

FURTHER MATHEMATICS

Units 3 & 4 – Written examination 1



2017 Trial Examination

SOLUTIONS

SECTION A: Core Multiple-choice questions (1 mark each)

Core: Data Analysis

Question 1.

Answer: **D**

Explanation:

- Apartment/Flat/Unit number – Although a number, it gives location which is **Categorical - ordinal**
- Street number, - once again gives location so **categorical - ordinal**
- Street name, - **categorical nominal**
- Suburb/locality, - **categorical nominal**
- State/territory, - **categorical nominal**
- Postcode, - another number which gives location, so **categorical nominal**
- Property/building name (if any). – **categorical nominal**

So there are 0 numerical variables and 5 nominal.

Question 2.

Answer: **B**

Explanation: $Q1 = 3.3 \text{ kg}$ $Q3 = \frac{3.9 + 4.1}{2} = 4.0 \text{ kg}$, so $IQR = 4.0 - 3.3 = 0.7$

Question 3.

Answer: **E**

Explanation: When considering shape, outliers need to be ignored. The data is symmetrical and it is most likely that premature births have meant that some birth weights are much lower than the normal.

Question 4.

Answer: **C**

Explanation: lower fence $= 3.3 - 1.5 \times 0.7 = 2.25$ upper fence $= 4.0 + 1.5 \times 0.7 = 5.05$ Since two of the birth weights are below 2.25 kg, and none above 5.05 kg, there are 2 outliers in this data set. Or sketch on CAS

Question 5.

Answer: **C**

Explanation: There are 4 babies with weights greater than 4.5 kg $\frac{4}{49} \times 100 = 8.2\%$

Question 6.**Answer: D**

Explanation: The graph only shows percentages, so without further information about the actual numbers of indigenous and non-indigenous women, the truth of A and E cannot be known. (In fact, in Australia, indigenous women are far fewer in number than non-indigenous, so there are actually many more non-indigenous females in this age bracket and A and E would both be false). Median and mean age of indigenous women is lower than for non-indigenous, thus answer D is correct.

Question 7.**Answer: E**

Explanation: 45 years is three standard deviations above the mean. This means that 0.15% of births are to women above 45 years of age. Since 2,538,700 births occurred over 10 years, the number of births for 1 year would be approximately 253,870. The number of mothers aged over 45 years giving birth would be $\frac{0.15}{100} \times 253870 = 380.8$ which is closest to 400, thus answer E.

Question 8.**Answer: A**

Explanation: Data needs to be entered into CAS and regression line calculated.

Stat	Value
"Title"	"Linear Regression (a+bx)"
"RegEqn"	"a+b·x"
"a"	47.6692
"b"	2.56264
"r²"	0.968138
"r"	0.98394
"Resid"	"{"...}"

Question 9.**Answer: D**

Explanation: From the calculation made for the previous question, we can see the product moment correlation coefficient, r , is 0.98394 which rounds to 0.984, answer D

Question 10.**Answer: B**

Explanation: 1965 will have a year value of 8.5, so
female life expectancy = $51.21 + 2.76 \times 8.5 = 74.67$

Question 11.**Answer: B**

Explanation: The data for female life expectancy needs to be entered on the CAS and the result is shown here:

LinRegBx *year,female,1: CopyVar stat.RegEqn*

"Title"	"Linear Regression (a+bx)"
"RegEqn"	"a+b·x"
"a"	51.2077
"b"	2.76044
"r ² "	0.968502
"r"	0.984125
"Resid"	"{...}"

Question 12.**Answer: E**

Explanation: Seasonal Index is calculated by taking historical data and finding the average “mean maximum temperature” and then dividing September’s mean by the yearly average.

$$\frac{21.4 + 21.6 + 20.3 + 18.0 + 15.6 + 13.7 + 13.0 + 13.8 + 15.2 + 17.0 + 18.3 + 19.9}{12} = 17.3167$$

$$\frac{15.2}{17.3167} = 0.8777$$

This answer rounds to 0.88.

