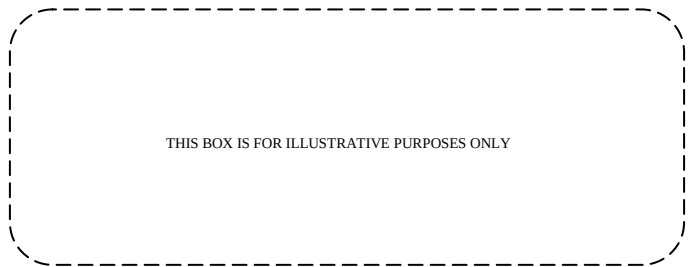




2018 Trial Examination



THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT  
NUMBER

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# FURTHER MATHEMATICS

## Units 3 & 4 – Written examination 2

Reading time: 15 minutes

Writing time: 1 hour and 30 minutes

### QUESTION AND ANSWER BOOK

#### Structure of book

| Section     | Number of questions | Number of questions to be answered | Number of modules | Number of modules to be answered | Number of marks |
|-------------|---------------------|------------------------------------|-------------------|----------------------------------|-----------------|
| A – Core    | 7                   | 7                                  |                   |                                  | 36              |
| B - Modules |                     |                                    | 4                 | 2                                | 24              |
|             |                     |                                    |                   |                                  | Total 60        |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

#### Materials supplied

- Question and answer book of 25 pages.

#### Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.**

**SECTION A – Core****Instructions for Section A**

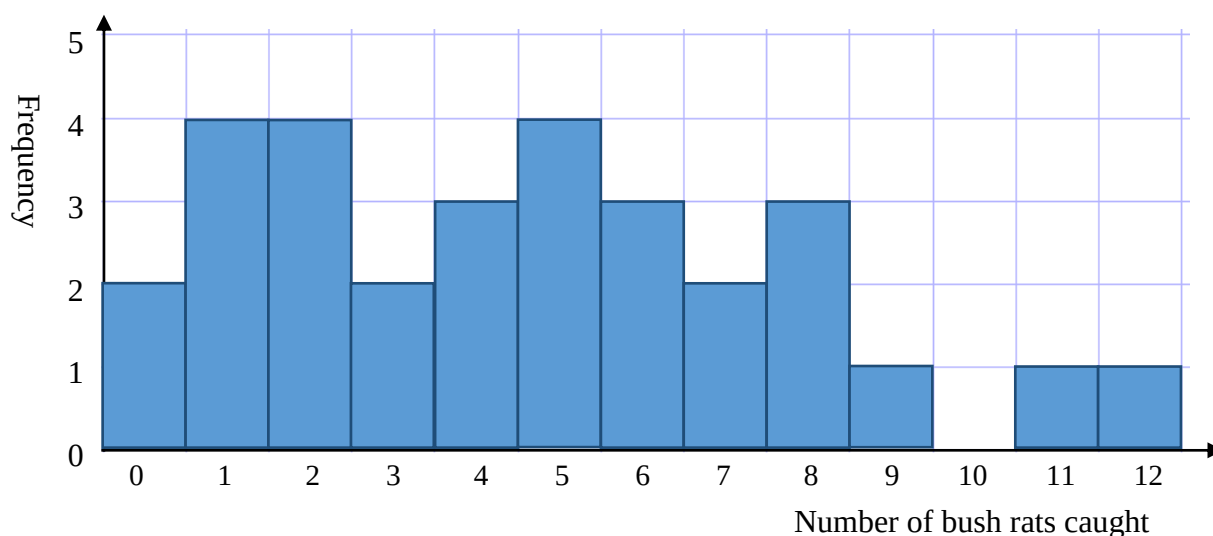
Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example,  $\pi$ , surds or fractions.

In ‘Recursion and financial modelling’, all answers should be rounded to the nearest cent unless otherwise instructed. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Data analysis****Question 1** (7 marks)

The following graph shows the number of bush rats captured in a 30 day study at Parker River.



- a.** On how many days were there more than 7 bush rats caught?

1 mark

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- b.** Write down the:

- i. median:

1 mark

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- ii. values of Q1 and Q3:

2 marks

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**SECTION A – Question 1 - continued**

- c. Calculate the interquartile range:

1 mark

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- d. Each day there were 15 traps set. Determine the percentage of days that more than 50% of the traps contained a bush rat.

