

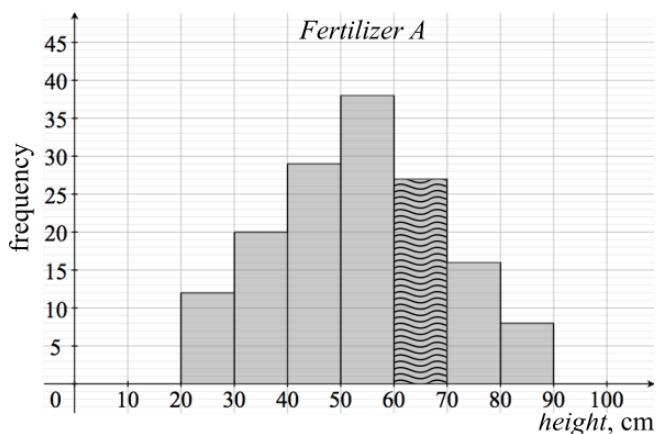
FURTHER MATHEMATICS – UNITS 3&4
2018
Written examination 2 Solutions

SECTION A – Core

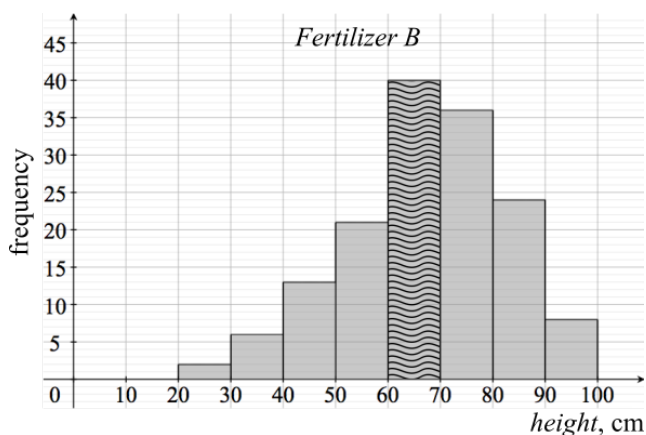
Data analysis

Question 1 (9 marks)

a.



1A



1A

b. Modal class = 50 –< 60 cm

1A

c. Negatively skewed.

1A

d.
$$\text{mean} = \frac{\sum(xf)}{n}$$

$$= \frac{2 \times 25 + 6 \times 35 + 13 \times 45 + 21 \times 55 + 40 \times 65 + 36 \times 75 + 24 \times 85 + 8 \times 95}{150}$$

$$= 67.33$$

$$\approx 67 \text{ cm}$$

1M

1A

e. i. $53.5 + 15.8 = 69.3 \text{ cm} \Rightarrow$ above one standard deviation to the right of the mean: $\frac{100 - 68}{2} = 16\%$

1A

ii. $53.5 - 2 \times 15.8 = 21.9 \text{ cm} \Rightarrow$ below two standard deviations to the left of the mean: $\frac{100-95}{2} = 2.5\%$ **1A**

f. $z\text{-score} = \frac{x - \mu}{\sigma}$
 $= \frac{26.6 - 53.5}{15.8}$
 $= -1.70$

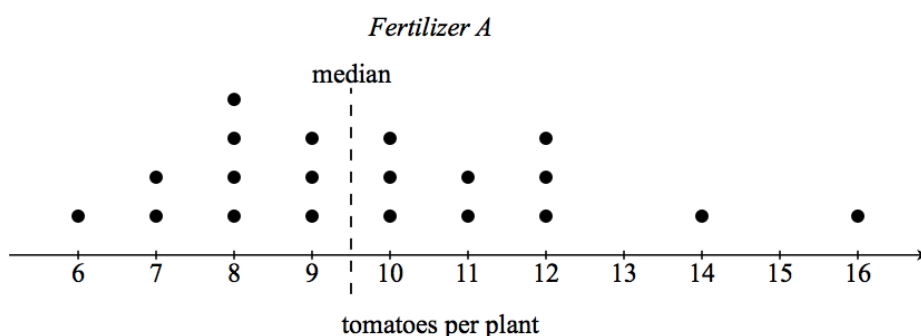
A height of 26.6 cm is 1.7 standard deviations to the left of the mean. **1A**

