



Trial Examination 2018

VCE Further Mathematics Units 3&4

Written Examination 2

Suggested Solutions

SECTION A – CORE

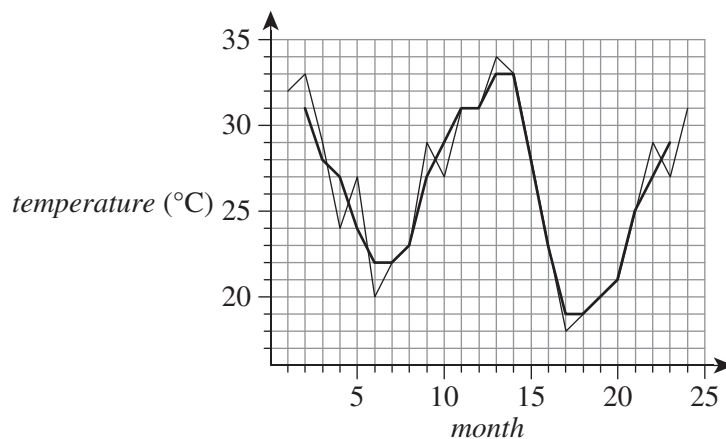
Data analysis

Question 1 (9 marks)

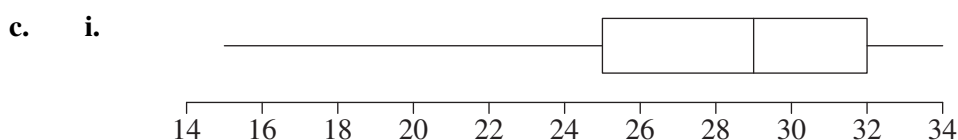
- a. i. $1 + 3 + 7 + 2 + 4 = 17$ months A1
- ii. $10\,000 = 10^4$, so on the log scale we are looking for the number of scores of 4 or more. This score has a frequency of 4 which is a percentage of $\frac{4}{17} \times 100 = 23.5$. Since 23.5 is less than 30 the island is suitable. A1

Note: Demonstration of knowledge of the log scale is required for mark.

- b. i. There is a seasonal pattern. A1
- ii.



correct method used A1
accurate graph A1



five-figure summary accuracy A1
accurate boxplot and scale A1

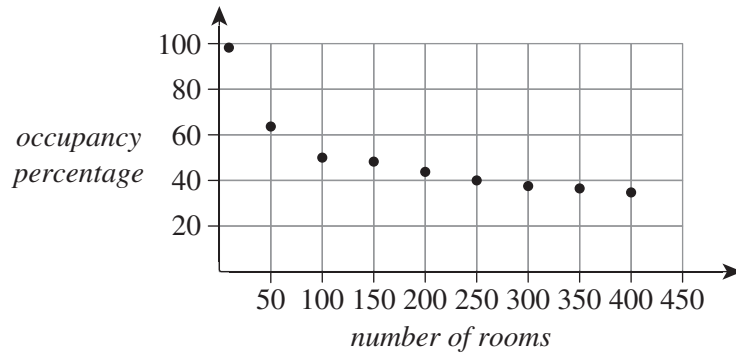
- ii. $IQR = 32 - 25 = 7$, the lower fence will be at $25 - 1.5 \times 7 = 14.5$. A1
- The value of 15 is inside the fence so not an outlier. A1

Question 2 (7 marks)

- a. The **number of rooms**, is the explanatory variable because **the percentage occupancy varies in response**. The occupancy rate does not cause extra rooms to be built or not built. A1

Note: An explanation similar to the above is required for mark.

b.



correct labelling A1

correct scatterplot information A1

- c. There is a strong non-linear trend. A1
- d. correlation coefficient = -0.84 A1
- e. i. correlation coefficient = -0.99 A1
- ii. The data more closely fits a $\log x$ relationship than a linear one. A1

Question 3 (3 marks)