Question 13.**Answer: D**

Explanation: Desasonalised temperature = $\frac{\text{actual temperature}}{\text{seasonal index}} = \frac{20.3}{1.17} = 17.3504$

This answer rounds to 17.4°C

Question 14.**Answer: D**

Explanation: Deseasonalised historic mean = $\frac{18.3}{1.06} = 17.264$

Mean max temp in Nov 2016 = $17.264 + 0.34 = 17.6^\circ\text{C}$

Question 15.**Answer: C**

Explanation: Care must be taken as this equation is a log transformation of population. Easiest way is to use solve on CAS as shown. The *year* is the number of centuries since 1500. $2020 - 1500 = 520$ years, which is 5.2 centuries.

$\text{solve}\left(\log_{10}(p) = 2.239 + 0.289 \cdot 5.2, p\right)$
 $p = 5518.23$

Question 16.

Answer: A

Explanation: $1980 - 1500 = 480$ years, or 4.8 centuries.
Calculate population predicted by rule, then find residual by calculating actual – predicted.

$$\text{solve}\left(\log_{10}(p) = 2.239 + 0.289 \cdot 4.8, p\right)$$

$$p = 4228.63$$

$4440 - 4228.63$	211.37
------------------	----------

Core: Recursion and financial modelling**Question 17.****Answer: B***Explanation:* Find 101.3% of the current price to determine the new price.

$$1180 \times \frac{101.3}{100} = \$1195.34$$

Question 18.**Answer: E**

Explanation: Using finance solver on CAS is the quickest and most efficient way. N is 160 days, PV should be entered as a negative since you are giving money to the bank, I is 3.5%, and Ppy/Cpy is 365 as the calculations are being made daily. FV returns a value of \$15231.9, which is closest to \$15232, answer E

Finance Solver	
N:	160
I(%):	3.5
PV:	-15000
Pmt:	0
FV:	15231.900280321
PpY:	365

Question 19.**Answer: C**

Explanation: The recurrence relations given here are either geometric or arithmetic, so we need to determine if the value is reducing by a constant amount or not. $\$2200 - \$1980 = \$220$, $\$1980 - \$1760 = \$220$, thus this is a flat rate reduction in value, thus answer C

Question 20.**Answer: E**

Explanation: To determine how much is paid in total, we need to know how much the repayments are. Finance solver is the quickest way of determining this. The amount “borrowed” is \$6900 less the deposit \$500, so \$6400 is borrowed, so PV is 6400. Repayments are \$283.65 each. Total paid is $283.65 \times 24 + 500 = 7307.6$, so total paid is closest to \$7308

Finance Solver	
N:	24
I(%):	6
PV:	6400
Pmt:	-283.65190561765
FV:	0
PpY:	12

Question 21.**Answer: B**

Explanation: The effective interest rate formula from the formula sheet is required here for converting a compounding interest rate to effective interest rate, or the CAS eff(function can be used as shown here. This function requires the input of compounding interest rate and the number of calculations per year.

$$\left(\left(1 + \frac{6}{100 \cdot 12} \right)^{12} - 1 \right) \cdot 100$$

6.16778

eff(6,12) 6.16778

Question 22.**Answer: B**

Explanation: If this was a perpetuity then $V_{n+1} = V_n$ so not A. The same is true for an interest only loan as the amount borrowed will remain after each payment, so not C. Reducing balance depreciation will not have a linear reduction component as these are geometric recursion equations where the previous value is multiplied by a number smaller than 1. This must be a reducing balance loan. If interest is compounding monthly then V_n would be multiplied by $1 + \frac{r}{12 \times 100}$ so the correct answer is B since

$$1 + \frac{8.4}{4 \times 100} = 1.021$$

Question 23.**Answer: A**

Explanation: Finance solver again. Use PV as -11183.75, since money is going into the bank and FV as 1925.67 and N as 12 since these values are 1 year apart. So answer is A \$800 repayments.