1 mark

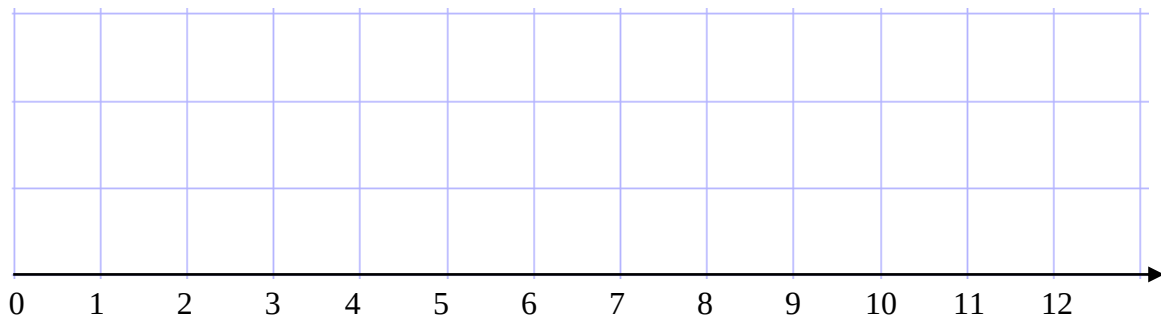
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- e. Use the grid below to construct a box and whisker plot which shows the number of bush rats caught during the study.

2 marks



**SECTION A – continued**  
**TURN OVER**

**Question 2** (7 marks)

During the study of bush rats at Parker River in 2017, weights and lengths were recorded. The following weights were obtained:

Weights of Bush rats

**Key** 1|07 = 107

| Stem | leaves               |
|------|----------------------|
| 1    | 07 12 26 30 37 46 49 |
| 1    | 52 56 61 61 87       |
| 2    | 01 14 15 32 34 48 48 |
| 2    | 61 71 91 95          |
| 3    | 08 12 27 27 39       |
| 3    | 68                   |
| 4    | 39                   |

- a.** Describe the distribution of weights in terms of shape, centre and spread. 2 marks

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- b.** Determine if 439 grams is an outlier for this data and show appropriate working to support your answer. 2 marks

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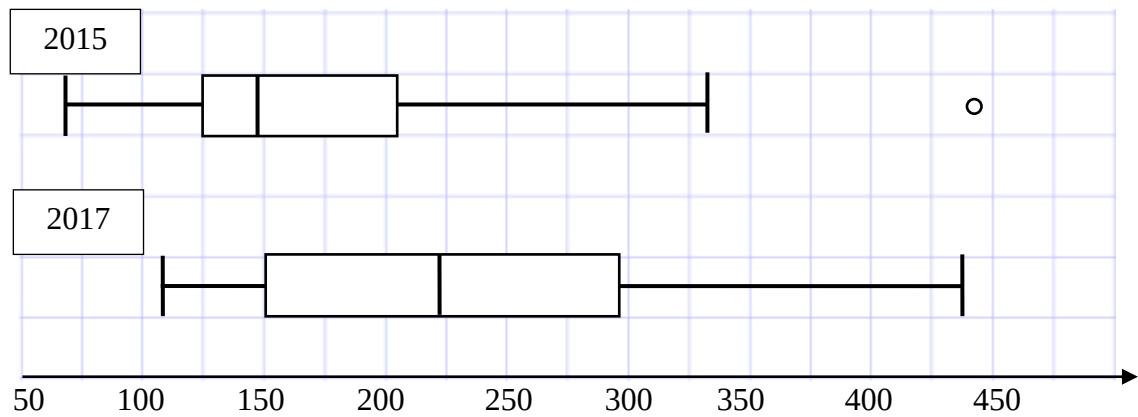


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- c. The box plots below show the weights distributions of bush rats at Parker River from studies conducted in 2015 and 2017.



Compare the populations of bush rats from the two studies in terms of shape, centre and spread. 3 marks

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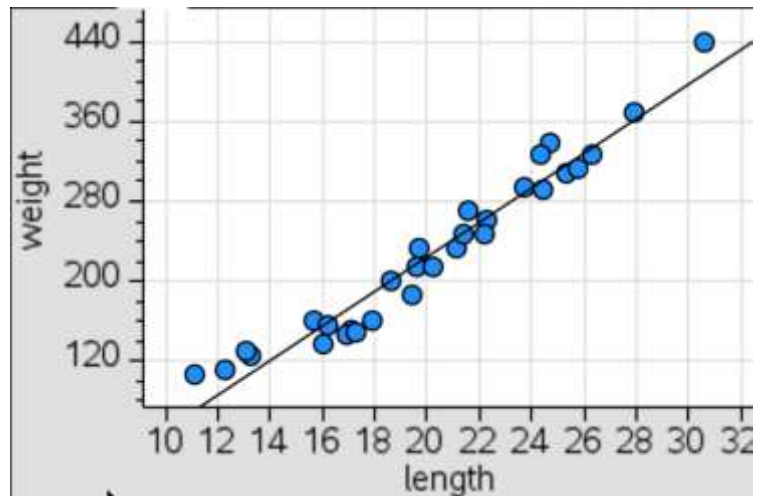
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**SECTION A – continued**  
**TURN OVER**

