR) for *beak length* for this sample of 108 birds is closest to

- A. 1.0 mm
- B. 1.5 mm
- C. 2.0 mm
- D. 2.5 mm
- E. 3.0 mm

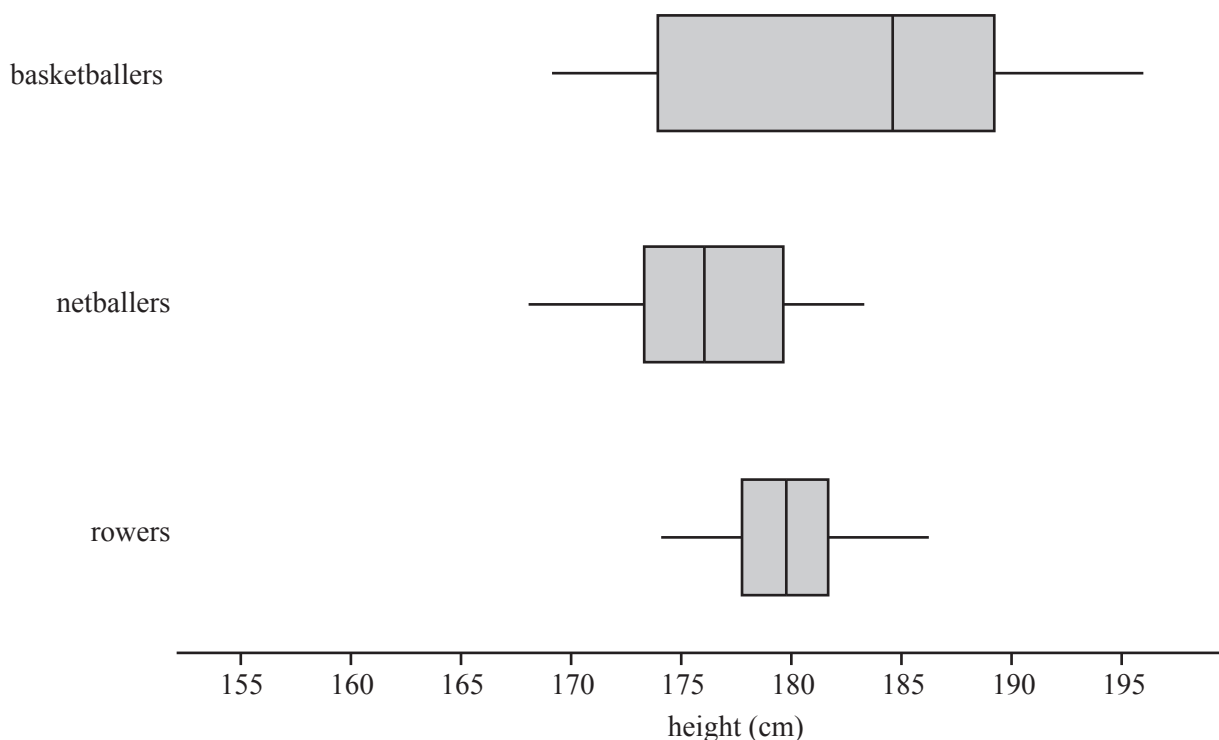
Question 5

The variables *height* (less than 1.83 m, 1.83 m and over) and *enthusiasm for playing basketball* (low, medium, high) are

- A. both ordinal variables.
- B. both nominal variables.
- C. a nominal and an ordinal variable respectively.
- D. an ordinal and a nominal variable respectively.
- E. a numerical and an ordinal variable respectively.

Question 6

The parallel boxplots below display the distribution of height for three groups of athletes: rowers, netballers and basketballers.



Which one of the following statements is **not** true?

- A. The shortest athlete is a netballer.
- B. The rowers have the least variable height.
- C. More than 25% of the netballers are shorter than all rowers.
- D. The basketballers are the tallest athletes in terms of median height.
- E. More than 50% of the basketballers are taller than any of the rowers or netballers.

Use the following information to answer Questions 7–10.

The table below shows the lean body mass (*LBM*), percentage body fat (*PBF*) and body mass index (*BMI*) of a sample of 12 professional athletes.

<i>LBM</i> (kg)	<i>PBF</i> (%)	<i>BMI</i> (kg/m ²)
63.3	19.8	20.6
58.6	21.3	20.7
55.4	19.9	21.9
57.2	23.7	21.9
53.2	17.6	19.0
53.8	15.6	21.0
60.2	20.0	21.7
48.3	22.4	20.6
54.6	18.0	22.6
53.4	15.1	19.4
61.9	18.1	21.2
48.3	23.3	22.0

Question 7

The mean, $\bar{x}$, and the standard deviation, s_x , for the lean body mass (*LBM*) of these athletes, in kilograms, are closest to

- A. $\bar{x} = 48.3$ $s_x = 4.6$
- B. $\bar{x} = 55.0$ $s_x = 4.6$
- C. $\bar{x} = 55.0$ $s_x = 4.8$
- D. $\bar{x} = 55.7$ $s_x = 4.6$
- E. $\bar{x} = 55.7$ $s_x = 4.8$

Question 8

A least squares line is fitted to the data using percentage body fat (*PBF*) as the response variable and body mass index (*BMI*) as the explanatory variable.

The equation of the least squares line is closest to

- A. $PBF = -4.7 + 1.2 \times BMI$
- B. $BMI = -4.7 + 1.2 \times PBF$
- C. $PBF = 17.8 + 1.7 \times BMI$
- D. $BMI = 17.8 + 1.7 \times PBF$
- E. $PBF = 23.6 - 0.1 \times BMI$

Question 9

The Pearson correlation coefficient, r , between lean body mass (LBM) and percentage body fat (PBF) is closest to

- A. -0.235
- B. -0.124
- C. 0.124
- D. 0.235
- E. 0.352

Question 10

A least squares line is fitted to the data using lean body mass (LBM) as the response variable and body mass index (BMI) as the explanatory variable.

The equation of this line is

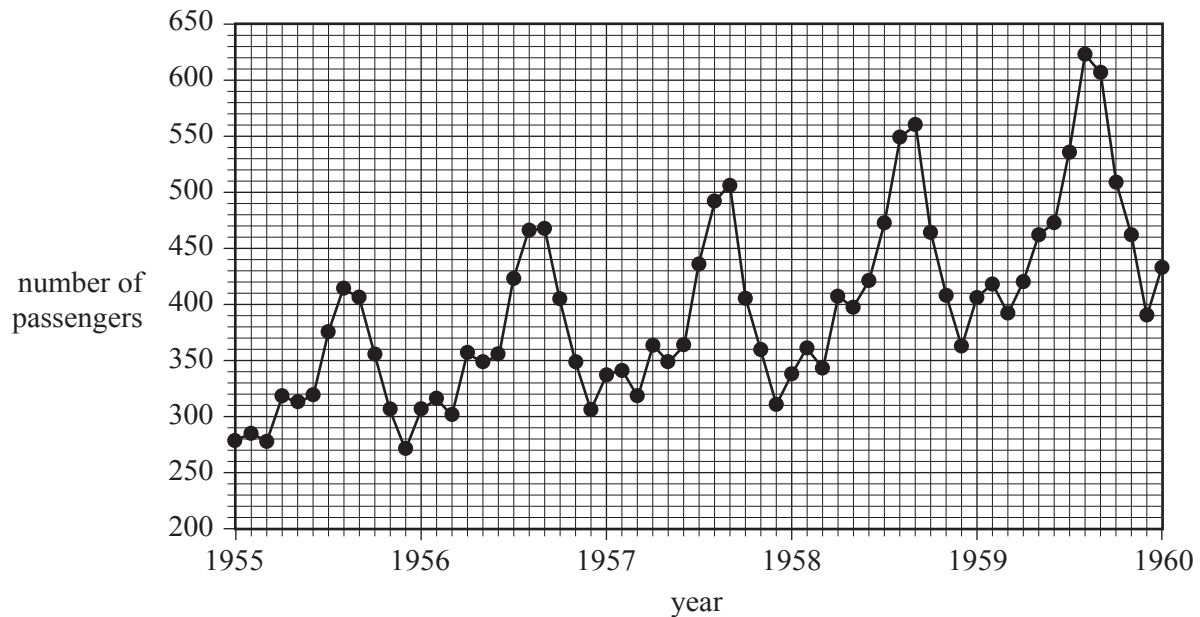
$$LBM = 48.9 + 0.320 \times BMI$$

When this line is used to predict the lean body mass (LBM) of an athlete with a body mass index (BMI) of 22.0, the residual will be closest to

- A. -7.6 kg
- B. -1.5 kg
- C. 1.5 kg
- D. 33.9 kg
- E. 55.9 kg

Use the following information to answer Questions 11 and 12.

