

The Mathematical Association of Victoria

Trial Examination 2017

FURTHER MATHEMATICS

Written Examination 2

STUDENT NAME: _____

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of Book

Section A - Core	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
	8	8	36
Section B - Modules	<i>Number of modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
	4	2	24
			Total 60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 29 pages
- Formula sheet.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are NOT drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A – CORE**Instructions for Section A**

Answer **all** questions in the space provided.

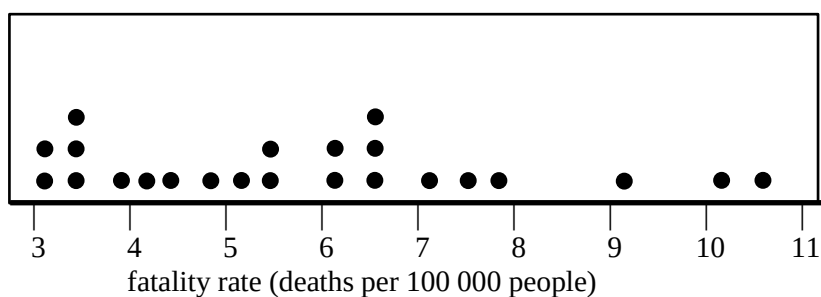
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include for example, π , surds or fractions.

In ‘Recursion and financial modelling’, all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are NOT drawn to scale.

Data Analysis**Question One (4 marks)**

The dot plot below shows the distribution of road fatality rates, in deaths per 100 000 people, for a sample of 23 countries.



Data based on WHO report, 2015

- (a) Circle, and clearly label, the point that is :

- (i) Q1
- (ii) median
- (iii) Q3

2 marks

- (b) Use the grid below to construct a histogram that displays the distribution of traffic fatality rates for this set of data. Use an interval width of one, with the first interval starting at 3.

2 marks



SECTION A - Question 1 – continued
TURN OVER

Question Two (4 marks)

This table contains the traffic fatality rates (fatalities per 100 000 people) for 20 Asian countries, including Australia and New Zealand.

Country	Traffic fatality rate
Singapore	3.6
Japan	4.7
Australia	5.4
New Zealand	6.0
Philippines	10.5
South Korea	10.5
Bangladesh	13.6
Pakistan	14.2
Laos	14.3
Indonesia	15.3
India	16.6
Timor-Leste	16.6
Papua New Guinea	16.8
Cambodia	17.4
Sri Lanka	17.4
China	18.8
Myanmar	20.3
Malaysia	24.0
Vietnam	24.5
Thailand	31.2

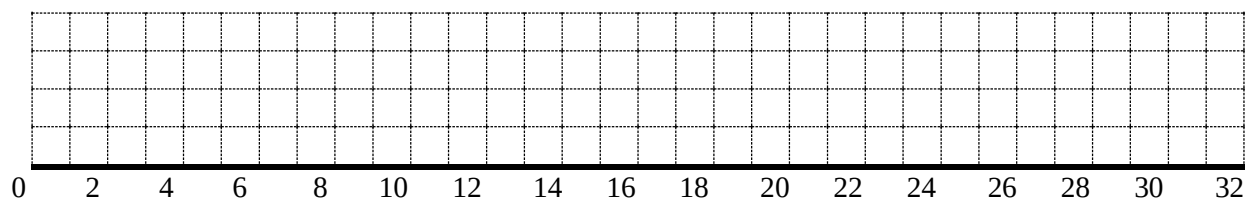
Data based on WHO report, 2015

- (a) **Show** that the value of the upper fence is 29.5.

1 mark

- (b) Draw a boxplot of this data on the grid provided below.

2 marks



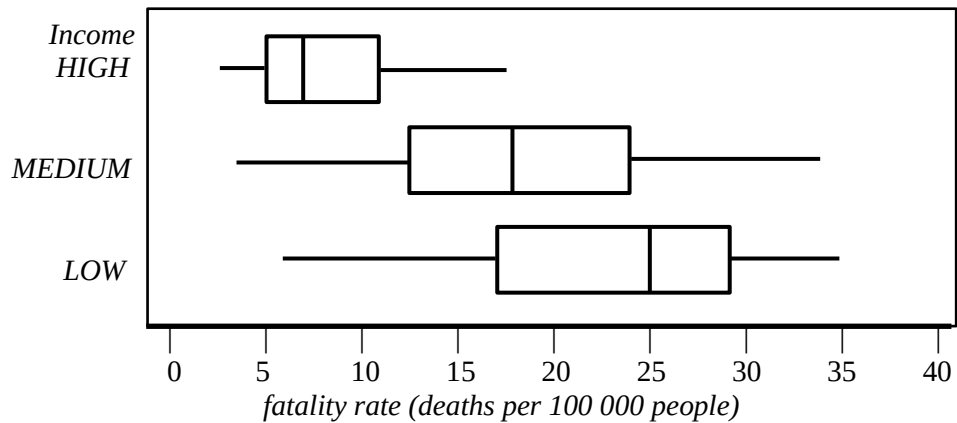
- (c) In which part of the boxplot does Australia's value lie ?