**Question 3** (8 marks)

The weights and lengths of bush rats in the 2017 study are given and plotted in the scatter plot below:

| Length | Weight | Length | Weight |
|--------|--------|--------|--------|
| 27.9   | 368    | 19.7   | 234    |
| 15.7   | 161    | 22.3   | 261    |
| 24.7   | 339    | 16.2   | 156    |
| 16.0   | 137    | 17.9   | 161    |
| 26.3   | 327    | 12.3   | 112    |
| 21.1   | 232    | 16.9   | 146    |
| 13.3   | 126    | 13.1   | 130    |
| 24.4   | 291    | 18.6   | 201    |
| 21.4   | 248    | 30.6   | 439    |
| 19.4   | 187    | 23.7   | 295    |
| 19.6   | 214    | 22.2   | 248    |
| 20.2   | 215    | 17.3   | 149    |
| 24.3   | 327    | 25.8   | 312    |
| 25.3   | 308    | 21.6   | 271    |
| 17.1   | 152    | 11.1   | 107    |



$n = 30$

$r^2 = 0.949$

- a. Use the scatter plot to describe the association between *weight* (in grams) and *length* (in cm) of bush rats in terms of strength, direction and form. 1 mark

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- b. i. Determine the equation of the least squares regression line that can be used to predict the weight of a bush rat from its length. Give the values of the intercept and slope correct to four significant figures. 3 marks

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- ii. Interpret the slope in terms of weight and length. 1 mark

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**SECTION A – Question 3 - continued**

- c. The coefficient of determination between weight and length is 0.949. Interpret the coefficient of determination in terms of weight and length. 1 mark

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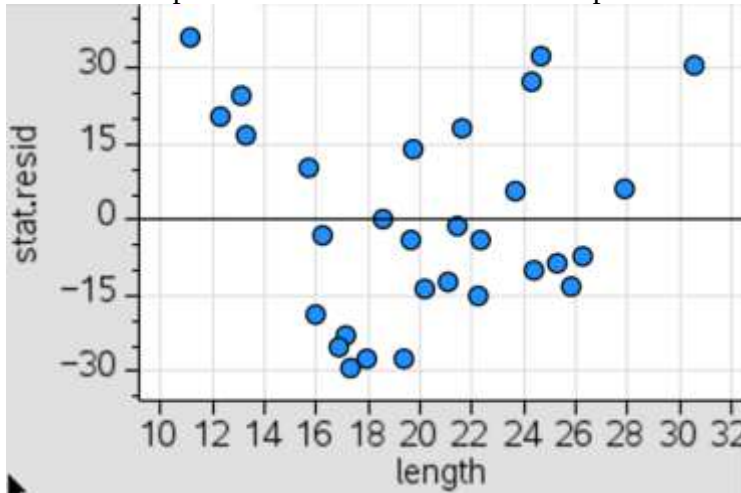


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- d. The residual plot for this data is shown in the plot below:



Does the residual plot above support the assumption of linearity for this relationship, and if not, what transformation would best linearise this data? 2 marks

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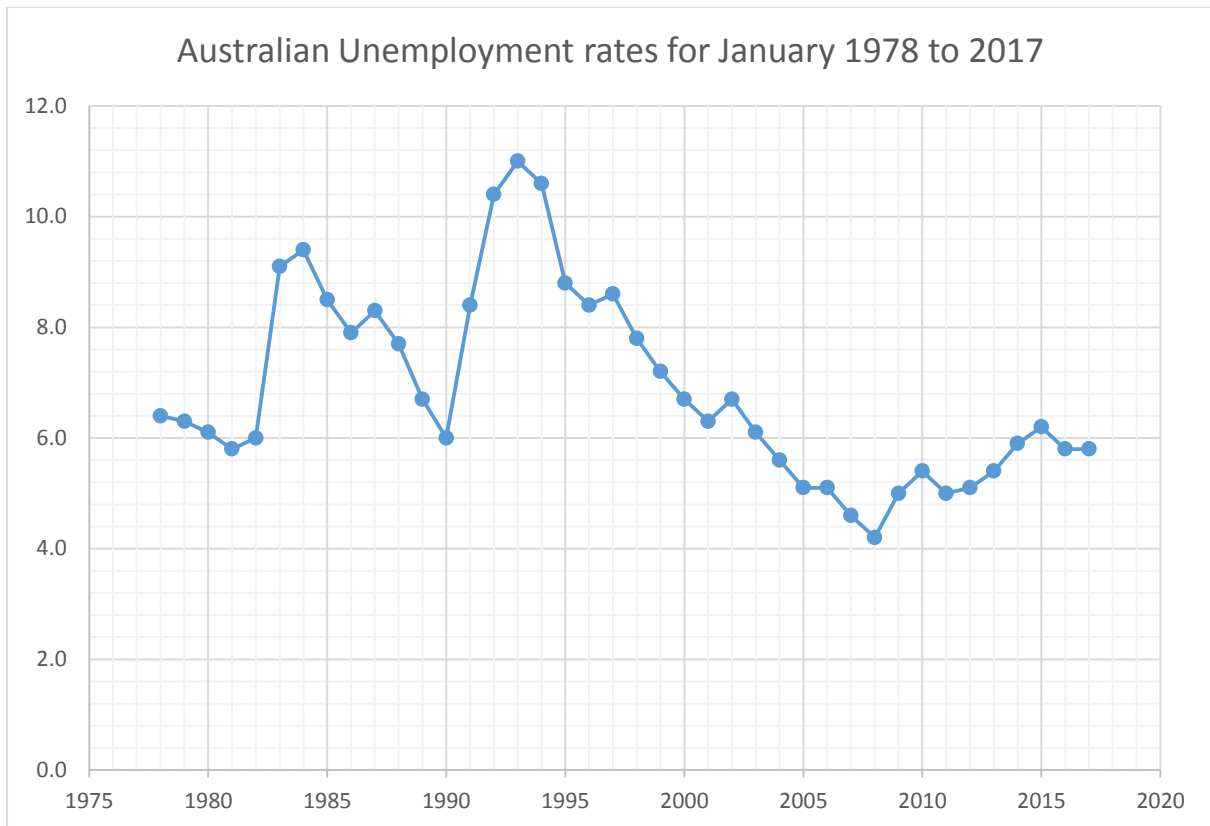