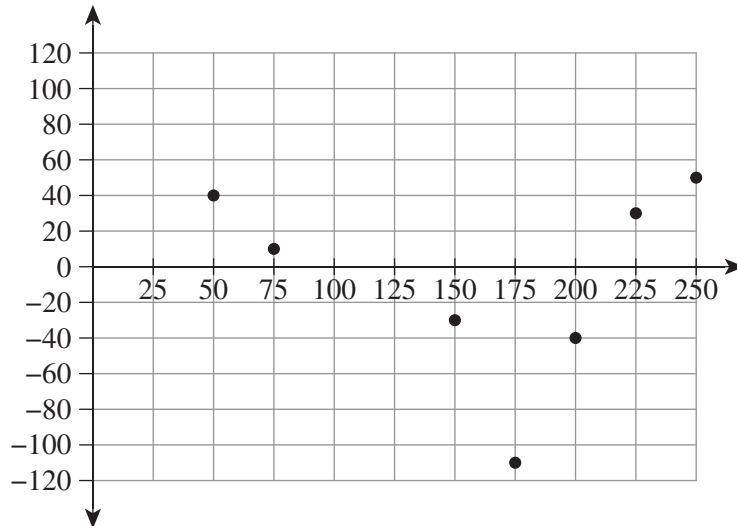
- a. 95% of the data lies within two standard deviations, which leaves 5% outside of this range; 2.5% at each of the higher and lower ends. $125 + 2 \times 40 = 205$.
The occupancy will exceed 205 rooms 2.5% of the time. A1
- b. 85 is one standard deviation below the mean of 125. 68% lies within one standard deviation of the mean (similar to **part a.**), so there will be $\frac{32}{2} = 16\%$ of the data below 85.
 $16\% \times 365 = 58.4$, so 58 days. A1
- c. $z = \frac{x - \mu}{\sigma}$
 $1.8 = \frac{x - 125}{40}$
rearranging gives $x = 197$ A1

Question 4 (5 marks)

- a. Reading from the graph the y-intercept is +100. A1

Using the points (0, 100) and (250, 400) the gradient is $\frac{300}{250} = 1.2$. The line of best fit equation is $y = 1.2x + 100$. A1

- b.



residual plot A1
accuracy A1

- c. There is a pattern in the residuals, so the assumption of a linear trend is not supported. A1

Recursion and financial modelling**Question 5** (3 marks)

- a. Use a calculator with $N = 1$, $I = 5.2\%$ and with the balance remaining at \$7 000 000.

Alternatively solve $A = 7\,000\,000 \left(1 + \frac{5.2}{12} \right)^{12}$.

The monthly interest only payment is \$30 333.33. A1

- b. i. $t_{n+1} = t_n \times 1.15$, $t_0 = 300\,000$ A1

- ii. Applying the regression formula shows that in year four the profit passes \$400 000. A1

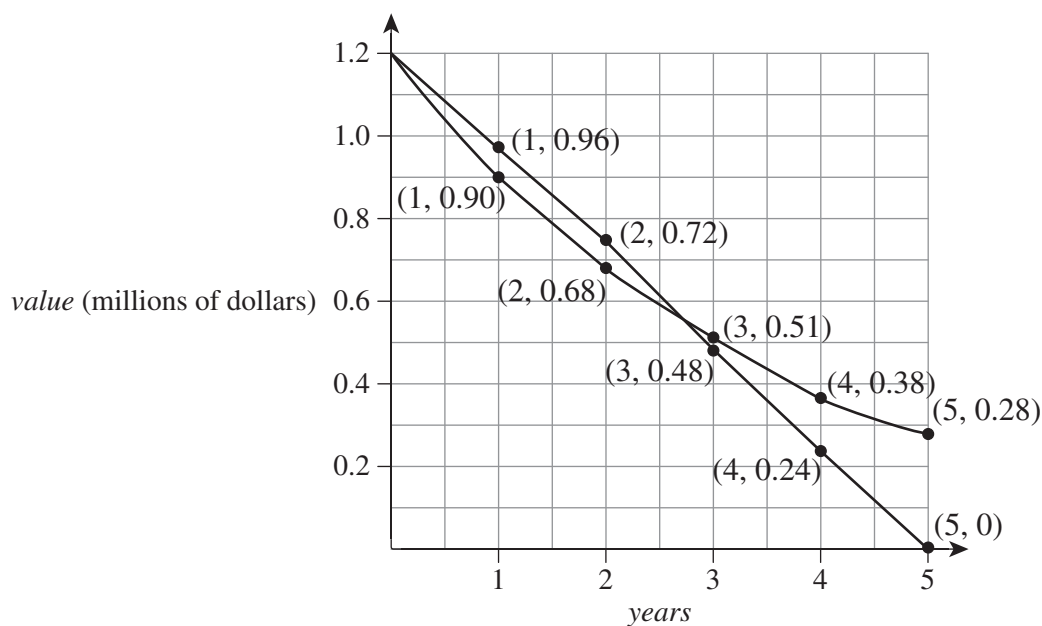
Year 1	t_0	\$300 000
Year 2	t_1	\$345 000
Year 3	t_2	\$396 750
Year 4	t_3	\$456 262.50