1 mark

SECTION A - continued

Question Three (3 marks)

The boxplots below display the distribution of road *fatality rates* (per 100 000 people) for countries classified as having *low* (less than \$6000), *medium* (between \$6000 and \$20 000) or *high* (above \$20 000) average *income* per person per year.



Data based on WHO report, 2015

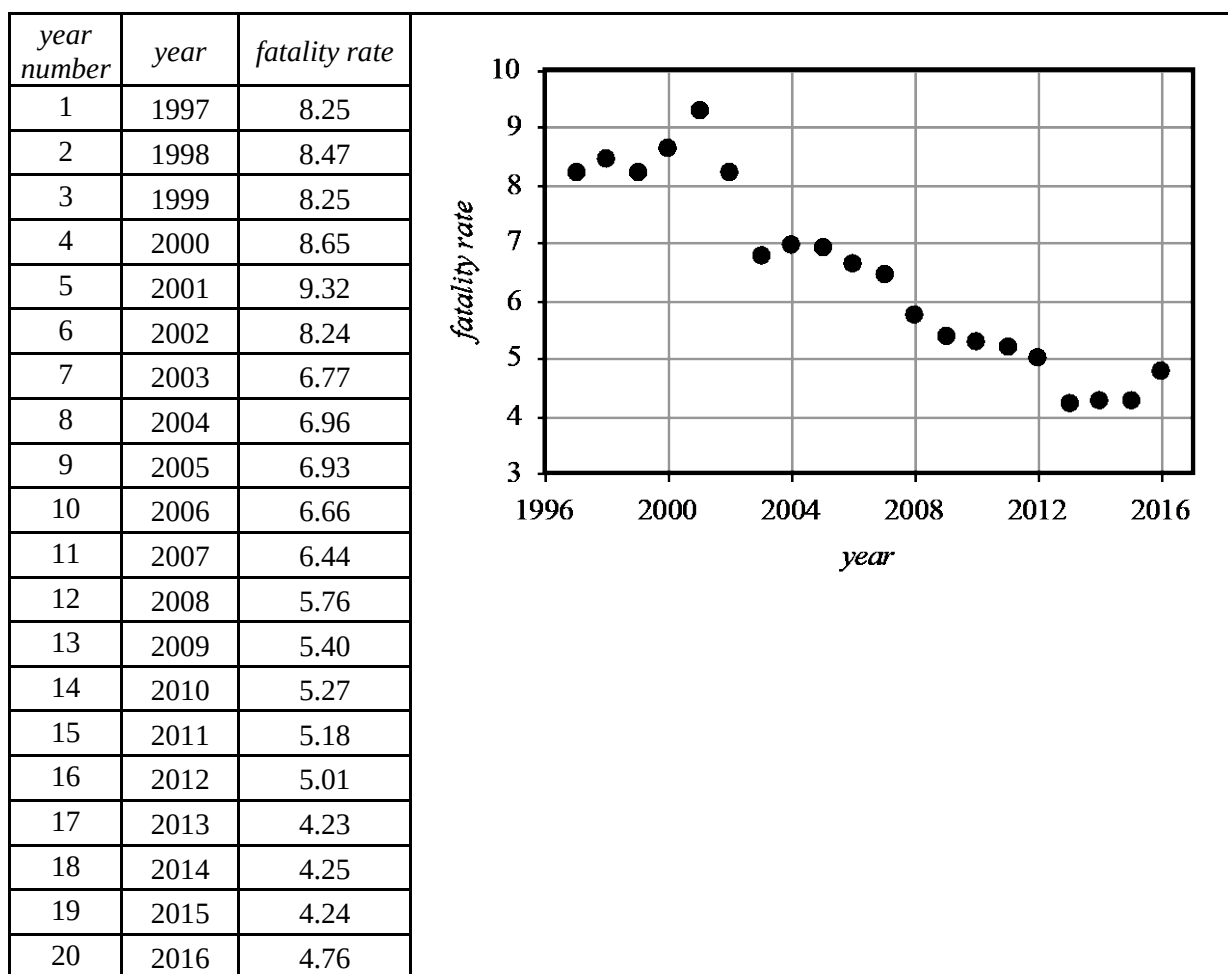
- (a) Describe the shape of the distribution of fatality rates for the medium average income group. **1 mark**
- (b) Using the information, explain the type of association that exists between fatality rates and average income. Quote the values of appropriate statistics in your response. **2 marks**