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**SECTION A – continued**  
**TURN OVER**

**Question 4** (2 marks)

The unemployment rates in Australia for the month of January over the years 1978 to 2017 are shown on the time series plot below:



- a.** Describe the general trend over the period from 2008 to 2015. 1 mark

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- b.** Mark on the graph above the 5 median smoothed value centred at 1990. 1 mark

**SECTION A** – continued



**Recursion and financial modelling****Question 5** (6 marks)

A company purchases a car for \$48,000 for one of their sales representatives. The car is depreciated using flat rate depreciation. The recurrence relation for the value of the car,  $V_n$ , after  $n$  years is given below:

$$V_0 = 48000, V_{n+1} = V_n - 5760$$

- a. What is the annual depreciation rate of the car? 1 mark

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- b. Write down the rule for calculating the value of  $V_n$  in terms of  $n$ . 1 mark

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- c. After how many years will the value of the car first fall below \$10000? 1 mark

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The car could be depreciated using a reducing balance method. Its value after  $n$  years is represented by  $C_n$ .

- d. Given that  $C_n$  first reaches a value less than \$10000 after 10 years and its value at this time is \$1769.97 **more than** the value of  $V_7$ , determine the depreciation rate for the reducing balance depreciation correct to the nearest whole number. 2 marks

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**SECTION A – Question 5 – continued**  
**TURN OVER**

- e. Write down the recurrence equation for  $C_n$  in terms of  $C_{n+1}$  and  $C_n$ . 1 mark

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**Question 6** (6 marks)

Chris wants to save money to purchase a caravan to travel around Australia in her retirement. She deposits \$20000 into a savings account which pays 5.1% p.a. interest compounded monthly.

- a. Write a recurrence relation which calculates the amount of savings,  $S_n$  in her account after  $n$  months in terms of  $S_{n+1}$  and  $S_n$ . 1 mark

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- b. Show that the amount in her savings account will be \$20342.17 after 4 months. 1 mark

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- c. How much money would Chris expect to have in her savings account after 10 years? 1 mark

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**SECTION A – Question 6 - continued**

After 6 months Chris begins to make monthly deposits into her savings account. She wants to have a balance of \$55000 in her savings account 6 years after her initial \$20000 deposit.

- d. If she deposits a whole dollar amount each month, what is the minimum amount she will need to add to her savings account each month to reach her target? 2 marks

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- e. How much less would she need to deposit each month if she had commenced deposits at the end of the first month of her investment? 1 mark

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**END OF SECTION A  
TURN OVER**

**SECTION B – Modules****Instructions for Section B**

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example  $\pi$ , surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

| <b>Contents</b>                                   | <b>Page</b> |
|---------------------------------------------------|-------------|
| Module 1 – Matrices .....                         | 13          |
| Module 2 – Networks and decision mathematics..... | 17          |
| Module 3 – Geometry and measurement .....         | 20          |
| Module 4 – Graphs and relations .....             | 23          |

**SECTION B – continued**

**Module 1 – Matrices****Question 1** (5 marks)

Long haul flights to the United Kingdom are available on Bragstar airlines jets. The table below details the number of seats in the three classes of seats available on a flight:

| Class    | No. of seats | Cost per seat (\$) |
|----------|--------------|--------------------|
| First    | 12           | \$15000            |
| Business | 40           | \$10000            |
| Economy  | 448          | \$2200             |

- a. State the order of a column matrix,  $N$ , which shows the number of seats in each class and write the matrix. 2 marks

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- b. Matrix  $C$ , is a  $1 \times 3$  matrix containing the costs of seats in each class. Write the matrix  $C$  below. 1 mark

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- c. Using matrix multiplication show the order that matrices  $N$  and  $C$  would need to be multiplied in order to determine the total revenue from sales of seats on the Bragstar jet if a flight to the United Kingdom is fully booked, and calculate what this total is. 2 marks