The time series plot below shows the number of passengers who flew with an airline each month over the period 1955–1960.



Data: GEP Box and GM Jenkins, *Time Series Analysis, Forecasting and Control*, revised edition, Holden-Day, 1976, p. 531

Question 11

The pattern shown in the time series plot above is best described as having

- A. no trend.
- B. irregular fluctuations only.
- C. an increasing trend with seasonality only.
- D. an increasing trend with irregular fluctuations only.
- E. an increasing trend with seasonality and irregular fluctuations.

Question 12

The dot above 1957 corresponds to the number of passengers in January 1957.

The five-median smoothed number of passengers for August 1957 is closest to

- A. 400
- B. 435
- C. 480
- D. 495
- E. 510

Question 13

A least squares line is used to model the trend in a time series displaying the *winning time*, in seconds, for the men's 1500 m race at each of the Olympic Games held during the period 1900–1952.

The explanatory variable is *year*.

The equation of the line is

$$\text{winning time} = 998.2 - 0.3968 \times \text{year}$$

The coefficient of determination is 0.8689

The Pearson correlation coefficient, r , is closest to

- A. -0.9321
- B. -0.7550
- C. 0.3968
- D. 0.7550
- E. 0.9321

Question 14

A company sells central heating systems.

The table below shows the quarterly seasonal indices for sales in the last three quarters of a year.

	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Seasonal index		1.33	1.45	0.58

The deseasonalised sales for Quarter 1 in 2018 were \$2.45 million.

The actual sales, in millions of dollars, were closest to

- A. 0.64
- B. 1.57
- C. 2.16
- D. 2.45
- E. 2.83

Use the following information to answer Questions 15 and 16.

The table below shows the number of surfboards that are repaired each month at a surfboard shop in 2017.

Month	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Number of surfboards	121	165	120	106	54	42	36	60	72	84	108	145

Question 15

The six-mean smoothed number of surfboards, with centring, repaired for June 2017 is closest to

- A. 42
- B. 57
- C. 62
- D. 66
- E. 70

Question 16

If the data in the table above is used to estimate the seasonal indices for the number of surfboards repaired each month, the seasonal index for May would be closest to

- A. 0.42
- B. 0.54
- C. 0.58
- D. 1.14
- E. 1.29

Recursion and financial modelling

Question 17

The first three lines of an amortisation table for a reducing balance loan are shown below.

Repayment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	250 000.00
1	945.00	791.67	153.33	249 846.67
2	945.00	791.18		249 692.85

What is the principal reduction from repayment number 2?

- A. \$153.33
- B. \$153.82
- C. \$791.21
- D. \$791.67
- E. \$945.00

Question 18

Consider the recurrence relation below.

$$B_0 = 12, \quad B_{n+1} = 2B_n - 14$$

Which term of the sequence generated by this recurrence relation is the first to be negative?

- A. B_1
- B. B_2
- C. B_3
- D. B_4
- E. B_5

Question 19

Cheryl invested \$175 000 in an annuity.

This investment earns interest at the rate of 4.8% per annum, compounding quarterly.

Immediately after the interest has been added to the account each quarter, Cheryl withdraws a payment of \$3500.

A recurrence relation that can be used to determine the value of Cheryl's investment after n quarters, V_n , is

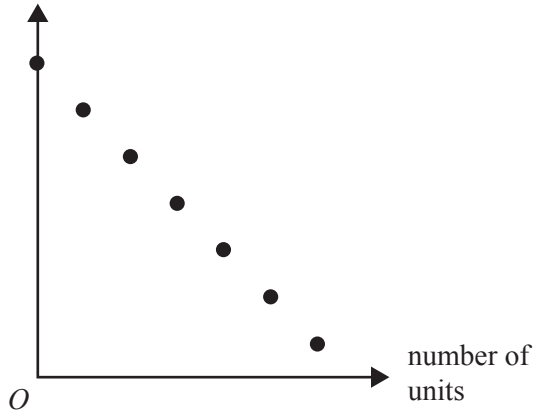
- A. $V_0 = 175\,000, \quad V_{n+1} = 0.952 V_n - 3500$
- B. $V_0 = 175\,000, \quad V_{n+1} = 0.988 V_n - 3500$
- C. $V_0 = 175\,000, \quad V_{n+1} = 1.004 V_n - 3500$
- D. $V_0 = 175\,000, \quad V_{n+1} = 1.012 V_n - 3500$
- E. $V_0 = 175\,000, \quad V_{n+1} = 1.048 V_n - 3500$

Question 20

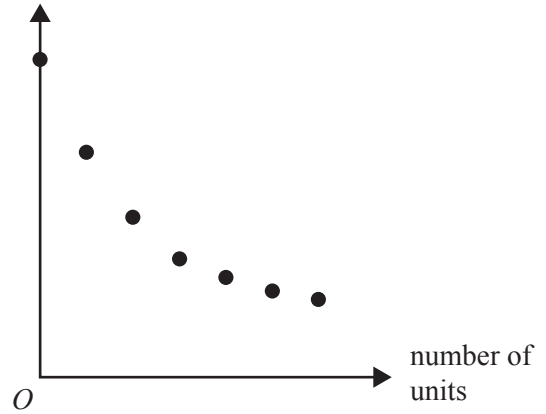
The value of a photocopier is depreciated using a unit cost method.

Which one of the following graphs could show the value of the photocopier as it depreciates?

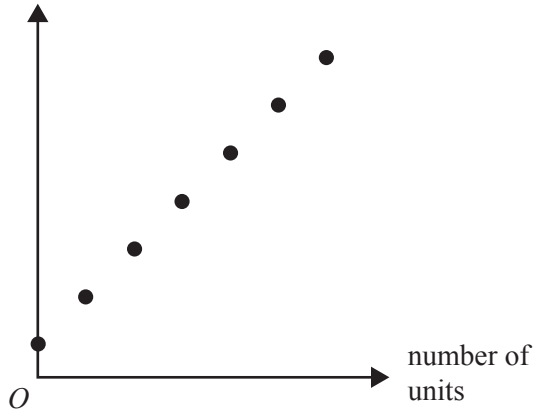
A. value



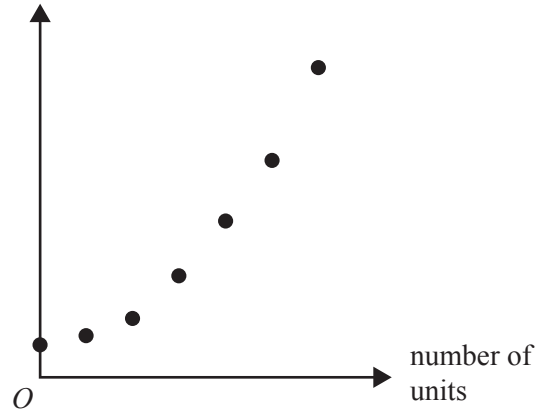
B. value



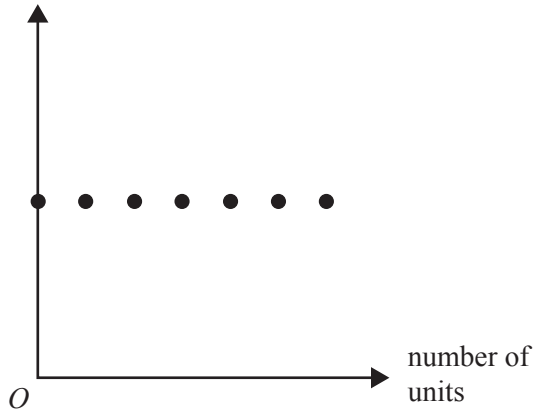
C. value



D. value



E. value



Question 21

An amount of money is deposited into an account that earns compound interest.