Question 6 (4 marks)

- a. The 1.06 factor is $1 + \frac{6}{100}$, so 6%. A1
- b. Using financial functions on your calculator with $N = 12$, amount borrowed is \$650 000, interest is 4.1% with zero payments. The current value is \$677 156.55, so the interest earned is \$27 156.55. A1
- c. Each year the rental figure has increased and the principal upon which the interest is earned has decreased, so the interest will be reduced. A2

*1 mark for each reason given.***Question 7** (5 marks)

a.

*method A1
accuracy A1*

- b. From the graph or otherwise, the flat depreciation model becomes the lower value between years 2 and 3. A1
- c. After five years the scrap value is \$100 000. Since the purchase price was \$1 200 000, the air-conditioning system has lost \$1 100 000 in value over five years or 5 000 000 hours use (based on 1 million hours per year.) A1
- depreciation per hour = $\frac{1\,100\,000}{5\,000\,000} = 22$ cents per hour
- d. After four years of use, the value will be $1\,200\,000 - 4\,000\,000 \times 0.22 = \$320\,000$. A1

SECTION B – MODULES**Module 1 – Matrices****Question 1** (5 marks)

- a. This diagonal represents a player playing against themselves. A1
- b. Both W and Z have a '1' recorded for their game. Since they cannot both win, one of the numbers should be a '0'. A1
- c. $20 + 30 + 20 + 30 = 100$ dresses A1

d.
$$\begin{bmatrix} 8 & 20 & 9 & 100 \\ 10 & 30 & 20 & 140 \\ 20 & 20 & 15 & 150 \\ 10 & 30 & 20 & 180 \end{bmatrix} \begin{bmatrix} 12 \\ 10 \\ 8 \\ 1.20 \end{bmatrix} = \begin{bmatrix} 8 \times 12 + 20 \times 10 + 9 \times 8 + 100 \times 1.20 \\ 10 \times 12 + 30 \times 10 + 20 \times 8 + 140 \times 1.20 \\ 20 \times 12 + 20 \times 10 + 15 \times 8 + 150 \times 1.20 \\ 10 \times 12 + 30 \times 10 + 20 \times 8 + 180 \times 1.20 \end{bmatrix}$$

M1

$$= \begin{bmatrix} 488 \\ 748 \\ 740 \\ 796 \end{bmatrix}$$

A1

= \$2272

Note: If a calculator is used, then the matrices being multiplied must be shown to get full marks.

Question 2 (3 marks)

- a. *For example:*
Commenting upon both reasons is not necessary. The second set of information is a multiple of the first. The inverse of the matrix produced cannot be found. A1
- b.
$$\begin{bmatrix} 3 & 5 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 27 & 500 \\ 22 & 000 \end{bmatrix}$$
 A1
- c. $P_1 = 2500, P_2 = 4000$ A1

Note: Both correct answers required for full marks.

Question 3 (4 marks)

- a. This can be done more efficiently on the calculator. Enter the transition matrix as matrix A and S_1 as matrix B and calculate A^2B .