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**SECTION B – MODULE 1 – continued**  
**TURN OVER**

**Question 2** (2 marks)

On a recent flight to the United Kingdom there were 6 first class, 27 business class and 392 economy seats sold. Matrix  $A$  is calculated using the following matrix multiplication:

$$A = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 392 \end{bmatrix} \times \begin{bmatrix} 15000 \\ 10000 \\ 2200 \end{bmatrix}$$

- a.** Explain what the information in matrix  $A$  represents. 1 mark

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- b.** Explain what the matrix product  $X \times A$  represents if  $X = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$  1 mark

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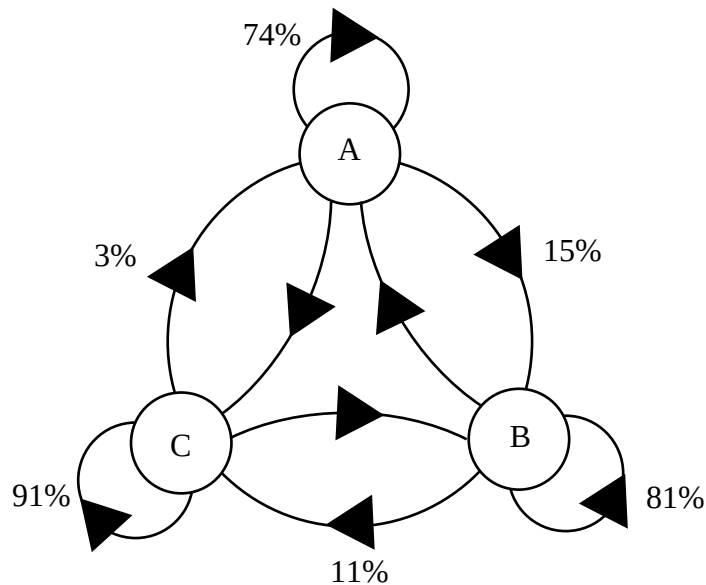
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**SECTION B – MODULE 1 – continued**

**Question 3** (5 marks)

There are three main airlines which fly directly from Melbourne to London each week and similar numbers of travellers fly each week. The airlines are Arewethereyet (A), Bragstar (B) and Conair (C). The transition diagram below shows how airline choices of passengers changes from one week to the next.



- a. Complete the transition matrix below:

1 mark

$$T = \begin{matrix} & \begin{matrix} \text{This week} \\ A & B & C \end{matrix} \\ \begin{bmatrix} 0.74 & \text{---} & 0.03 \\ 0.15 & 0.81 & \text{---} \\ \text{---} & 0.11 & 0.91 \end{bmatrix} & \begin{matrix} A \\ B \\ C \end{matrix} \end{matrix} \begin{matrix} \\ \text{next week} \\ \end{matrix}$$

- b. Last week Conair had 396 passengers, Arewethereyet had 654 passengers, and Bragstar had 421 passengers. Write this information in an initial state matrix.

1 mark

- c. How many passengers in total travel the Melbourne to London route each week?

1 mark

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**SECTION B – MODULE 1 – Question 3 - continued**  
**TURN OVER**

- d.** In the long term, how many passengers are expected to travel Bragstar each week? 1 mark

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- e.** The largest aircraft owned by Bragstar airlines can seat a total of 500 passengers only. Does this model predict that this number will be exceeded at any time? 1 mark

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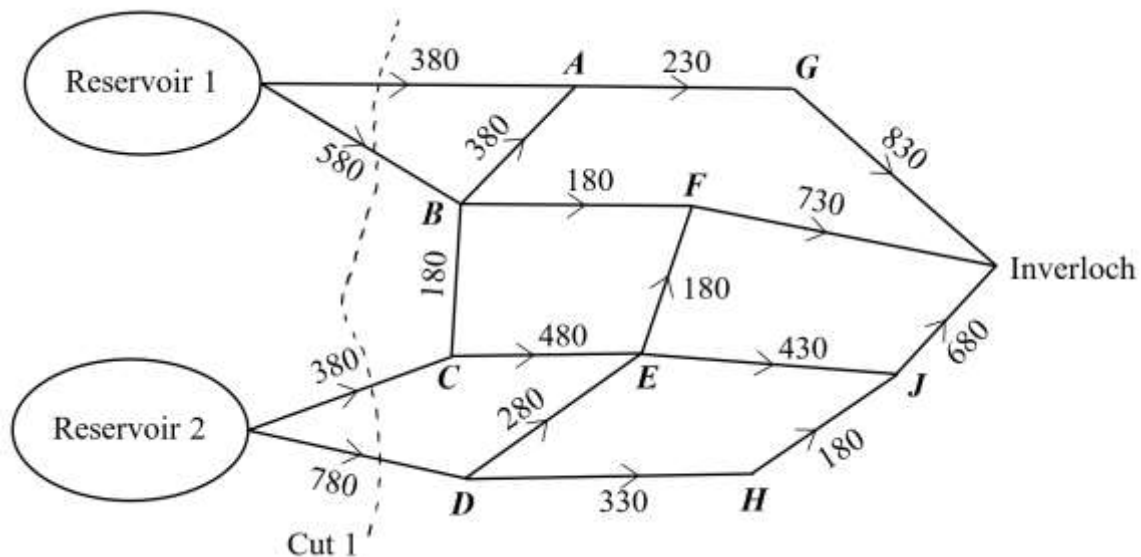
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**END OF MODULE 1 - SECTION B – continued**