Which combination of interest rate and compounding period has the largest effective interest rate?

- A. 3.7% per annum, compounding weekly
- B. 3.7% per annum, compounding monthly
- C. 3.7% per annum, compounding quarterly
- D. 3.8% per annum, compounding monthly
- E. 3.8% per annum, compounding quarterly

Question 22

Amir invested some money in a perpetuity from which he receives a monthly payment of \$525.

The perpetuity pays interest at an annual rate of 4.2%, paid monthly.

How much money did Amir invest in the perpetuity?

- A. \$12 500
- B. \$22 500
- C. \$52 500
- D. \$120 500
- E. \$150 000

Question 23

An annuity investment earns interest at the rate of 3.8% per annum, compounding monthly.

Cho initially invested \$85 000 and will add monthly payments of \$1500.

The value of this investment will first exceed \$95 000 after

- A. five months.
- B. six months.
- C. seven months.
- D. eight months.
- E. nine months.

Question 24

Indira borrowed \$29 000 to buy a car and was charged interest at the rate of 12.5% per annum, compounding monthly.

For the first year of the loan, Indira made monthly repayments of \$425.

For the second year of the loan, Indira made monthly repayments of \$500.

The total amount of interest that Indira paid over this two-year period is closest to

- A. \$2500
- B. \$4300
- C. \$5900
- D. \$6800
- E. \$7700

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

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Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

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Module 1 – Matrices

Before answering these questions, you must **shade** the ‘Matrices’ box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 8 & 5 & 1 \end{bmatrix}$ is an example of a

- A. unit matrix.
- B. diagonal matrix.
- C. triangular matrix.
- D. symmetric matrix.
- E. communication matrix.

Question 2

Consider the matrix equation below.

$$2 \times \begin{bmatrix} 3 & 0 \\ 4 & -1 \end{bmatrix} + W = \begin{bmatrix} 6 & 2 \\ 7 & 0 \end{bmatrix}$$

Which one of the following is matrix W ?

- A. $\begin{bmatrix} 0 & 2 \\ 1 & -2 \end{bmatrix}$ B. $\begin{bmatrix} 0 & 2 \\ -1 & 2 \end{bmatrix}$ C. $\begin{bmatrix} 0 & -2 \\ 1 & -2 \end{bmatrix}$ D. $\begin{bmatrix} 12 & 2 \\ 15 & -2 \end{bmatrix}$ E. $\begin{bmatrix} 12 & -2 \\ 15 & -2 \end{bmatrix}$

Question 3

Hayley uses the password H8\$P@2 to unlock her computer.

She uses a permutation matrix to scramble this password, as shown in the matrix product below.

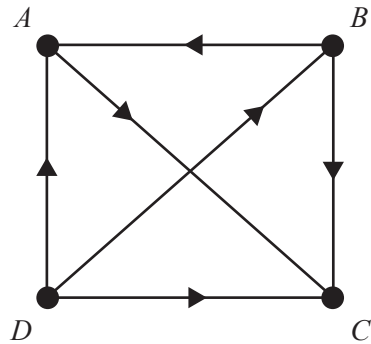
$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \text{H} \\ 8 \\ \$ \\ \text{P} \\ @ \\ 2 \end{bmatrix}$$

Which one of the following is the scrambled password?

- A. 2@PH\$8
- B. H8\$P@2
- C. P@2H\$8
- D. P2@\$8H
- E. P8@\$2H

Question 4

The diagram below shows the results of a chess competition between four players: Asha (A), Bai (B), Cam (C) and Drika (D).



Each competitor played each of the other competitors only once.

The arrows in the diagram indicate the winner of each match.

For example, the arrow from A to C shows that Asha defeated Cam.

The two-step dominances in this competition can be shown as a matrix where the *winner* is the person who has two-step dominance over the *loser*.

The matrix that shows the two-step dominances is

A.	<i>loser</i>					B.	<i>loser</i>				
	A	B	C	D			A	B	C	D	
<i>winner</i>	A	0	0	0	0	<i>winner</i>	A	0	0	0	0
	B	0	0	1	0		B	0	0	1	0
	C	0	0	0	0		C	0	0	0	0
	D	1	0	2	0		D	1	1	1	0

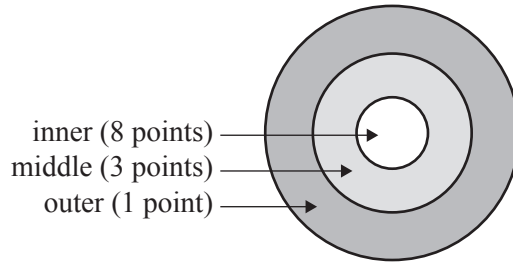
C.	<i>loser</i>					D.	<i>loser</i>				
	A	B	C	D			A	B	C	D	
<i>winner</i>	A	0	0	1	0	<i>winner</i>	A	0	0	1	0
	B	1	0	1	0		B	1	0	2	0
	C	0	0	0	0		C	0	0	0	0
	D	1	1	1	0		D	2	1	3	0

E.	<i>loser</i>				
	A	B	C	D	
<i>winner</i>	A	0	0	1	0
	B	1	0	2	0
	C	0	0	0	0
	D	2	1	2	0

Question 5

The target in a game is a circle divided into three regions: inner, middle and outer.

The diagram below shows these regions and the number of points awarded for hitting each region with a ball.



The number of inner, middle and outer regions hit by Mustafa and Neville in one game is shown in the table below.

Player	Inner	Middle	Outer
Mustafa	6	7	2
Neville	5	9	7

Neville was the winner of this game.

Which one of the following matrix calculations shows the difference between the winning and losing scores?

A. $\begin{bmatrix} 8 \\ 3 \\ 1 \end{bmatrix} \times ([5 \ 9 \ 7] - [6 \ 7 \ 2])$

B. $[5 \ 9 \ 7] - [6 \ 7 \ 2] \times \begin{bmatrix} 8 \\ 3 \\ 1 \end{bmatrix}$

C. $([5 \ 9 \ 7] - [6 \ 7 \ 2]) \times \begin{bmatrix} 8 \\ 3 \\ 1 \end{bmatrix}$

D. $\left(\begin{bmatrix} 5 \\ 9 \\ 7 \end{bmatrix} - \begin{bmatrix} 6 \\ 7 \\ 2 \end{bmatrix} \right) \times [8 \ 3 \ 1]$

E. $\begin{bmatrix} 8 \\ 3 \\ 1 \end{bmatrix} \times [5 \ 9 \ 7] - \begin{bmatrix} 8 \\ 3 \\ 1 \end{bmatrix} \times [6 \ 7 \ 2]$

Question 6

The solution to a system of simultaneous linear equations is determined by evaluating the matrix product

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

One of these linear equations could be

- A. $-4x + 3y = 3$
- B. $-4x - 2y = 3$
- C. $2x + 3y = 3$
- D. $3x + 3y = -1$
- E. $3x + 4y = 3$

Use the following information to answer Questions 7 and 8.

A company has selected 200 of its regular customers to rate its performance every month.

The rating given by a customer can be poor (P), good (G) or excellent (E).