Finance Solver	
N:	12
I(%):	4.9
PV:	-11183.75
Pmt:	799.99986732246
FV:	1925.67
PpY:	12

Question 24.**Answer: C**

Explanation: The initial investment can be found by finding PV, given FV is either 11183.75 with N of 12 months or, FV is 1925.67 with N of 24 months. Each will give the same answer.

Finance Solver	
N:	12
I(%):	4.9
PV:	-20000.000178584
Pmt:	800
FV:	11183.75
PpY:	12

SECTION B: Modules Multiple-choice questions (1 mark each)**Module 1 - Matrices****Question 1.**

Answer: **B**

Explanation: If $A \times B \times C$ is defined and B is 2×3 , then A must be a $? \times 2$ matrix. Similarly if $C \times A \times B$ is defined then C must be a $3 \times ?$, so there must be the same number of columns in C as there are rows in A . Answer B is correct because $A \times B \times C$ will be $? \times 2 \times 2 \times 3 \times 3 \times ?$ resulting in a $? \times ?$ dimensional matrix, so must be a square matrix.

Question 2.

Answer: **B**

Explanation: multiplying these matrices can be done on CAS, but knowing that the row matrix will be multiplied by each column, you can determine the result simply by hand.

$$[s \ t \ e \ m] \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad [e \ s \ m \ t]$$

Question 3.

Answer: **E**

Explanation: To get the total number of jumpers, blazers and shirts you need to multiply the information matrix by the column matrix to sum the rows. Thus $S \times C$ which means the answer will be either C or E. $S \times C \times P$ would result in a $3 \times 3 \times 3 \times 1 \times 1 \times 3 = 3 \times 3$ matrix. To get the total value of all stock we require a 1×1 answer. $P \times S \times C$ gives the 1×1 matrix we require, thus answer E.

Question 4.

Answer: **C**

Explanation: Simply take away the percentage of bicycles going from C to A and B.
 $100\% - 25\% - 9\% = 66\%$

Question 5.

Answer: **E**

Explanation: Remember that “from” goes across the top and “to” goes down the side. The columns should also add to 1 in a transition matrix. The difference between answers C and E are that the entries in row 2 column 3 and row 3 column 2 have been swapped. So make sure you check all entries.

Question 6.

Answer: **C**

Explanation: You need to determine the number of bicycles at each depot at the end of 5 weeks. $T^5 \times S$ where S is the initial state matrix will determine this. This leaves 122 bicycles at depot A, so 22 will need to be relocated to B and C.

$$\begin{bmatrix} 0.72 & 0.14 & 0.25 \\ 0.18 & 0.68 & 0.09 \\ 0.1 & 0.18 & 0.66 \end{bmatrix}^5 \cdot \begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix} = \begin{bmatrix} 121.74701056 \\ 92.34126848 \\ 85.91172096 \end{bmatrix}$$

Question 7.

Answer: **E**

Explanation: Multiplying the two digits in each diagonal of the matrix should result in the same number if a 2x2 square matrix is singular. After substitution of $m = -1$ the only matrix for which this is true is E.

Question 8.

Answer: **A**

Explanation: The quickest way to find the long run proportion is to put the transition matrix to a large power and multiply by the initial state matrix. It would actually not matter what you entered as the initial state matrix. The top number represents the number attending the Gym (answer B), but we require the number who will attend aerobics, so closest to 104 is 105, thus answer A is correct.

$$\begin{bmatrix} 0.68 & 0.4 \\ 0.32 & 0.6 \end{bmatrix}^{50} \cdot \begin{bmatrix} 163 \\ 72 \end{bmatrix} = \begin{bmatrix} 130.55555556 \\ 104.44444444 \end{bmatrix}$$

Module 2 – Networks and decision mathematics

Question 1.