**Module 2 – Networks and decision mathematics****Question 1** (6 marks)

The township of Inverloch has been undergoing significant growth over the past few years which has placed its water supply under some pressure. There is a secure water supply for the town, but the infrastructure which delivers water is in need of upgrading and this has become



a hot topic in politics in the lead up to the election.

- a. Cut 1 represents the total water that can be diverted directly from the two reservoirs. Determine the capacity of Cut 1. 1 mark

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- b. The pipe connection points **B** and **C** allows water to equalise, so water may flow in either direction along this section of pipe. By analysing the network, determine if the water is more likely to flow towards **B** or towards **C** and give reasons for your answer. 2 marks

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**SECTION B – MODULE 2 – Question 1 - continued**  
**TURN OVER**

- c. Determine the maximum flow of the water network shown above and show the minimum cut clearly on the flow diagram. 2 marks

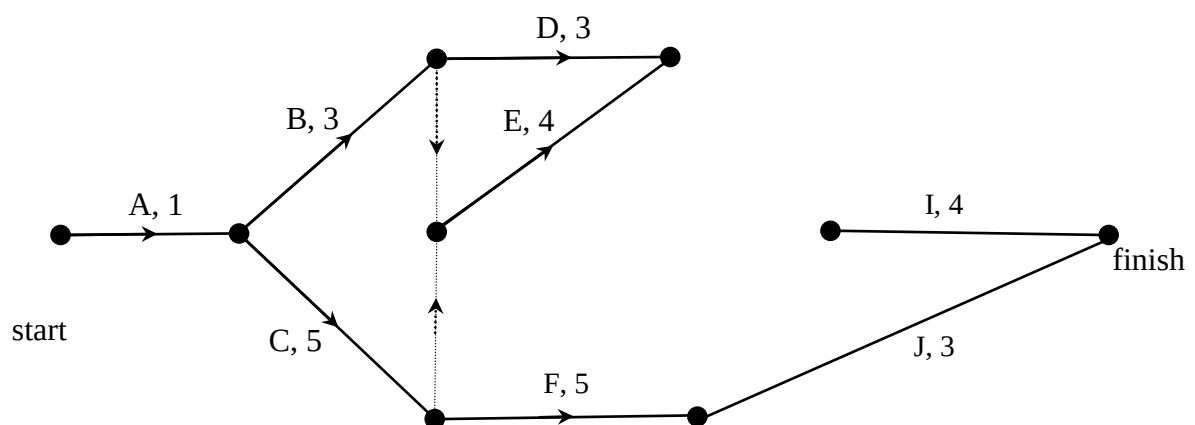
- d. An upgrade is proposed to increase the flow rate in the network above. Which one section of pipe should be upgraded first to give maximum increase in the flow of water to Inverloch? 1 mark

### Question 2 (6 marks)

A live show is to be telecast by a local television station. The entire process of setting up for the telecast involves ten activities, and these activities, together with their immediate predecessors are shown in the following table.

| Activity | Time (hrs) | Immediate predecessors |
|----------|------------|------------------------|
| A        | 1          | –                      |
| B        | 3          | A                      |
| C        | 5          | A                      |
| D        | 3          | B                      |
| E        | 4          | B, C                   |
| F        | 5          | C                      |
| G        | 3          | D, E                   |
| H        | 4          | F                      |
| I        | 4          | G, H                   |
| J        | 3          | F                      |

- a. The activity network has been started for you below. Complete the activity network in the space provided: 1 mark



### SECTION B – MODULE 2 – Question 2 - continued

- b. Determine the minimum time it will take to set up for the telecast and state the critical path. 2 marks

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- c. What is the latest starting time of activity B? 1 mark

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The following activities could be reduced in duration by employing another worker at a cost of \$50 per hour.

| Activity | Original duration | Maximum Reduction | Minimum duration |
|----------|-------------------|-------------------|------------------|
| C        | 5                 | 1.5               | 3.5              |
| E        | 4                 | 1                 | 3                |
| F        | 5                 | 2                 | 3                |
| J        | 3                 | 0.5               | 2.5              |

- d. Determine the minimum **completion time** now possible and identify **which activities** should be reduced and at what **cost**. Below is additional space for you to redraw the activity network. 2 marks