Customers are expected to change their rating from month to month as shown in the transition matrix T below.

$$T = \begin{array}{c} \begin{array}{ccc} & \text{this month} & \\ P & G & E \\ \begin{bmatrix} 0.2 & 0.3 & 0.2 \\ 0.7 & 0.3 & 0.5 \\ 0.1 & 0.4 & 0.3 \end{bmatrix} & \begin{array}{l} P \\ G \\ E \end{array} & \text{next month} \end{array} \end{array}$$

The expected number of each rating received after n months can be determined by the recurrence relation

$$S_0 = \begin{bmatrix} 40 \\ 110 \\ 50 \end{bmatrix} \begin{array}{l} P \\ G \\ E \end{array}, \quad S_{n+1} = TS_n$$

where S_0 is the state matrix for January.

Question 7

What percentage of these 200 customers are **not expected** to change their rating in February?

- A. 28%
- B. 40%
- C. 43%
- D. 56%
- E. 80%

Question 8

In the long term, the number of these 200 customers who are **expected** to change their rating each month from excellent to good is closest to

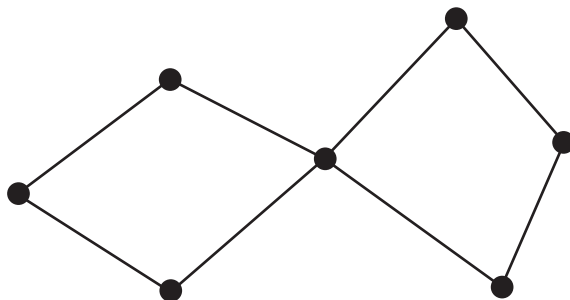
- A. 28
- B. 60
- C. 66
- D. 88
- E. 92

Module 2 – Networks and decision mathematics

Before answering these questions, you must **shade** the ‘Networks and decision mathematics’ box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

Consider the following graph.



Which one of the following statements is **not** true for this graph?

- A. This graph has seven vertices.
- B. There are no isolated vertices.
- C. All vertices have an even degree.
- D. Six of the vertices have the same degree.
- E. The sum of the degrees of the vertices is 14.

Question 2

A puzzle has four parts.

Four friends, Audrey, Bruce, Christie and Darren, are solving the puzzle together.

The table below shows the time it takes each person to complete each part of the puzzle, in minutes.

	Part 1	Part 2	Part 3	Part 4
Audrey	5	5	5	5
Bruce	3	4	5	6
Christie	6	5	4	3
Darren	5	5	5	5

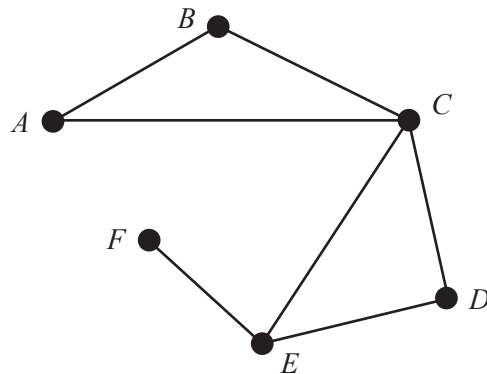
The parts of the puzzle must be solved one after the other, with each friend completing one part.

The minimum time taken to complete the entire puzzle, in minutes, is

- A. 14
- B. 16
- C. 18
- D. 20
- E. 22

Question 3

A graph has six vertices and seven edges, as shown in the diagram below.



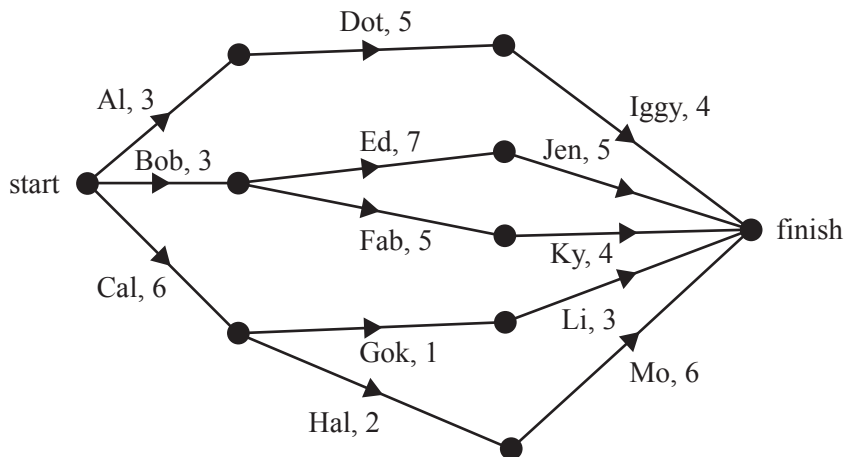
For the graph shown above, it is possible to find

- A. an Eulerian trail and an Eulerian circuit.
- B. an Eulerian trail and a Hamiltonian path.
- C. an Eulerian trail and a Hamiltonian cycle.
- D. a Hamiltonian path and an Eulerian circuit.
- E. a Hamiltonian path and a Hamiltonian cycle.

Question 4

The network below shows the sequence of activities required to complete a project.

The name of the person completing each activity and the duration of the activity, in hours, are also shown on the network.



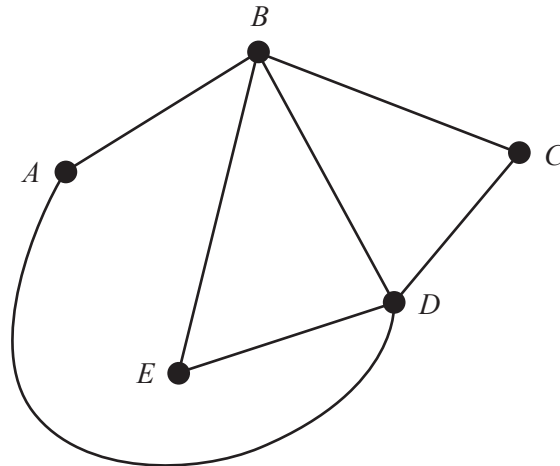
The project is to be completed in the minimum time possible.

The activity that has the latest starting time of 12 hours is completed by

- A. Iggy.
- B. Jen.
- C. Ky.
- D. Li.
- E. Mo.

Question 5

The friendships between five children are summarised in the graph below.



The vertices A , B , C , D and E in the graph above represent these children.

Each edge between two vertices indicates that the two children are friends.

For example, the edge between vertex B and vertex C shows that child B and child C are friends.

An adjacency matrix that summarises the friendships can also be constructed.

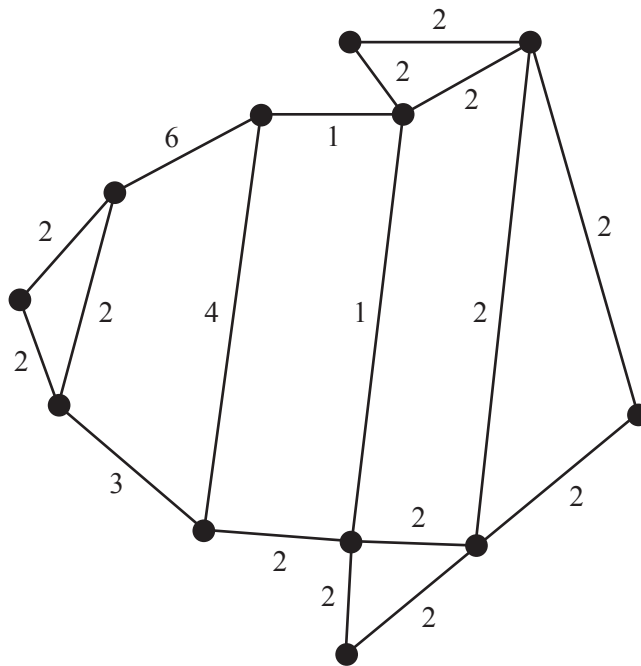
In this matrix, a '0' indicates no friendship and a '1' indicates a friendship.

How many zero elements will this adjacency matrix have?

- A. 7
- B. 8
- C. 9
- D. 10
- E. 11

Question 6

A minimal spanning tree is to be drawn for the weighted graph below.



How many edges with weight 2 will **not** be included in the minimal spanning tree?

- A. 3
- B. 4
- C. 5
- D. 6
- E. 7

Question 7

A simple connected graph has at least one vertex with an odd degree and at least one vertex with an even degree.