Question 2 (5 marks)

a. The variable *Type of fertilizer* (*Fertilizer A* and *Fertilizer B*) is categorical nominal. **1A**

The variable *Number of tomatoes per plant* is numerical discrete. **1A**

b.



median = $\frac{9+10}{2} = 9.5$ **1A**

c. $IQR = Q_3 - Q_1$
 $= 14 - 11$
 $= 3$ **1A**

Lower fence = $11 - 1.5 \times 3$
 $= 11 - 4.5$
 $= 6.5 \Rightarrow 6 < 6.5$, hence it is an outlier ... as required **1A**

Question 3 (5 marks)

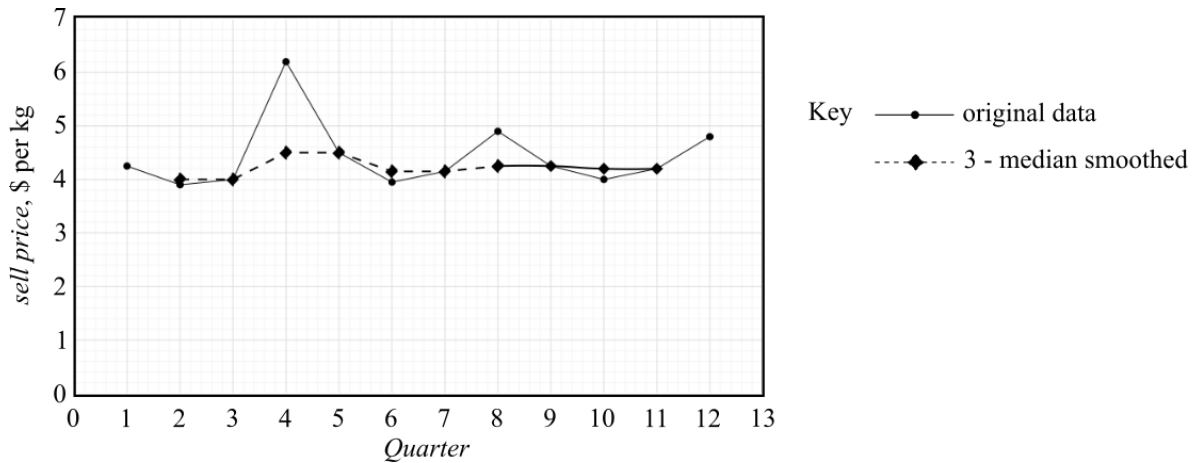
a. Using available CAS technology, use the *diameter* as the explanatory variable and *weight* as the response variable.

$weight = 36.9 + 12.3 \times diameter$ **2A**

b. 86.6% of the variation in the *weight* of tomatoes can be explained by the variation in the *diameter* of tomatoes. **1A**

c. Substitute $diameter = 5.4$ into the equation $weight = 42.6 + 11.1 \times diameter$.
 $weight = 42.6 + 11.1 \times 5.4$
 $= 102.54 \text{ grams}$ **1A**

d. Residual = actual – predicted
 $= 98 - 102.54$
 $= -4.54$ **1A**

Question 4 (5 marks)**a.** \$4.25**1A****b.****1A****c.**

<i>Year</i>	<i>Season</i>	<i>Quarter</i>	<i>Price, \$</i>	<i>4-mean smoothing</i>	<i>Centring</i>
2016	Spring	1	4.25		
	Summer	2	3.90		
	Autumn	3	4.00	4.5875	4.61875
	Winter	4	6.20	4.65	
2017	Spring	5	4.50		
	Summer	6	3.95		
	Autumn	7	4.15		
	Winter	8	4.90		
2018	Spring	9	4.25		
	Summer	10	4.00		
	Autumn	11	4.20		
	Winter	12	4.80		

\$4.62

1A**d.** $0.98 + S + 0.93 + 1.20 = 4 \Rightarrow S = 0.89$ **1A**

e. Re-seasonalised data = predicted (de-seasonalised) $\times$ seasonal index
 $= 5.10 \times 0.98$
 $= \$4.998$
 $\approx \$5$