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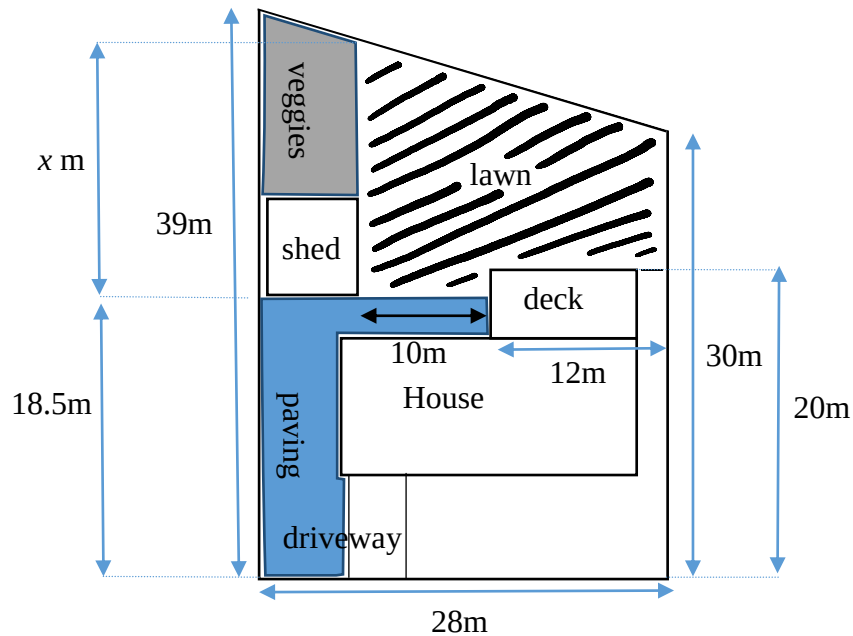


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**END OF MODULE 2 - SECTION B – continued**  
**TURN OVER**

**Module 3 – Geometry and measurement****Question 1** (6 marks)

A house block is represented in the diagram below. A new home has been built, a shed erected, paving laid and a vegetable patch has been planted out. The back yard still needs to be sown with lawn seed.



- a. Calculate the area of the entire house block. 1 mark

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- b. Calculate the length of the distance from the end of the vegetable patch to the end of the shed represented by distance  $x$  on the diagram above. Find the length correct to the nearest 10 centimetres. 2 marks

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**SECTION B – MODULE 3 – Question 1 - continued**

- c. Determine the area to be sown with lawn seed giving your answer in square metres correct to one decimal place. 2 marks

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- d. A packet of lawn seed will cover 50 square metres of ground. How many packets of lawn seed would need to be purchased to sow down the back yard? 1 mark

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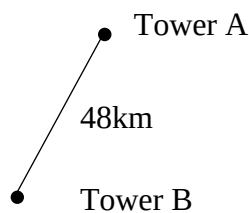
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**Question 2** (4 marks)

Two fire look outs have observed the same plume of smoke rising from a remote area of bush. Tower B is located 48 km from tower A, on a bearing of  $203^\circ\text{T}$ . The observer in tower A reports the direction of the smoke to be  $236^\circ\text{T}$ , while the observer from tower B reports the direction of the smoke to be  $267^\circ\text{T}$ .



- a. On the diagram above, show the position of the smoke in relation to towers A and B. 1 mark

**SECTION B – MODULE 3 – Question 2 - continued**  
**TURN OVER**

- b. Let X represent the location of the fire. Determine the size of the angle  $\angle AXB$  1 mark

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- c. Calculate the distance of towers A and B from the fire. Give answers in kilometres correct to two decimal places 2 marks

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**Question 3** (2 marks)

Melbourne is located at  $38^{\circ}\text{S } 145^{\circ}\text{E}$ . A plane travels 2700 km due west from Melbourne and drops a beacon. Assuming the earth's radius is 6400 km, determine the coordinates of the position of the beacon. 2 marks

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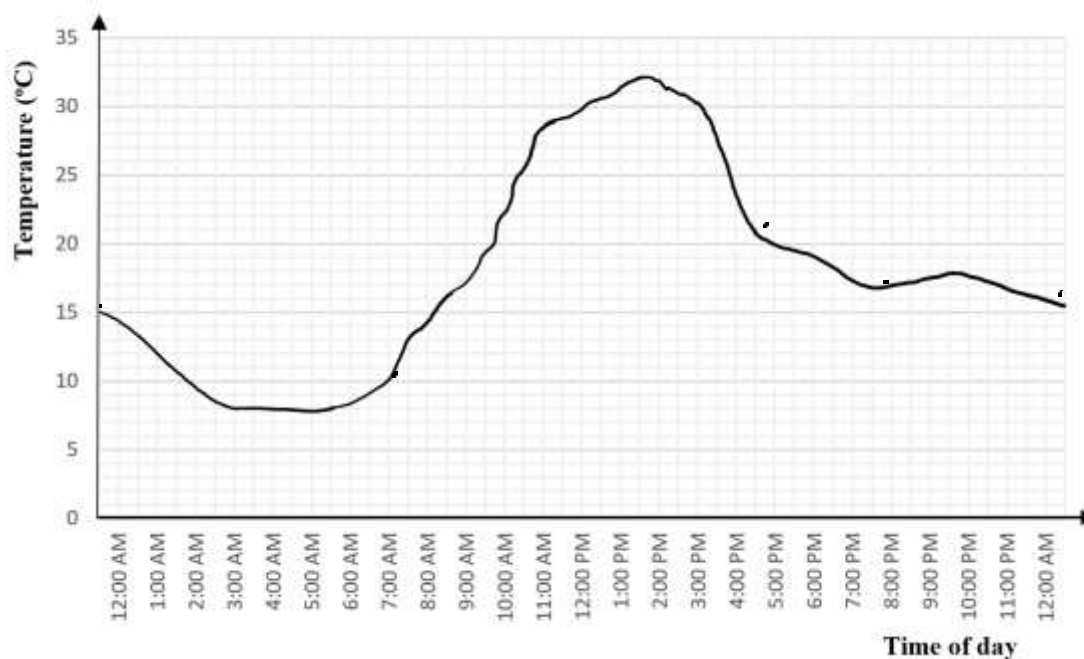
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**END OF MODULE 3 – SECTION B – continued**