Which one of the following could be the number of vertices with an odd degree in this graph?

- A. 3
- B. 4
- C. 5
- D. 7
- E. 9

Question 8

A project consists of eight activities, A to H .

The table below shows the immediate predecessor(s) and earliest starting time, in hours, of each activity.

Activity	Immediate predecessor(s)	Earliest starting time
A	–	0
B	–	0
C	A	4
D	B	12
E	C	15
F	D	19
G	E	27
H	F, G	36

It is known that activity G has a completion time of three hours.

The project can still be completed in minimum time if activity C is delayed.

The maximum length of the delay for activity C is

- A. two hours.
- B. four hours.
- C. five hours.
- D. six hours.
- E. nine hours.

Module 3 – Geometry and measurement

Before answering these questions, you must **shade** the ‘Geometry and measurement’ box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

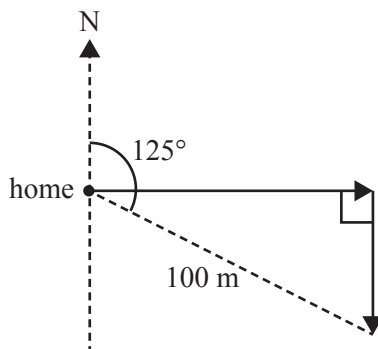
Which one of the following locations is closest to the equator?

- A. $12^\circ \text{ S}, 135^\circ \text{ E}$
- B. $25^\circ \text{ N}, 100^\circ \text{ W}$
- C. $40^\circ \text{ S}, 72^\circ \text{ W}$
- D. $50^\circ \text{ N}, 40^\circ \text{ E}$
- E. $70^\circ \text{ N}, 20^\circ \text{ E}$

Question 2

Alan walked directly east from his home and then directly south.

His final position was 100 m from his home on a bearing of 125° , as shown in the diagram below.

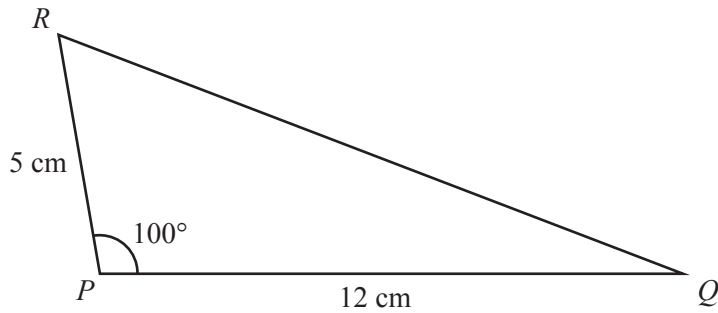


The distance, in metres, that Alan walked directly east is closest to

- A. 42.8
- B. 57.3
- C. 81.9
- D. 142.8
- E. 181.9

Question 3

The triangle PQR is shown in the diagram below.



The length of the side PQ is 12 cm.

The length of the side PR is 5 cm.

The angle QPR is 100° .

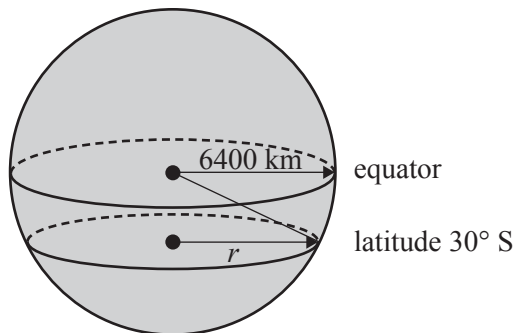
The perimeter of triangle PQR , in centimetres, is closest to

- A. 8
- B. 13
- C. 14
- D. 30
- E. 31

Question 4

The diagram below shows the small circle of Earth with latitude 30° S.

The radius of this circle is labelled r .



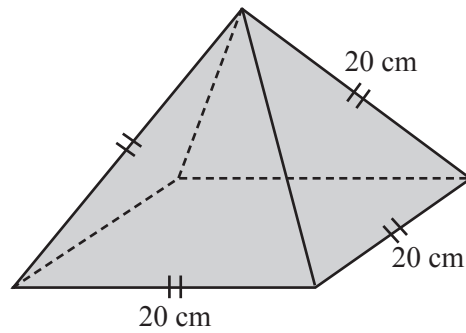
Assume that the radius of Earth is 6400 km.

Which one of the following calculations will give the radius of this small circle, r , in kilometres?

- A. $\sin(30^\circ) \times 6400$
- B. $\sin(60^\circ) \times 6400$
- C. $\cos(60^\circ) \times 6400$
- D. $\frac{\sin(30^\circ)}{6400}$
- E. $\frac{\cos(60^\circ)}{6400}$

Question 5

A square-based pyramid is shown in the diagram below.



The base lengths of this pyramid are 20 cm.

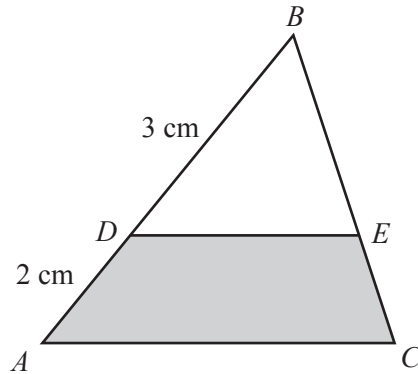
The slant edges of this pyramid are 20 cm.

Which one of the following calculations gives the volume of this pyramid in cubic centimetres?

- A. $\frac{1}{3} \times 20 \times 20 \times 20$
- B. $\frac{1}{3} \times 10 \times 10 \times 10 \times \sqrt{2}$
- C. $\frac{1}{3} \times 10 \times 10 \times 20 \times \sqrt{2}$
- D. $\frac{1}{3} \times 20 \times 20 \times 10 \times \sqrt{2}$
- E. $\frac{1}{3} \times 20 \times 20 \times 20 \times \sqrt{2}$

Question 6

The triangle ABC is divided into two regions, as shown in the diagram below.



The line DE is parallel to the line AC .

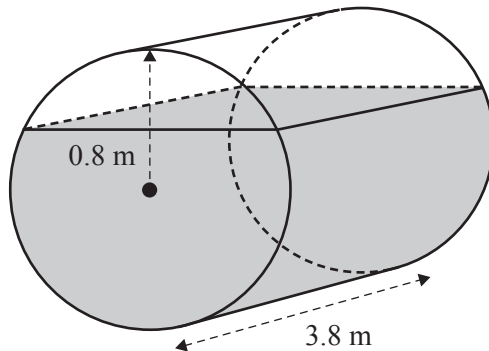
The length of AD is 2 cm and the length of DB is 3 cm.

The ratio of the unshaded area, BDE , to the shaded area, $ADEC$, is

- A. 3:2
- B. 3:5
- C. 9:4
- D. 9:16
- E. 9:25

Question 7

A cylindrical fuel tank is shown in the diagram below.



The radius of the fuel tank is 0.8 m.

The length of the fuel tank is 3.8 m.

The depth of fuel in the tank is 1.2 m.

One thousand litres of fuel has a volume of 1 m^3 .

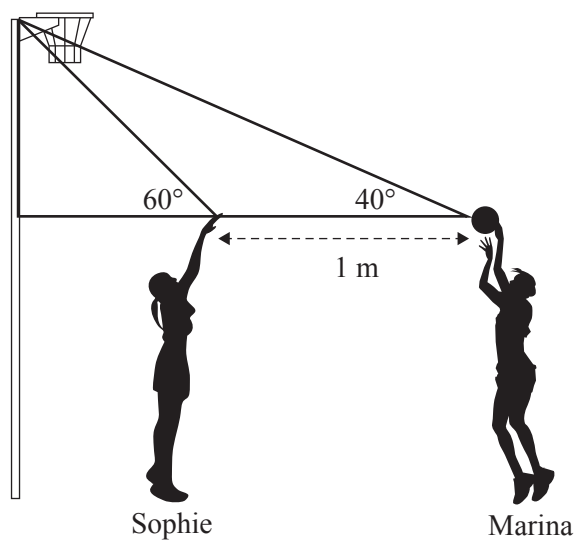
The amount of fuel in this tank is closest to

- A. 5094 litres.
- B. 5730 litres.
- C. 6147 litres.
- D. 6587 litres.
- E. 7420 litres.