Answer: C

Explanation: There are three possible paths from Cobden to the 12 Apostles. The shortest of 50km, the second shortest of $(26+27+3) = 56$ km, and the longer route of $(26+35+35+3) = 99$ km. Any other route would require Melanie to pass by another point of interest or use an intersection more than once.

Question 2.

Answer: C

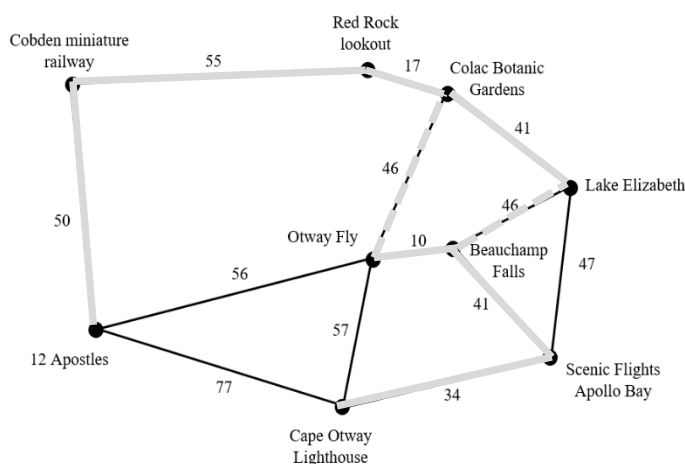
Explanation: There are a lot! This table shows the number of unique routes between each point of interest.

Point of Interest	2	3	4	5	6	7	8	9	Sum
1 Cobden Railway	1	2		2	2		2	3	12
2 Red Rock		1							1
3 Colac Gardens			1	1	1		1	2	6
4 Lake Elizabeth				1		1			2
5 Apollo Bay					1	1	1	2	5
6 Cape Otway							1	2	3
7 Beauchamp Falls							1		1
8 Otway Fly								2	2
9 12 Apostles									0
Total									32

Question 3.

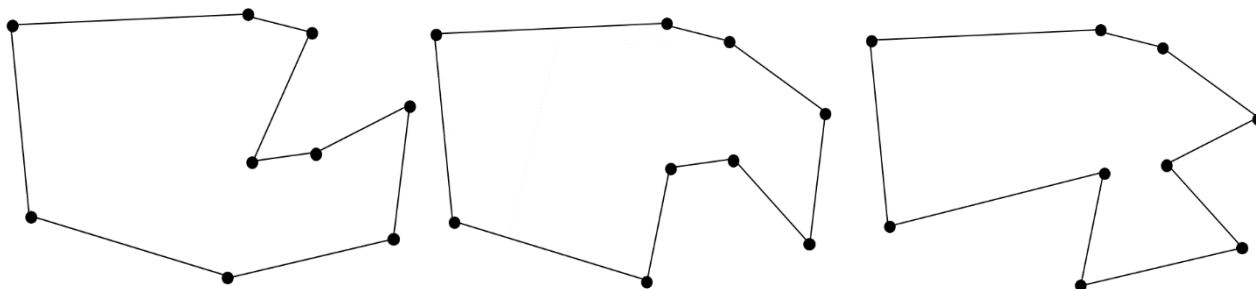
Answer: A

Explanation: The minimum spanning tree is shown here. Because the two dotted lines are both weighted as 46, either one of them would complete the minimum spanning tree. Total length is $(50+55+17+41+10+41+34+46) = 294$ km in total. Thus answer A.



Question 4.**Answer: B**

Explanation: As the network contains 6 odd vertices, it is impossible to have an Eulerian path or circuit, so answers A and C are out. Two of the answers relate to Hamiltonian cycles, so investigation of these is the next priority. For a cycle, it does not matter where you start. There are three possible Hamiltonian cycles so the answer is B.