SECTION A - continued
TURN OVER

Question Four (8 marks)

The traffic fatality rates (given as deaths per 100 000 people) for a twenty-year period for the state of Victoria are given in the table below, and a scatterplot of the data is shown alongside.

The data will be used to investigate the association between the variables *fatality rate* and *year*.



Based on data from gapminder.org, TAC

- (a) Use the scatterplot to describe the association between *year* and *fatality rate* in terms of direction, strength and form. **1 mark**
- (b) Determine the equation of the least squares regression line that can be used to predict the *fatality rate* from the *year number*, where 1997 is *year number* 1. Write the values of the intercept and slope of this least squares line in the appropriate spaces provided below. Round your answers to three significant figures. **1 mark**

Fatality rate = × *year number*

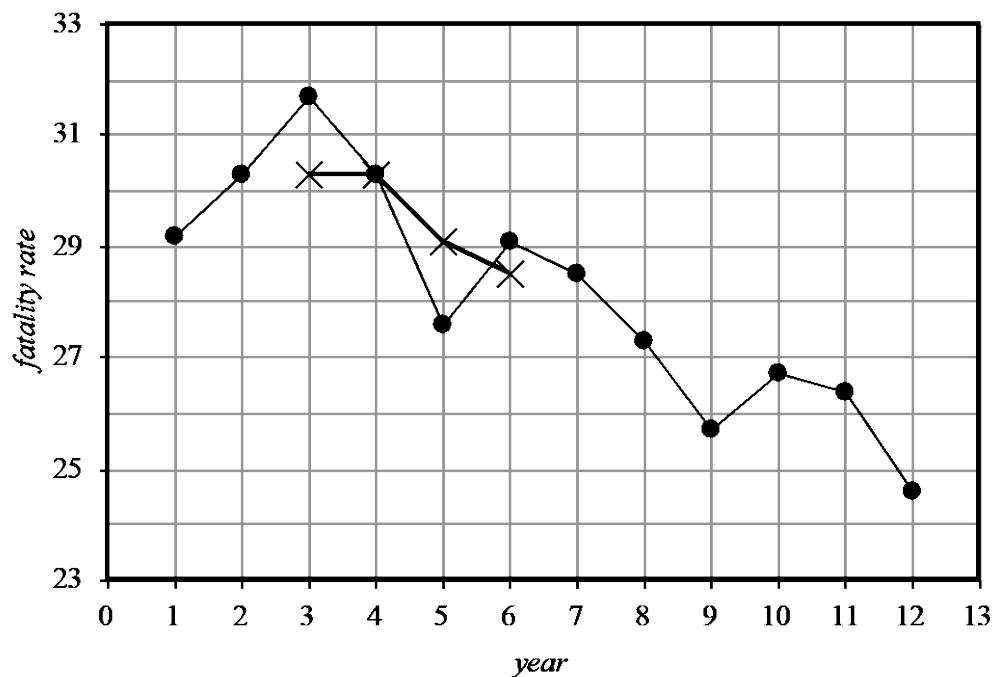
SECTION A – Question 4 - continued

- (c) Draw the regression line on the scatterplot above. **1 mark**
Show the calculations for the key points to place the line.
- (d) Interpret the slope of the least squares line in terms of the variables *fatality rate* and *year number*. **1 mark**
- (e) Assuming that the least squares regression equation from (b) can apply to data for 1994, predict the *fatality rate* for 1994, correct to three significant figures. **1 mark**
- (f) The actual *fatality rate* for 1994 was 8.45.
Calculate the residual value for your prediction, correct to three significant figures. **1 mark**
- (g) What does this residual value tell us about the predicted value for 1994 ? **1 mark**
- (h) Why is the prediction for 1994 unreliable ? **1 mark**

SECTION A - continued
TURN OVER

Question Five (1 mark)

The time series plot below shows the fatality rate per 100 000 people for the whole of Australia over a twelve-year period at about the time when stronger safety measures (such as seat belts) were starting to be implemented.