Question 8

Sophie and Marina are on opposite teams in a netball match.

As Marina is about to shoot for goal, Sophie's hand is 1 m in front of the ball, as shown in the diagram below.



The angle of elevation from the ball to the top of the goalpost is 40° .

The angle of elevation from Sophie's hand to the top of the goalpost is 60° .

The horizontal distance of the ball from the goalpost is closest to

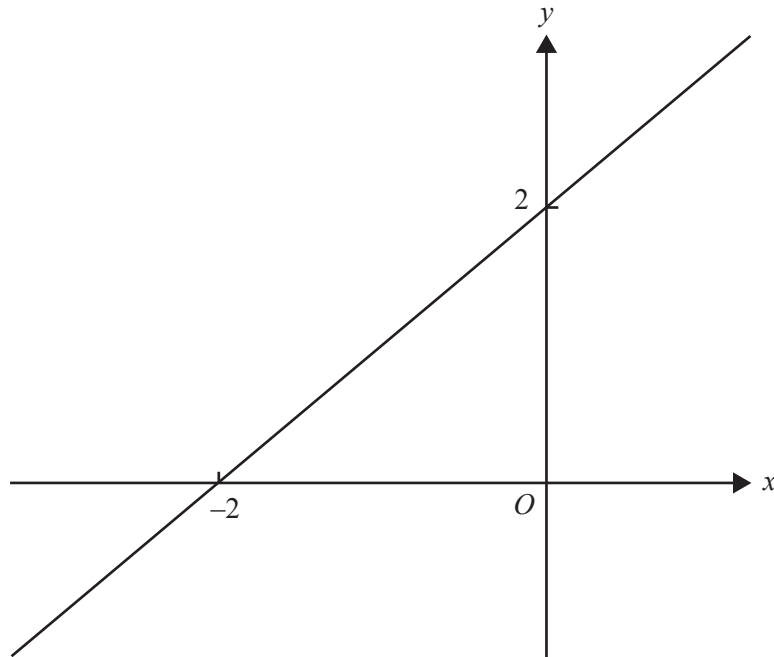
- A. 0.84 m
- B. 0.88 m
- C. 1.00 m
- D. 1.88 m
- E. 1.94 m

Module 4 – Graphs and relations

Before answering these questions, you must **shade** the ‘Graphs and relations’ box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

A straight line passes through the points $(-2, 0)$ and $(0, 2)$, as shown in the diagram below.



The equation of this straight line is

- A. $y = 2x$
- B. $y = x + 2$
- C. $y = 2x + 2$
- D. $y = x - 2$
- E. $y = 2x - 2$

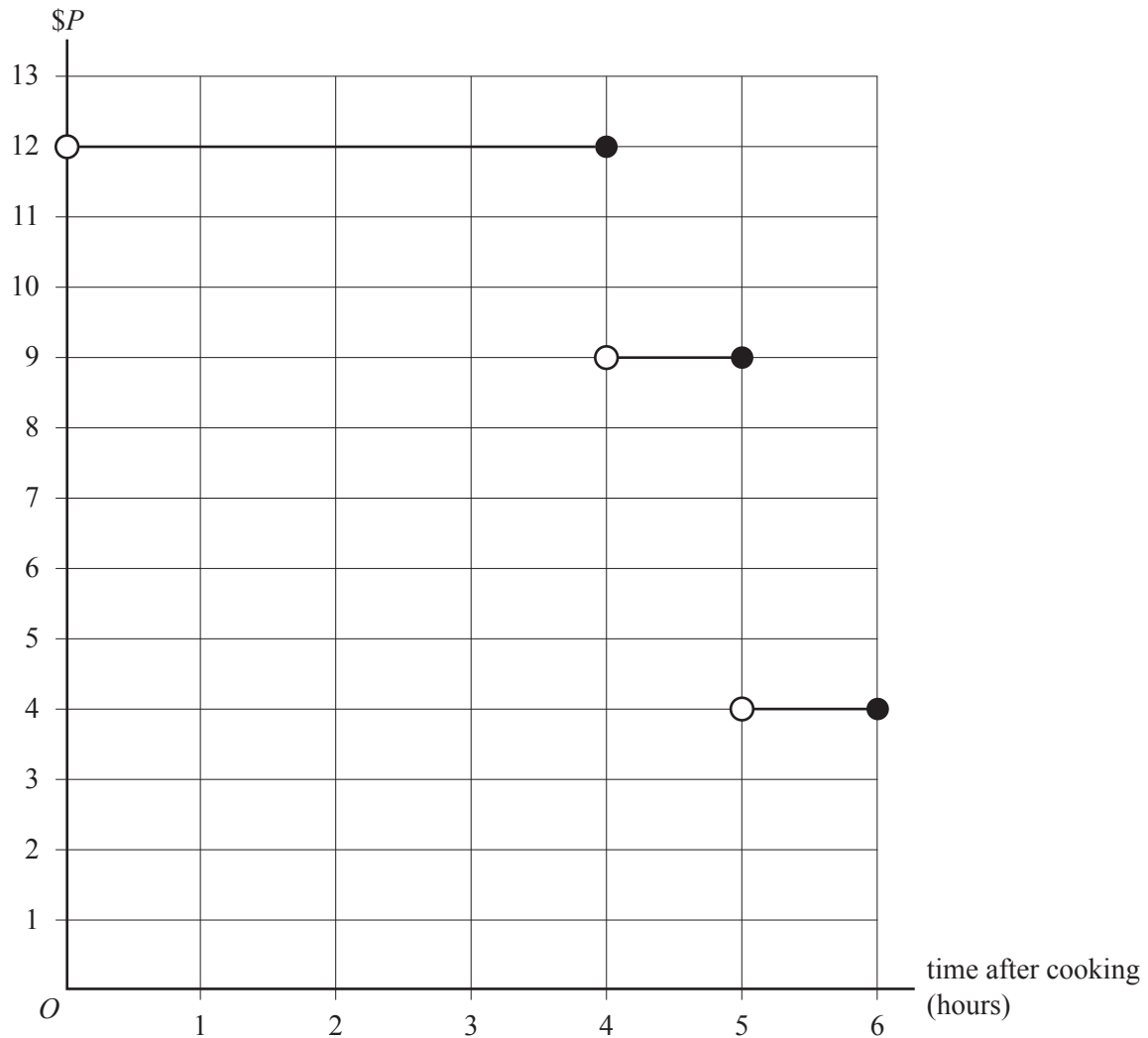
Question 2

A supermarket sells roasted chickens.

For the first four hours after cooking, the roasted chickens are sold at full price.

After this time, the selling price of each roasted chicken is reduced.

The price of a roasted chicken, $\$P$, at any time up to six hours after cooking is shown in the step graph below.



A roasted chicken is sold five hours after cooking.

By how much has the full price of the roasted chicken been reduced?

- A. \$3
- B. \$4
- C. \$8
- D. \$9
- E. \$12

Question 3

The total assembly time for six tables and 12 chairs is 720 minutes.

The total assembly time for eight tables and 20 chairs is 1040 minutes.

All tables take the same time to assemble and all chairs take the same time to assemble.

Let x be the assembly time of a table, in minutes.

Let y be the assembly time of a chair, in minutes.