**Question 5.****Answer: D**

Explanation: As there are 6 odd vertices, you would need to remove all odd vertices by removing 3 edges for an Eulerian circuit to exist.

Question 6.**Answer: A**

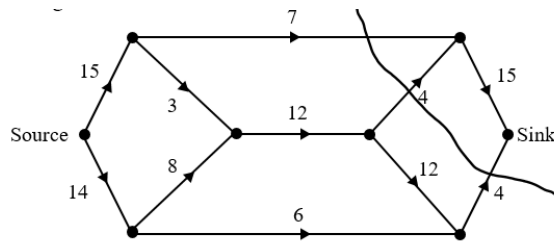
Explanation: Using Dijkstra's method for finding the shortest distance, the second row of the table would contain a next smallest value of 55 representing the distance to Red Rock. Thus Red Rock would be entered in the third row of the table.

	Red Rock	Colac Botanic Gardens	Lake Elizabeth	Apollo Bay	Cape Otway	Beauchamp Falls	Otway Fly	12 Apostles
Cobden Miniature Railway	55	*	*	*	*	*	*	50
12 Apostles	55	*	*	*	77	*	56	50
Red Rock	55							50

Question 7.

Answer: A

Explanation: Maximum flow = minimum cut. Minimum cut is shown here. Cut is $(7 + 4 + 4) = 15$, so the answer is A



Question 8.

Answer: D

Explanation: Earliest finish time is 64 days because the critical path is A-B-G-C-H-I-L-M with a total duration of $10+12+10+12+6+4+8+2=64$

Module 3 – Geometry and measurement

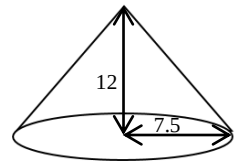
Question 1.

Answer: **D**

Explanation: Using Pythagoras theorem, the length of the slant edge

$$l = \sqrt{12^2 + 7.5^2} = 14.15 \text{ cm}$$

which rounds to 14.2 cm



Question 2.

Answer: **C**

Explanation: The area of triangle PQR is smaller than PST, so it cannot be answer B. The ratio of lengths is 1:2, thus the ratios of areas must be $1^2 : 2^2$ or 1:4. The area of triangle PQR is $\frac{1}{4}$ of the area of PST and is therefore 64 square centimetres.

Question 3.

Answer: **D**

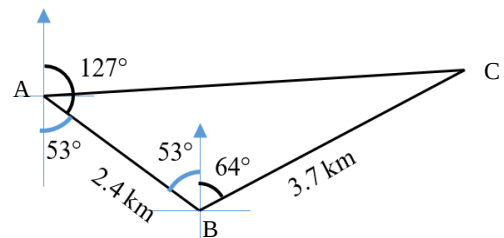
Explanation: Draw a diagram showing the directions travelled. Now the cosine rule can be used to find the unknown distance. The angle at B is $(53 + 64) 117^\circ$, side length c is 2.4 km and side length a is 3.7 km.

$$\text{Using } b^2 = a^2 + c^2 - 2ac \cos(B)$$

$$b = \sqrt{2.4^2 + 3.7^2 - 2 \times 2.4 \times 3.7 \cos 117^\circ}$$

$$b = 5.2453 \text{ km}$$

Thus the distance from base camp is 5.25 km



Question 4.

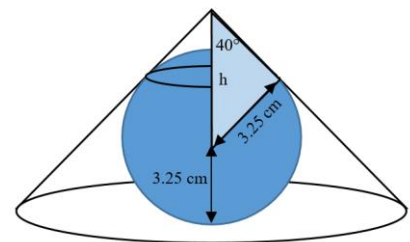
Answer: **A**

Explanation: First calculate the length h shown in the diagram at right.

$$h = \frac{3.25}{\sin(40^\circ)} = 5.056 \text{ cm}$$

The height of the cone is equal to the length h plus the radius of the ball.

$$\text{Height of cone} = 5.056 + 3.25 = 8.306 \approx 8.31 \text{ cm}$$

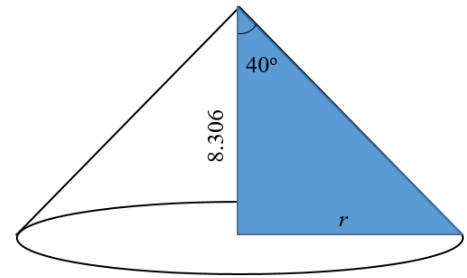


Question 5.