A1

OR

$$S_2 = \begin{bmatrix} 0.2 & 0.3 & 0.4 \\ 0.3 & 0.4 & 0.4 \\ 0.5 & 0.3 & 0.2 \end{bmatrix} \begin{bmatrix} 850 \\ 1510 \\ 340 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2 \times 850 + 0.3 \times 1510 + 0.4 \times 340 \\ 0.3 \times 850 + 0.4 \times 1510 + 0.4 \times 340 \\ 0.5 \times 850 + 0.3 \times 1510 + 0.2 \times 340 \end{bmatrix}$$

$$= \begin{bmatrix} 759 \\ 995 \\ 946 \end{bmatrix}$$

$$S_3 = \begin{bmatrix} 0.2 & 0.3 & 0.4 \\ 0.3 & 0.4 & 0.4 \\ 0.5 & 0.3 & 0.2 \end{bmatrix} \begin{bmatrix} 759 \\ 995 \\ 946 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2 \times 759 + 0.3 \times 995 + 0.4 \times 946 \\ 0.3 \times 759 + 0.4 \times 995 + 0.4 \times 946 \\ 0.5 \times 759 + 0.3 \times 995 + 0.2 \times 946 \end{bmatrix}$$

$$= \begin{bmatrix} 828.7 \\ 1004.1 \\ 867.2 \end{bmatrix}$$

A1

- b. Find $S_{60} = A^{60}B$ and $S_{61} = A^{61}B$ are both equal to $\begin{bmatrix} 816.8 \\ 998.3 \\ 884.9 \end{bmatrix}$.

A1

Since S_{60} and S_{61} are equal a steady state situation has been reached.

A1

- c. $S_{n+1} = \begin{bmatrix} 0.2 & 0.3 & 0.4 \\ 0.3 & 0.4 & 0.4 \\ 0.5 & 0.3 & 0.2 \end{bmatrix} S_n + \begin{bmatrix} 30 \\ 30 \\ 50 \end{bmatrix}$

A1

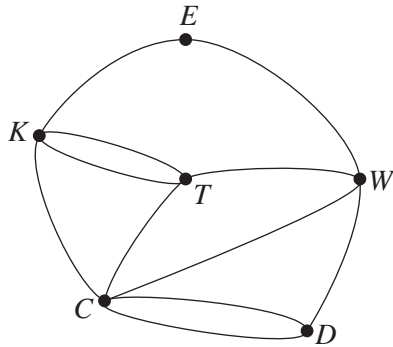
Module 2 – Networks and decision mathematics**Question 1** (2 marks)

- a. echidnas and cassowaries

A1

Note: Students can also select Tasmanian devils and not have marks deducted.

- b. Consider this graph representing the map:



The two pairs are $K-T$ and $C-D$.

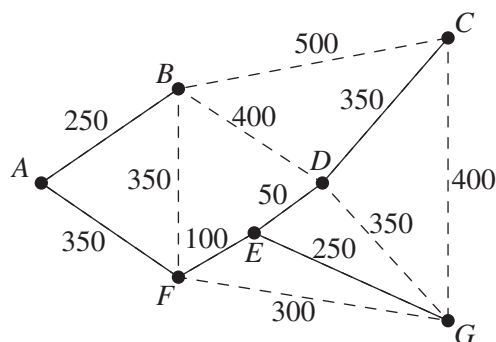
A1

Question 2 (2 marks)

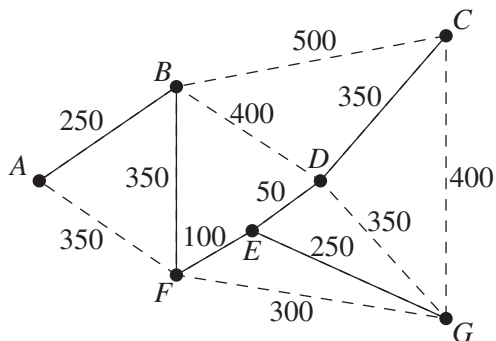
- a. minimum length = $250 + 350 + 100 + 50 + 250 + 350 = 1350$

A1

- b.