The set of simultaneous linear equations that can be solved to find the assembly time of a table and the assembly time of a chair is

A. $6x + 12y = 1040$
 $8x + 20y = 720$

B. $12x + 6y = 1040$
 $20x + 8y = 720$

C. $6x + 12y = 720$
 $20x + 8y = 1040$

D. $6x + 12y = 720$
 $8x + 20y = 1040$

E. $12x + 6y = 720$
 $20x + 8y = 1040$

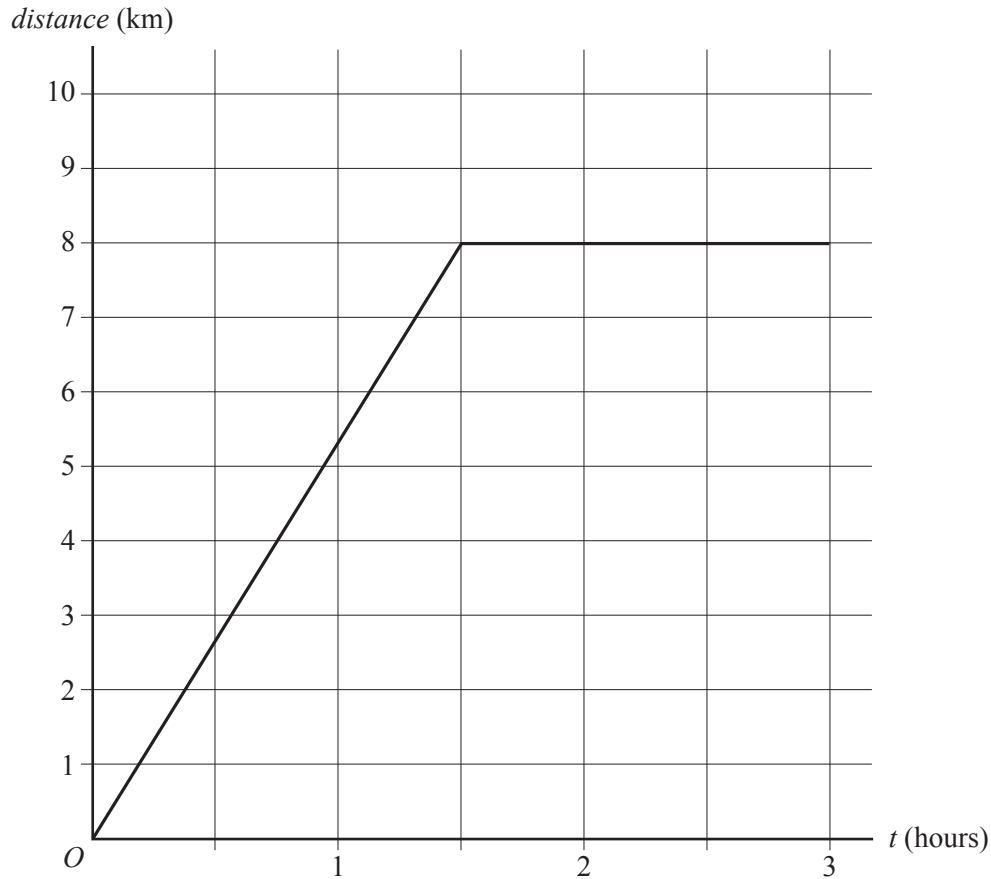
Question 4

A car park and a waterfall in a national park are 8 km apart.

James and Camille walked from the car park to the waterfall.

James and Camille left the car park at the same time, but walked at different speeds.

The graph below shows the *distance*, in kilometres, that James was from the car park after t hours.



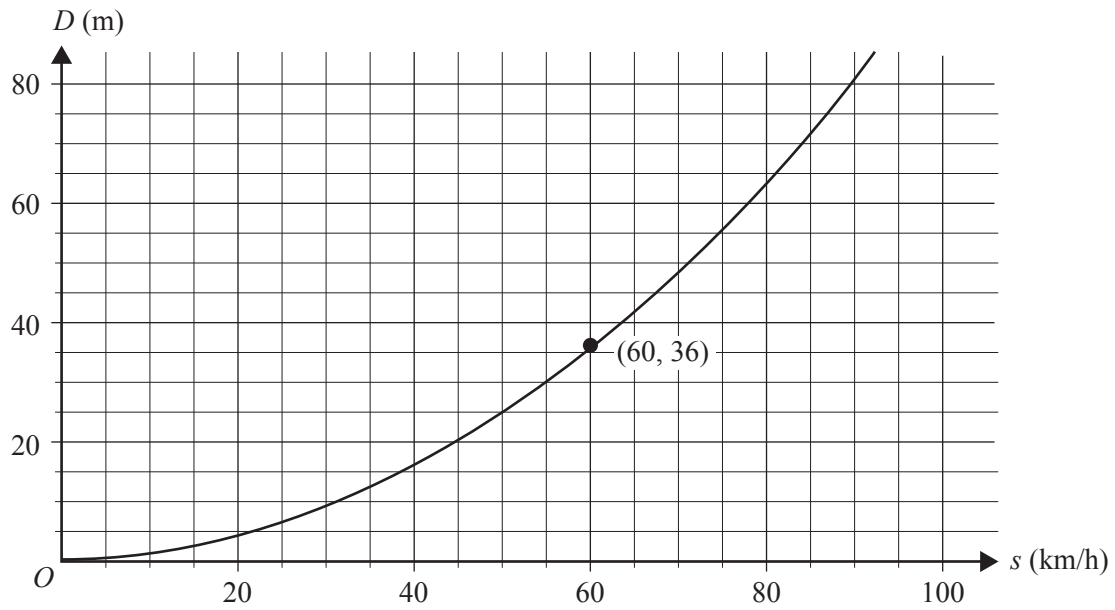
The *distance*, in kilometres, that Camille was from the car park after t hours is given by the rule $\text{distance} = 4 \times t$.

Which one of the following statements is true?

- A. It took Camille 90 minutes to walk to the waterfall.
- B. Camille arrived at the waterfall 30 minutes after James.
- C. James waited three hours for Camille to get to the waterfall.
- D. James and Camille both arrived at the waterfall at the same time.
- E. James walked past Camille during their walk to the waterfall.

Question 5

The graph below shows the braking distance, D metres, for a car travelling at speed s kilometres per hour.



The braking distance can be calculated using the rule $D = ks^2$.

The value of k is

- A. $\frac{36}{60}$
- B. $\frac{60}{36}$
- C. $\frac{36}{60^2}$
- D. $\frac{60}{36^2}$
- E. 60×36^2

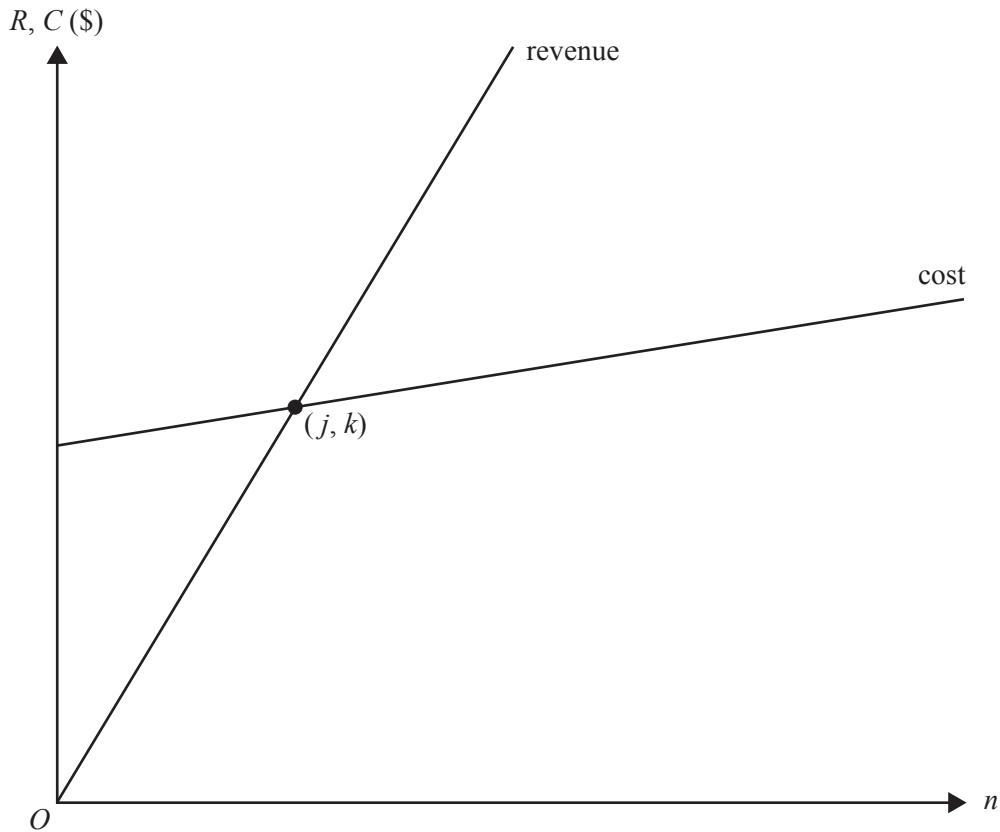
Question 6

A company makes and sells beds.