Answer: D

Explanation: Using the height of the cone as the adjacent side, we can use the tan ratio to calculate the radius.

$$r = 8.306 \tan(40^\circ) = 6.96956 \approx 6.97 \text{ cm}$$



Question 6.

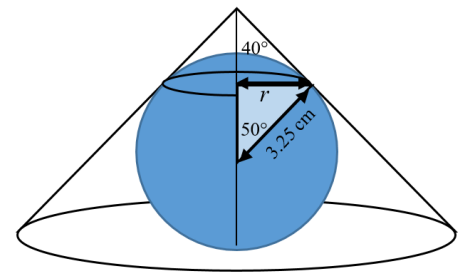
Answer: A

Explanation: First of all we need to calculate the radius of the smaller circle.

$$r = 3.25 \sin(50^\circ) = 2.4896 \text{ cm}$$

The length of the line of paint is the circumference of this smaller circle.

$$C = 2\pi r = 2\pi \times 2.4896 = 15.64 \text{ cm}$$



Question 7.

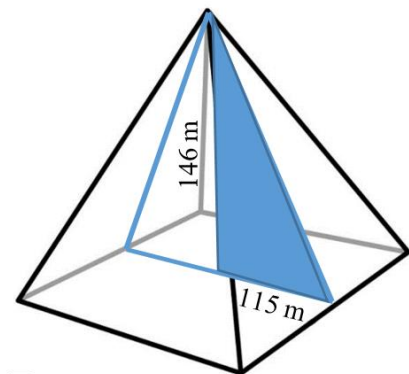
Answer: A

Explanation: Using the right angle triangle shown in this diagram and Pythagoras theorem, we can calculate the length of the slant edge of the Great Pyramid.

$$h = \sqrt{146^2 + 115} = 185.85 \text{ m}$$

Thus the total length along the line is

$$230 + (2 \times 185.85) = 601.70 \text{ m}$$



Question 8.

Answer: C

Explanation: Because these are similar figures the ratio of the lengths of line will be equal to the ratio of the heights of the two pyramids. These can be used to determine the height of the model

$$\frac{\text{line on model}}{\text{line on pyramid}} = \frac{\text{height of model}}{\text{height of pyramid}}$$

$$\frac{61}{601.70} = \frac{h}{146}$$

$$\therefore h = \frac{146 \times 61}{601.70} = 14.8 \text{ cm}$$

Module 4 – Graphs and relations

Question 1.

Answer: **D**

Explanation: Graph has intercepts of (0,-4) and (2,0). Gradient is 2, thus equation is $y = 2x - 4$ which rearranges to give $2x - y = 4$

Question 2.

Answer: **E**

Explanation: The equation of the line with x intercept of -2 and y intercept of 3 is $y = \frac{3}{2}x + 3$ or

$2y - 3x = 6$. Substituting in values for x and y will determine the correct answer