Data from gapminder.org

Five-median smoothing has been used to smooth the time series plot above.

The first four points are shown as X.

Complete the five-median smoothing by marking smoothed values with X on the time series plot above.

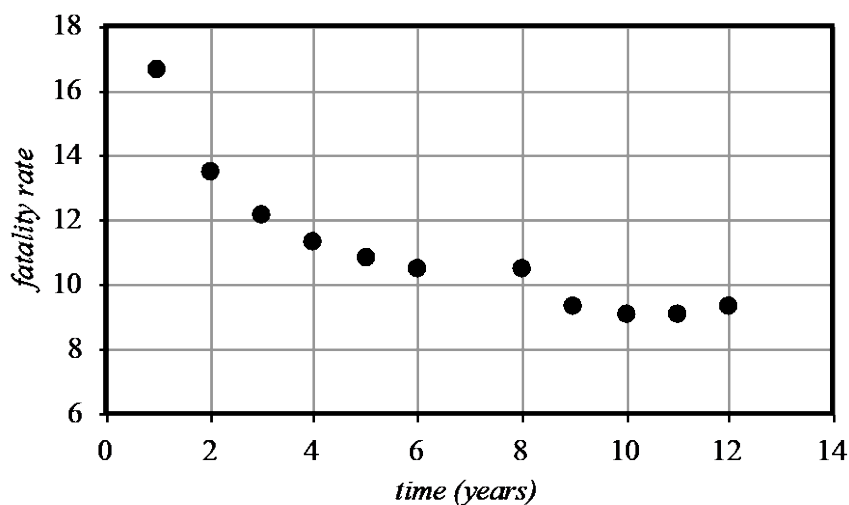
1 mark

SECTION A - continued

Question Six (4 marks)

The traffic *fatality rates* for a twelve-year *time* period for the whole of Australia are given in the table below, and a scatterplot of the data is shown alongside. Note that the *fatality rate* value for the seventh year is NOT given.

<i>time (years)</i>	<i>fatality-rate</i>
1	16.7
2	13.5
3	12.2
4	11.3
5	10.8
6	10.5
8	10.5
9	9.3
10	9.1
11	9.1
12	9.3



Data from gapminder.org

For this data set, the relationship between *fatality rate* and *time* is non-linear.

A reciprocal transformation can be applied to the variable *fatality rate* to linearise the scatterplot.

- (a) Apply the reciprocal transformation to the data and determine the equation of the least squares regression line that allows the reciprocal of the fatality-rate to be predicted from the time.

Write the values of the intercept and slope of this least squares line in the appropriate spaces provided below.

Round your answers to three decimal places.

2 marks

$$\frac{1}{\text{fatality rate}} = \dots \times \text{time}$$

- (b) Use this regression equation to predict the *fatality rate* in the seventh year. Write your answer to one decimal place.

1 mark

- (c) Plot this point on the graph.

1 mark

**SECTION A - continued
TURN OVER**

Recursion and financial modelling**Question Seven (6 marks)**

Ellie has opened a savings account to save money for the deposit for a home. The amount of money in the savings account after n years, H_n can be modelled by the recurrence relation shown below:

$$H_0 = 22\,000, \quad H_{n+1} = 1.035 \times H_n$$

- (a) What is the annual percentage compound interest rate for Ellie's savings account? **1 mark**
- (b) Use recursion to write down calculations that show that the amount of money in Ellie's savings account after two years will be \$23 566.95. **1 mark**
- (c) The amount of money in the account after n years, H_n , can also be determined using a rule. Complete the rule below by writing the appropriate numbers in the brackets provided. **1 mark**
- $$H_n = (\text{.....})^n \times (\text{.....})$$
- (d) How many years will it take for Ellie to save at least a \$30 000 deposit in her savings account? **1 mark**
- (e) Ellie is advised to ask for monthly compounds instead. How many months sooner will she have at least \$30 000 given that all other conditions remain the same? **1 mark**
- (f) What is the effective rate of interest if the original interest rate is compounded monthly? Give your answer correct to two decimal places. **1 mark**

SECTION A - continued

Question Eight (6 marks)

Ellie is taking out a loan for her first home. The loan is for \$370 000 over a period of 25 years at 4.2% per annum with monthly compounds and monthly payments.