Let C be the cost of making n beds.

Let R be the revenue received from making and selling n beds.

The lines representing the equation for revenue and the equation for cost are shown on the graph below.



The graphs of the equations for revenue and cost intersect at the point (j, k) .

Which one of the following statements is **not** true?

- A. A profit is made when $n > j$.
- B. The cost of producing j beds is k .
- C. The company will break even when $n = j$.
- D. The revenue from selling j beds is k .
- E. Exactly j beds need to be sold to make a profit.

Question 7

A recipe for fruit juice contains both pears and apples.

Let x be the number of pears required to make the juice.

Let y be the number of apples required to make the juice.

For every three pears that are used, at least five apples must be used.

An inequality representing this situation is

- A. $y \geq \frac{3x}{5}$
- B. $y \geq \frac{5x}{3}$
- C. $y \geq \frac{x}{3} + 5$
- D. $y \geq \frac{x}{5} + 3$
- E. $y \leq \frac{x}{5} + 3$

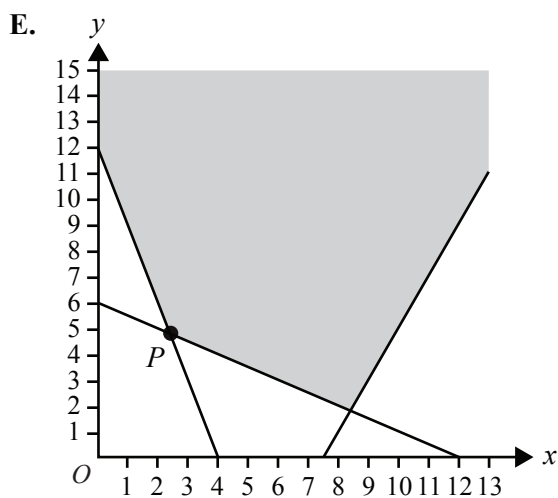
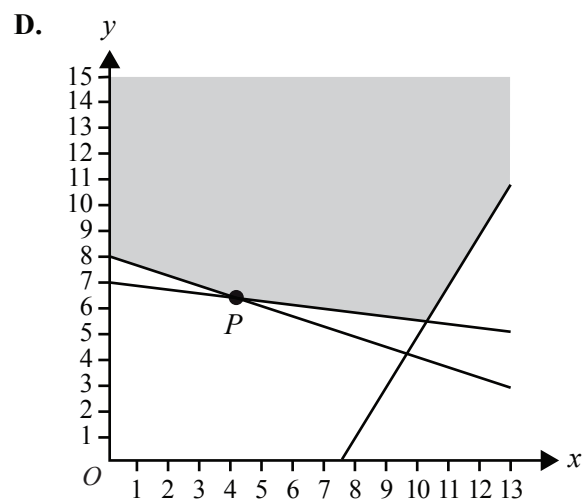
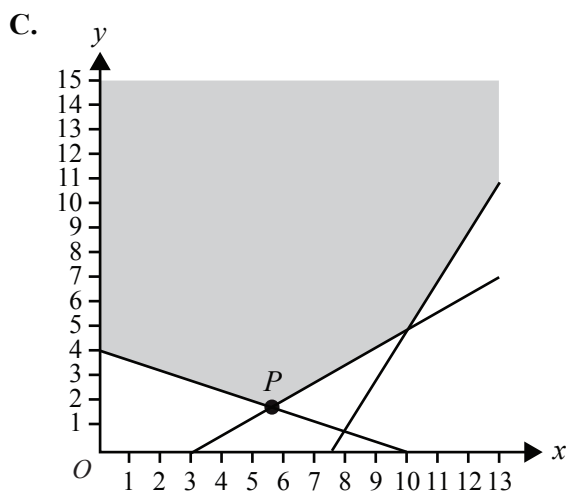
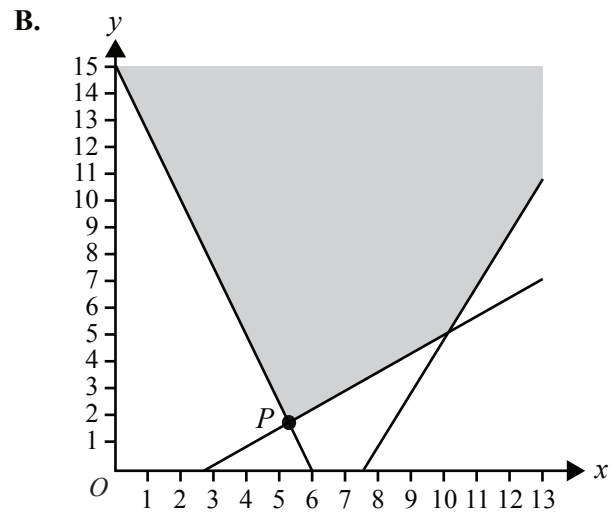
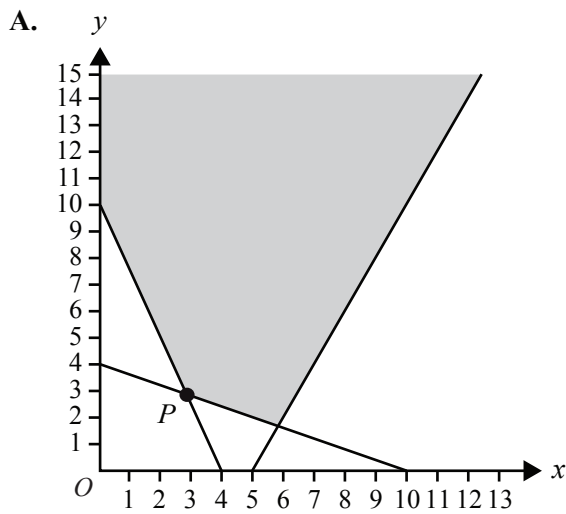
Question 8

A linear programming problem has the objective function $Z = 5x + 2y$.

The minimum value of Z occurs only at point P .

For each of the following graphs, the feasible region has been shaded.

Which graph could represent this linear programming problem?



**Victorian Certificate of Education
2018**

FURTHER MATHEMATICS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A multiple-choice question book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Further Mathematics formulas

Core – Data analysis

standardised score	$z = \frac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r \frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	seasonal index = $\frac{\text{actual figure}}{\text{deseasonalised figure}}$

Core – Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{\text{effective}} = \left[\left(1 + \frac{r}{100n} \right)^n - 1 \right] \times 100\%$

Module 1 – Matrices

determinant of a 2×2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix	$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \quad \text{where} \quad \det A \neq 0$
recurrence relation	$S_0 = \text{initial state}, \quad S_{n+1} = TS_n + B$

Module 2 – Networks and decision mathematics

Euler's formula	$v + f = e + 2$
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Module 3 – Geometry and measurement

area of a triangle	$A = \frac{1}{2}bc \sin(\theta^\circ)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}, \quad \text{where} \quad s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc \cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta^\circ$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta^\circ}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base $\times$ height
volume of a pyramid	$\frac{1}{3} \times \text{area of base} \times \text{height}$

Module 4 – Graphs and relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$