1A

Recursion and financial modelling

Question 5 (6 marks)

a. $d = \frac{h_4 - h_0}{4} = \frac{470 - 350}{4}$

$$d = 30 \text{ mm}$$

1A

b. $h_{n+1} = h_n + 30$

1A

c. Substitute h_0 and h_1 into $h_1 = ah_0 + b \Rightarrow 440 = 500a + b \dots [1]$

1M

Substitute h_1 and h_2 into $h_2 = ah_1 + b \Rightarrow 392 = 440a + b \dots [2]$

Use available CAS technology to solve the system of simultaneous equations [1] and [2].

$$a = 0.8 \text{ and } b = 40$$

$$h_{n+1} = 0.8h_n + 40 \dots \text{as required}$$

1A

Alternative 'by hand' method

Subtract [1] – [2]: $48 = 60a \Rightarrow a = \frac{48}{60} = 0.8$

Substitute $a = 0.8$ into [1]: $440 = 500 \times 0.8 + b \Rightarrow b = 440 - 400 = 40$

$$h_{n+1} = 0.8h_n + 40 \dots \text{as required}$$

d. The 4th picket has height $h_3 = 0.8h_2 + 40$.

$$h_3 = 0.8 \times 392 + 40$$

$$= 353.6 \text{ mm}$$

1A

The 5th picket has height $h_4 = 0.8h_3 + 40$.

$$h_4 = 0.8 \times 353.6 + 40$$

$$= 322.88$$

$$\approx 323 \text{ mm}$$

1A

Question 6 (3 marks)

a. $26000 - 16250 = \$9750$

1A

b. $16250 \div 5 = \$3250 \text{ per year} \Rightarrow \text{Depreciation per unit} = \frac{3250}{15000}$
 $= \$0.217$
 $\approx \$0.22$

1A

c. $26000 \div 3250 = 8 \text{ years}$

$$8 - 5 = 3 \text{ years}$$

1A

Question 7 (3 marks)

a. Using finance solver on available CAS technology

$$N: 36$$

$$I(\%): 6.2$$

$$PV: -14000$$

$$Pmt: -100$$

$$FV: 20800$$

$$PpY: 12$$

$$CpY: 12$$

1A

- b. account balance = $20800 \left(1 + \frac{8.5}{400} \right) + 300$ or account balance = $20800 \times 1.02125 + 300$ 1A
- c. Using finance solver on available CAS technology
 N: 1
 I(%): 8.5
 PV: -20800
 Pmt: -300
FV: 21542
 PpY: 4
 CpY: 4 1A

SECTION B – Applications

Module 1 – Matrices

Question 1 (5 marks)

- a.
- $$W_1 = \begin{bmatrix} & T & U & V \\ 12.50 & \underline{13.75} & \underline{17.20} & \end{bmatrix} \quad 1A$$
- $$W_2 = \begin{bmatrix} & T & U & V \\ \underline{18.00} & 21.50 & \underline{25.60} & \end{bmatrix} \quad 1A$$
- b. Order = 1×5 1A
- c. $W_2 R = \begin{bmatrix} 188.05 & 251.40 \end{bmatrix}$ 1A
- d. \$188.5 = total amount of money earned by the three shop assistants on Saturday
 \$251.40 = total amount of money earned by the three shop assistants on Sunday 1A

Question 2 (4 marks)

- a. $22 + 106 + 30 = 158$ items 1A
- b. $\det(M) = 2622 \neq 0 \Rightarrow$ the matrix is invertible 1A
- c. $\begin{bmatrix} c \\ s \\ j \end{bmatrix} = \begin{bmatrix} \underline{17} & \underline{-10} & \underline{-3} \\ \underline{-13} & \underline{8} & \underline{2} \\ \underline{21} & \underline{-13} & \underline{-3} \end{bmatrix} \times \begin{bmatrix} \underline{1127} \\ \underline{1517} \\ \underline{1287} \end{bmatrix}$ 1A
- d. Using available CAS technology solve the matrix equation from **part c.** for c , s and j .
 $c = 128$, $s = 59$ and $j = 85$ 1A