**Module 4 – Graphs and relations****Question 1** (4 marks)

The graph above shows the changing temperature on a December day. After very cold overnight temperatures, the morning was overcast until the sun started to break through the cloud cover and temperatures began to rise. In the afternoon a cooler change brought some relief from high temperatures.

- a. What was the lowest temperature recorded over the 24 hour period, and at what time of the day did it first reach this temperature? 2 marks

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- b. How long during the day did the temperature exceed 30°C? 1 mark

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- c. What is the average rate of increase in temperature between 7 am and midday? 1 mark

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**SECTION B – MODULE 4 – continued**  
**TURN OVER**

**Question 2** (4 marks)

Organisers of a handicap event need to calculate starting times for 3 boats so that all three boats will complete a set course at the same time. The slowest boat starts first and the fastest boat starts last. The average speed of the slowest boat is 15 km/hr, the next boat has an average speed of 24 km/hr, and the last boat has an average speed of 27 km/hr.

- a.** How long will it take the third boat to complete a course of 2500 metres? Give your answer to the nearest second. 1 mark

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- b.** How many seconds after the first boat leaves will the third boat need to start if they are to finish at the same time? 1 mark

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- c.** If the race is due to start at 2pm, at what time of the day will the second boat start? 1 mark

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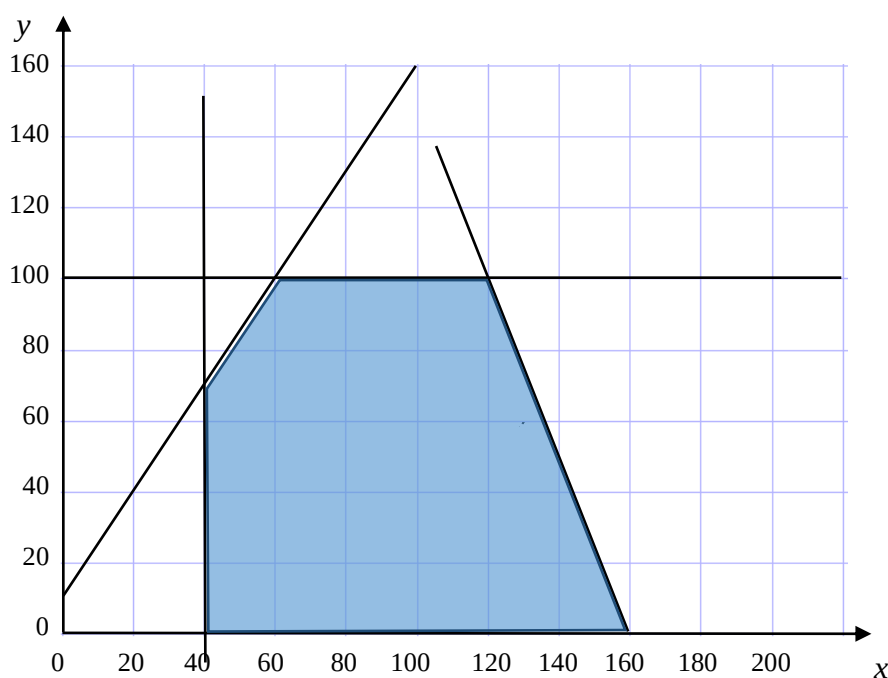
- d.** At what time are the boats due to cross the finish line? 1 mark

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**SECTION B – MODULE 4 – continued**



**Question 3** (4 marks)

The graph above shows the feasible region for a set of four inequalities.

- a.** Write down each of the four inequations. 2 marks

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- b.** The profit function is  $P = 2x + 3y$ . Calculate the values of  $x$  and  $y$  that will maximise the profit function and remain in the feasible region. 2 marks

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**END OF QUESTION AND ANSWER BOOK**