- (a) Ellie is offered a two year interest only period at the start of her loan.
What is the monthly payment that Ellie would make during this period? **1 mark**
- (b) Ellie will still need to complete her loan within the 25 year period, so she starts making equal monthly payments of both principal and interest after two years that reduce her balance to zero after another 23 years.
What monthly payment would Ellie need to make? **1 mark**
- (c) Alternatively Ellie could start making equal monthly payments of both interest and principal from the beginning of her loan over the full 25 year period.
What payment would Ellie make under these circumstances? **1 mark**
- (d) Show that it will cost Ellie more overall if she takes the interest only option.
State the extra cost correct to 3 significant figures. **2 marks**
- (e) Explain why the loan with the interest only period is more expensive overall. **1 mark**

**END OF SECTION A
TURN OVER**

SECTION B - Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include for example, π , surds or fractions.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

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Module 4 – Graphs and relations.....	25

SECTION B - continued

Module 1 – Matrices**Question One (4 marks)**

The After-Sales Accessory Department is an additional source of income for a car retailer. At Scotty Moffit Motors in Wombat Flat, Alana meets with each customer after they have purchased a vehicle to try to sell them window tinting (W), panel protection coating (P) and/or seat stain protection (S).

Matrix C contains the cost in dollars (including GST) of each of these accessories.

$$C = \begin{bmatrix} & P & S & W \\ 1050 & 350 & 675 \end{bmatrix}$$

- (a) Write down the order of matrix C .

1 mark

Matrix M shows the number of sales for each accessory last month.

$$M = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} \begin{matrix} P \\ S \\ W \end{matrix}$$

To find the total value of her sales for last month, Alana suggests the following calculation :

$$R = C \times M$$

- (b) **Show** the calculation, with answer, for $R = C \times M$.

2 marks

When her offsider, Barry, performed the calculation, he obtained the answer $[9275]$. Alana told him that his answer was incorrect, and that he had switched two of the values in matrix M .

- (c) By filling the values, show the calculation that Barry would have carried out.

1 mark

$$R = C \times M = \begin{bmatrix} 1050 & 350 & 675 \end{bmatrix} \times \begin{bmatrix} \dots\dots\dots \\ \dots\dots\dots \\ \dots\dots\dots \end{bmatrix} = [9275].$$

SECTION B – Module 1 - continued
TURN OVER

Question Two (3 marks)

The staff at Scotty Moffit Motors have a darts competition every Friday afternoon between teams led by Alana (*A*), Barry (*B*), Chris (*C*) and Dana (*D*). The results after two rounds have been played (where every game has a winner and a loser) are shown in the one-step dominance matrix below.

		loser			
		<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
winner	<i>A</i>	0	1	0	0
	<i>B</i>	0	0	1	0
	<i>C</i>	0	0	0	1
	<i>D</i>	1	0	0	0

- (a) Which teams will play each other in the third round, so that each team has played each of the others once and once only. **1 mark**

If Chris' team wins their third round game, they will have a chance to win the competition as the best team as determined by the sum of the one-step and two-step dominances. However, if a particular team wins the other third round game, Chris' team will finish second.

- (b) Write down the leader of the team that by winning their third round game will finish first ahead of Chris' team. **1 mark**

- (c) Write down the final dominances table for this situation. **1 mark**

Question Three (3 marks)

The sales team at Scotty Moffit Motors have noticed that only four of the many colours available for the new cars that they sell are preferred by their customers (blue (B), green (G), red (R) and yellow (Y)), and these colours can change with each subsequent purchase of a new car. Matrix T contains the preferences of customers who are expected to change their choice of colour from car to car.

$$T = \begin{array}{ccccc} & \begin{array}{c} \text{this car} \\ B \quad G \quad R \quad Y \end{array} & & & \\ \begin{array}{c} B \\ G \\ R \\ Y \end{array} & \begin{bmatrix} 0.65 & 0.10 & 0.10 & 0.10 \\ 0.05 & 0.60 & 0.10 & 0.05 \\ 0.10 & 0.10 & 0.70 & 0.10 \\ 0.20 & 0.20 & 0.10 & 0.75 \end{bmatrix} & \begin{array}{c} B \\ G \\ R \\ Y \end{array} & \begin{array}{c} \text{next car} \end{array} \end{array}$$