Question 3 (3 marks)

- a. $\begin{bmatrix} 0.32 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.10 \end{bmatrix} \times \begin{bmatrix} 200 \\ 380 \\ 160 \end{bmatrix} = \begin{bmatrix} 64 \\ 19 \\ 16 \end{bmatrix} \Rightarrow 19 \text{ shirts}$ 1A

$$\text{b. } B = \begin{bmatrix} 200 \\ 380 \\ 160 \end{bmatrix} - \begin{bmatrix} 64 \\ 19 \\ 16 \end{bmatrix} = \begin{bmatrix} 136 \\ 361 \\ 144 \end{bmatrix}$$

1A

$$\begin{aligned} \text{c. } S_1 &= TS_0 + B \\ S_2 &= TS_1 + B \\ &= T(TS_0 + B) + B \\ &= T^2S_0 + TB + B \end{aligned}$$

1A

Module 2 – Networks and decision mathematics

Question 1 (3 marks)

a.

	Leona	Max	Natalie	Oliver
Route A	0	1	3	5
Route B	4	0	0	1
Route C	2	0	0	0
Route D	0	0	1	4

The minimum number of lines that cover the 0s is four, which is equal to the number of rows.

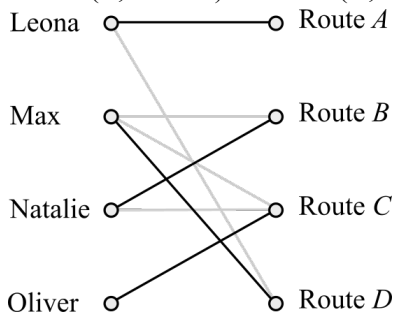
1A

- b. Oliver can be assigned a route first because there is only one 0 in his column.
If the lines are crossed horizontally, then Leona can be assigned a route first because there is only one 0 in her row.

1A

- c. Leona (A; 10 min) + Oliver (C; 9 min) + Natalie (B; 12 min) + Max (D; 12 min) = 43 min

1A

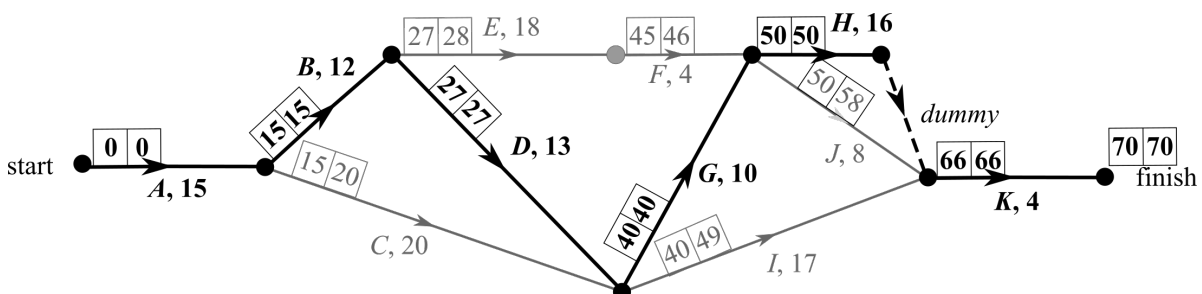


Question 2 (5 marks)

- a. The *dummy* activity is introduced to clearly show the precedence of activity H to activity K.
- b. The critical path includes activities A, B, D, G, H, *dummy* and K.

1A

1A



The completion time is 70 minutes or 1 hour and 10 minutes. Therefore the organisers will be ready to start the race as planned since they have 1 hour and 20 minutes available.