A (1,5) results in $2 \times 5 - 3 \times 1 = 7$ incorrect

B (-3,-1) results in $2 \times (-1) - 3 \times (-3) = 7$ incorrect

C (14,25) results in $2 \times 25 - 3 \times 14 = 8$ incorrect

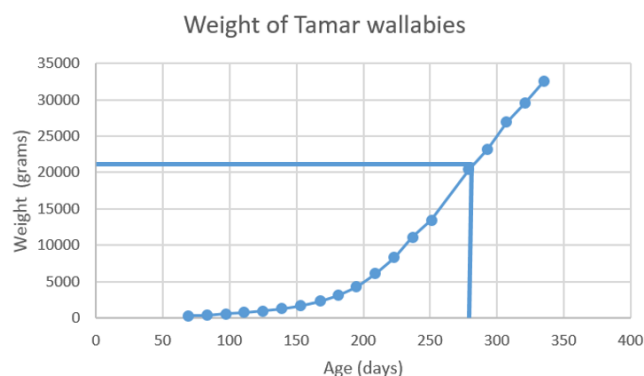
D (12,20) results in $2 \times 20 - 3 \times 12 = 4$ incorrect

E (-12,-15) results in $2 \times (-15) - 3 \times (-12) = 6$ Correct

Question 3.

Answer: **D**

Explanation: Interpreted from graph. It can be seen the weight is over 20,000 grams or 20 kg, so 21 kg is the closest estimate.



Question 4.

Answer: **C**

Explanation: Total hours worked is given by $1 \times x + 2.5 \times y$ which must be less than or equal to 42 hours.

$x + 2.5y \leq 42$ so answers A, and D are incorrect.

Rearranging the inequality gives $2.5y \leq 42 - x$ or $42 - x \geq 2.5y$ so not B

Rearranging also gives $-x - 2.5y \geq -42$ so E is incorrect

Rearranging also gives $x \leq 42 - 2.5y$ so answer C is correct.

Question 5.**Answer: E**

Explanation: Can be determined from the slope of the objective function. The last point for it to pass through when translated right will be (55, 60).

Question 6.**Answer: C**

Explanation: Since the graph goes through the point (4, 1/10), substitute into each equation to see which work.

A. $y = \frac{1}{5x}$, $y = \frac{1}{5 \times 4} = \frac{1}{20}$ incorrect.

B. $y = \frac{8}{5x^2}$, $y = \frac{8}{5 \times 4^2} = \frac{8}{80} = \frac{1}{10}$, but when $x = 1$, $y = \frac{8}{5}$ however $y < 1$ on graph

C. $y = \frac{2}{5x}$, $y = \frac{2}{5 \times 4} = \frac{1}{10}$ and when $x = 1$, $y = \frac{2}{5}$ Correct answer.

D. $y = \frac{3}{5x}$, $y = \frac{3}{5 \times 4} = \frac{3}{20}$ which is incorrect.

E. $y = \frac{2}{5x^2}$, $y = \frac{2}{5 \times 16} = \frac{1}{40}$ which is incorrect.

Question 7.**Answer: B**

Explanation: Need to set up simultaneous equations from the information given. Shirts sold 2 for \$50 means each shirt sold for \$25. x for number of shirts, y for number of ties.

All stock sold gives

$$25x + 15y = 10000$$

If only half of the shirts, ($x/2$), and three fifths of the ties, ($3y/5$) are sold, then

$$25 \times \frac{x}{2} + 15 \times \frac{3y}{5} = 5150$$

$$\therefore \frac{25x}{2} + 9y = 5150$$

Can now be solved on CAS

$$\text{solve} \left(\begin{cases} 25 \cdot x + 15 \cdot y = 10000 \\ \frac{25 \cdot x}{2} + 9 \cdot y = 5150 \end{cases}, x \right)$$

$x=340$ and $y=100$

Question 8.

Answer: B

Explanation: Need to set up simultaneous equations and solve either using CAS or by hand.

$$C = an + b$$

$$n = 300, C = 450$$

$$450 = 300a + b$$

$$n = 280, C = 426$$

$$426 = 280a + b$$

solving with CAS

$$\text{solve}\left(\begin{cases} 450 = 300 \cdot a + b \\ 426 = 280 \cdot a + b \end{cases}, a\right) \quad a = 1.2 \text{ and } b = 90.$$