A1

OR

A1

Question 3 (3 marks)

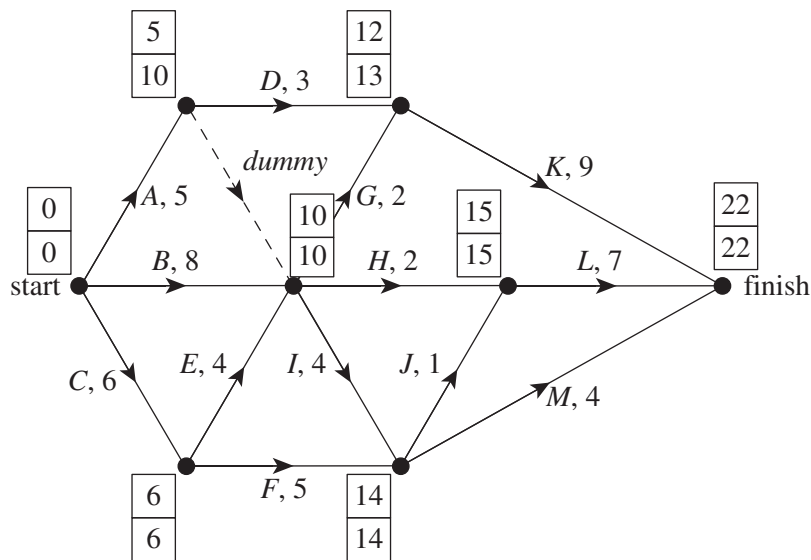
a. 30 A1

b. The number of lines required to cover the '0's in the table is 4. A1

c.

Worker	Task
A	1
B	4
C	3
D	2

A1

Question 4 (5 marks)

(The earliest start time and latest start time at each vertex is shown.)

a. A, B and E A1

b. C-E-I-J-L A1

c. Activity D could have its time extended since it has earliest start time = 5 days and latest start time = 10 days (a float time of 5 days), and similarly activity A with earliest start time = 0 and latest start time = 5. A1

d. i. Reducing C by 2 days would result in C-E-I-J-L remaining a critical path and B-I-J-L becoming a critical path, both with a completion time of 20 days. Reducing L by 2 days would result in C-E-G-K becoming the critical path with a completion time of 21 days. Therefore, the critical paths resulting in the minimum completion times are C-E-I-J-L and B-I-J-L. A1

ii. The completion time would be 20 days. A1

Module 3 – Geometry and measurement**Question 1** (2 marks)

$$\begin{aligned} \text{a.} \quad \frac{3}{10} \times \text{circumference} &= \frac{3}{10} \times \pi \times 9 \\ &= 8.4823 \\ &= 8.48 \text{ cm} \end{aligned}$$

A1

OR

$$\begin{aligned} \frac{4.5 \times \pi}{180} \times 108 &= 8.4823 \\ &= 8.48 \text{ cm} \end{aligned}$$

A1

$$\begin{aligned} \text{b.} \quad \frac{1}{3} \times \pi \times (4.5)^2 \times 21 &= 445.321 \\ &= 445.32 \text{ cm}^3 \end{aligned}$$

A1

Question 2 (2 marks)

Use Pythagoras to find the base length of the right-angled triangle with AB as the hypotenuse:

$$\sqrt{10^2 - 8^2} = 6$$

M1

Create a right-angled triangle using CD as the hypotenuse.

The base length is $27 - (18 + 6) = 3$

Using trigonometry (adjacent = 3, opposite = 8):

$$\tan(x) = \frac{8}{3}$$

$$\begin{aligned} \tan^{-1}\left(\frac{8}{3}\right) &= 69.444 \\ &= 69.4^\circ \end{aligned}$$

A1

Question 3 (3 marks)

$$\text{a.} \quad \text{area scale factor} = 2$$

$$\text{length scale factor} = \sqrt{2}$$

$$\text{volume scale factor} = (\sqrt{2})^3$$

M1

$$(\sqrt{2})^3 = 2.82843$$

$$30 \times 2.8284271247462 = 84.8528$$

$$= 85.0 \text{ L}$$

A1

$$\text{b.} \quad h = 60$$

$$r = 15$$

$$2 \times \pi \times 15^2 + 2 \times \pi \times 15 \times 60 = 7068.58$$

$$= 7068.6 \text{ cm}^2$$

A1

Question 4 (3 marks)

a. Use cosine rule:

$$\sqrt{7^2 + 6^2 - 2 \times 7 \times 6 \times \cos(70)} = 7.50135$$

$$= 7.5 \text{ km}$$

A1

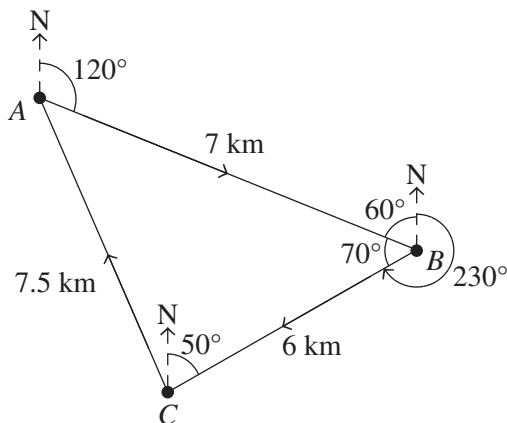
b. Find the angle at point C using sine rule:

$$\frac{7 \times \sin(70)}{7.5} = 0.877046$$

$$\sin(0.87704644606683) = 61.2881$$

M1

Find the bearing:



Using the north direction as a parallel lines and then using co-interior angles in parallel lines summing to 180.

$$60 + 70 = 130$$

$$180 - 130 = 50$$

Subtract 50 from the angle at C to get the anti-clockwise angle.

$$61.288104878574 - 50 = 11.2881$$

Subtract this value from 360 to get clockwise bearing.

$$360 - 11.288104878574 = 348.712$$

$$= 349^\circ$$

A1

Question 5 (2 marks)Find the radius at 38° S using trigonometry:

$$6400 \times \cos(38) = 5043.27$$

M1

Find the length of the arc:

$$174 - 144 = 30$$

$$\frac{5043.268823083 \times 30 \times \pi}{180} = 2640.65$$

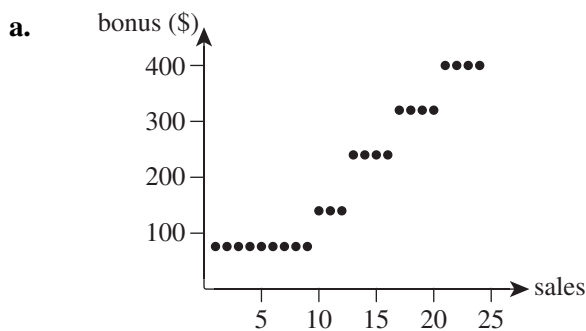
$$= 2640 \text{ km}$$

A1

Module 4 – Graphs and relations**Question 1** (3 marks)

a. $\frac{50}{1.6} = 31.25 \text{ km/h}$ A1

b. John = $10 + 17t$
 Priti = $26t$
 Solve $10 + 17t = 26t$ for t :
 $t = 1.11111$ A1
 $60 \times 1.11111 = 66.66667$
 Therefore 67 minutes. A1

Question 2 (4 marks)

A1

- b. Divide bonus amount by 15 to find an integer solution as sales need to be a whole number.

$$\frac{80}{15} = 5.33$$

$$\frac{140}{15} = 9.33$$

$$\frac{240}{15} = 16$$

$$\frac{320}{15} = 21.33$$

$$\frac{400}{15} = 26.67$$

The answer will therefore be 16 sales.

A1

c. $800 + 300(11) = \$4100$ A1

d. Solve $800 + 300n = 500n$ for n .
 $n = 4$ weeks A1

Question 3 (5 marks)

a. *For example:*

The number of freezer units produced must be no more than one-third of the number of fridge units produced.

A1

b. $y \geq 10$

A1

c. Maximum profit occurs at (90, 30).

$$130 \times 90 + 150 \times 30 = \$16\,200$$

A1

d. Use the sliding rule concept:

Maximum profit now occurs at (100, 20).

The profit line equation has the same gradient (−1).

Therefore, m and n are equal.

$$Q = 100m + 20n$$

$$Q = 120m$$

$$120m = 18\,000$$

M1

$$m = \frac{18\,000}{120}$$

$$m = n = 150$$

A1