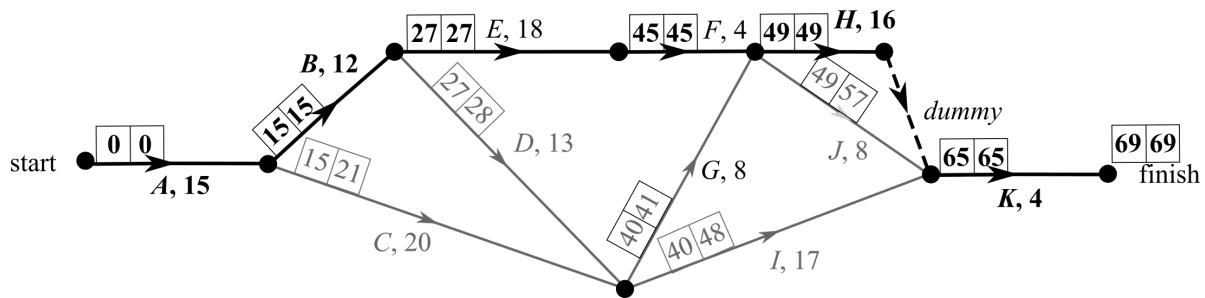
1A

c. Activities *E* and *F* have a float time of 1 minute.

1A

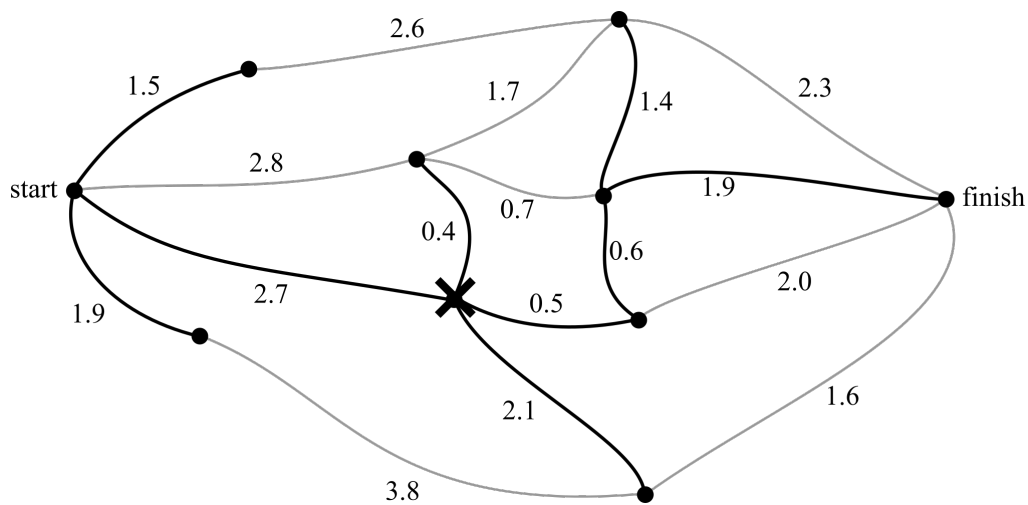
d. Reduces the completion time by 1 minute and the critical path to *A, B, E, F, H, dummy* and *K*.

1A



Question 3 (4 marks)

a.



2A

b. No. The minimum spanning tree should have edge 1.6 instead of edge 2.1.

1A

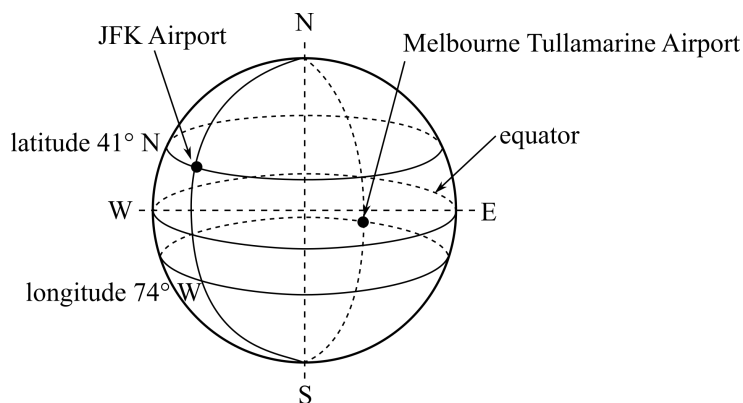
c. Hamiltonian circuit.

1A

Module 3 – Geometry and trigonometry

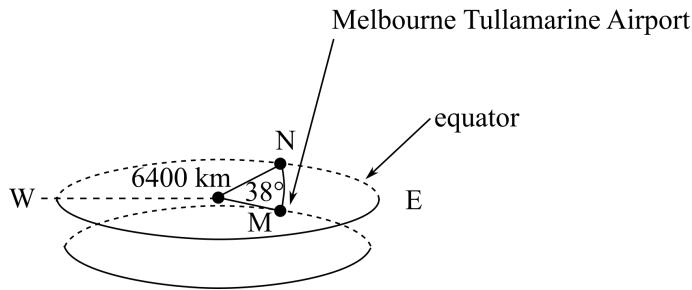
Question 1 (5 marks)

a.



2A

b.



$$\begin{aligned}
 \text{Arc length } MN &= 2\pi r \frac{\theta^\circ}{360^\circ} \\
 &= 2\pi \times 6400 \times \frac{38^\circ}{360^\circ} \\
 &= 4244.64 \text{ km}
 \end{aligned}$$

1A

- c. Difference in longitude = $145^\circ - (-74^\circ)$
 $= 219^\circ$

$$1 \text{ hour} \approx 15^\circ \Rightarrow \frac{219^\circ}{15^\circ} = 14.6 \approx 15 \text{ hours}$$

1A

- d. Melbourne is about 15 hours ahead of New York. This means that when Corrine's flight leaves Melbourne, the time in New York is 11:15 am – 15 hours = 8:15 pm on the 20th of December. From 8:15 pm on the 20th of December to 4:30 pm on the 21th of December there are 20 hours 15 minutes. Flight time = 20 hours 15 minutes

1A

Question 2 (4 marks)

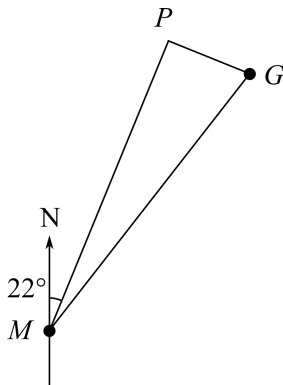
- a. $\theta = 360^\circ - 65^\circ = 295^\circ\text{T}$

1A

- b. Using Pythagoras theorem, $MG = \sqrt{1.2^2 + 0.24^2} = 1.22 \text{ km}$

1A

c.



$$\angle PMG = \tan^{-1}\left(\frac{0.24}{1.20}\right) = 11^\circ$$

1A

$$\text{Bearing} = 22^\circ + 11^\circ = \text{N}33^\circ\text{E}$$

1A

Question 3 (3 marks)

- a. $\text{Area}_{\text{large sector}} = \frac{104^\circ}{360^\circ} \times \pi \times 30^2 = 816.814 \text{ m}^2$

$$\text{Area}_{\text{small sector}} = \frac{104^\circ}{360^\circ} \times \pi \times 16^2 = 232.338 \text{ m}^2$$

$$\begin{aligned}
 \text{Area} &= \text{Area}_{\text{large sector}} - \text{Area}_{\text{small sector}} \\
 &= 816.814 - 232.338 \\
 &= 584.476 \text{ m}^2
 \end{aligned}$$

1A

$$\begin{aligned}
 \text{Volume} &= 584.476 \times 0.1 \\
 &= 58.4476 \\
 &\approx 58 \text{ m}^3
 \end{aligned}$$

1A

b. Arc length = $\frac{\theta^\circ}{360^\circ} \times 2\pi \times \text{radius}$

$$\theta^\circ = \frac{360^\circ \times \text{Arc length}}{2\pi \times \text{radius}}$$

$$= \frac{360^\circ \times 9}{2\pi \times 65}$$

$$= 7.93$$

$$\approx 8^\circ$$

1A

Module 4 – Graphs and relations

Question 1 (4 marks)

a. Time = 1 hour and 36 minutes = 1.6 hours

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$= \frac{120}{1.6}$$

$$= 75 \text{ kmh}^{-1}$$

1A

b. The volume of petrol per kilometre = $\frac{10-5}{0-50}$

$$= -0.1$$

$$= \text{decrease of } 0.1 \text{ L per km}$$

1A

c. Before 50 km, from **part b.**, the change in the volume of petrol per kilometre = -0.1

$$\text{After 50 km, the change in the volume of petrol per kilometre} = \frac{25-20}{50-120}$$

$$= -0.07 \text{ L per km}$$

The car used more petrol per kilometre in the first 50 km.

1A

d. The tank was filled with petrol from 5 L to 25 L.

1A

Question 2 (4 marks)

a. Substitute $t = 1$ and $d = 3.5$ into $d = kt^2$.

$$3.5 = k \times 1^2$$

$$k = 3.5$$

1A

b. Average speed = $\frac{87.5-0}{5-0}$

$$= 17.5 \text{ ms}^{-1}$$

1A

c. $56 = 3.5 \times t^2$

$$t^2 = \frac{56}{3.5}$$

$$= 16$$

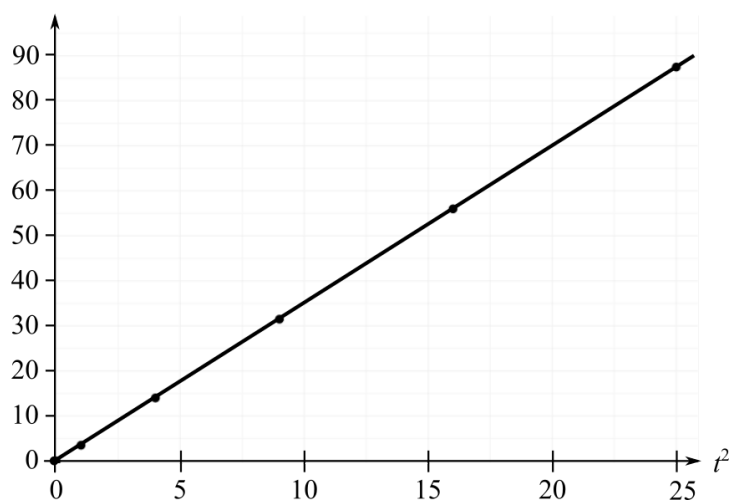
$$t = \sqrt{16}$$

$$= 4 \text{ s}$$

1A

d.

distance, m



1A

Question 3 (4 marks)

a. Revenue = $300 \times 150 + 500 \times 50 = \70000

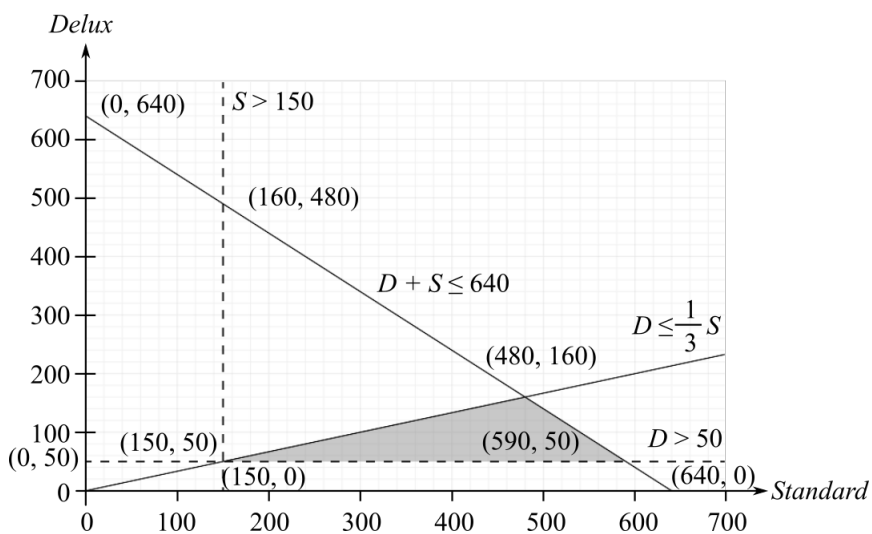
Cost = $45000 + 120 \times 150 + 140 \times 50 = \70000

Cost = Revenue for 150 *standard* snowboards per day and 50 *delux* snowboards per day as required. 1M&A

b. $S \geq 3D$

1A

c.



2A