Let S_0 show the number of customers who chose each colour for their last purchase.

$$S_0 = \begin{bmatrix} 20 \\ 20 \\ 40 \\ 40 \end{bmatrix} \begin{array}{c} B \\ G \\ R \\ Y \end{array}$$

Matrix S_1 will show the number of customers who chose each colour for their next purchase.

$$S_1 = TS_0 = \begin{bmatrix} \dots\dots\dots \\ \dots\dots\dots \\ 36 \\ \dots\dots\dots \end{bmatrix} \begin{array}{c} B \\ G \\ R \\ Y \end{array}$$

- (a) Fill in the values for S_1 , correct to the nearest whole number. **1 mark**
- (b) From the matrix given above, write a calculation that shows that 36 customers chose Red as the colour for their next purchase. **1 mark**
- (c) In the long term, which colour will be most favoured for the cars purchased from Scotty Moffit Motors. **1 mark**

SECTION B – Module 1 - continued
TURN OVER

Question Four (2 marks)

The drivers around Wombat Flat are not very careful and they often damage the paintwork on their cars.

Scotty Moffit Motors runs a repair shop and the manager has developed the following matrix equation to order the paint (in litres) required each year.

$$P_{2017} = T \times P_{2016} + B$$

Where

		<i>this year</i>				
		<i>B</i>	<i>G</i>	<i>R</i>	<i>Y</i>	
	$T =$	0.55	0.05	0.05	0.05	<i>B</i>
		0.15	0.65	0.10	0.05	<i>G</i>
		0.10	0.15	0.65	0.20	<i>R</i>
		0.20	0.15	0.20	0.70	<i>Y</i>
						<i>next year</i>

$$P_{2016} = \begin{bmatrix} 140 \\ 160 \\ 160 \\ 200 \end{bmatrix} \begin{matrix} B \\ G \\ R \\ Y \end{matrix} \qquad B = \begin{bmatrix} 20 \\ 25 \\ 25 \\ 40 \end{bmatrix} \begin{matrix} B \\ G \\ R \\ Y \end{matrix}$$

$$B = \begin{bmatrix} 20 \\ 25 \\ 25 \\ 40 \end{bmatrix} \begin{matrix} B \\ G \\ R \\ Y \end{matrix}$$

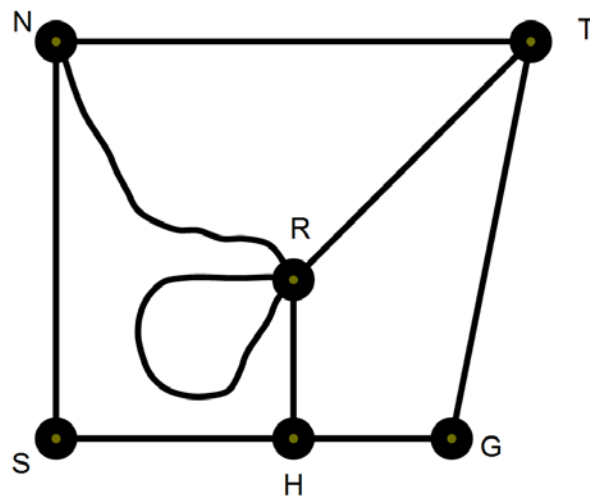
Determine the amount of Yellow paint required in 2018, correct to the nearest litre.

2 marks

END OF MODULE 1 - SECTION B – continued

Module 2 – Networks and decision mathematics**Question One (3 marks)**

Jack is going for his licence. His driving instructor shows him a section of the test course that he should practice before his test. The section has a number of intersections with a roundabout (R), a handbrake start (H), a give-way sign (G), a stop sign (S), a traffic light (T) and an intersection with no signs (N) as shown below:



Jack is to start at the traffic lights (T) and he must go through every intersection exactly once and then return to T.

- (a) State a route that starts and ends at T passing through all other intersections exactly once. **1 mark**
- (b) State the mathematical name given to the route that Jack will take. **1 mark**
- (c) Explain why Jack could not drive along every edge exactly once in this network. **1 mark**

SECTION B – Module 2 - continued
TURN OVER

Question Two (3 marks)

Alf, Barbara, Clarice and Donald all work at the VicRoads office. They each serve at the counter and can process registrations (R), licences (L), learner's permits (P) and roadworthiness certificates (W). Their average times in minutes for each task for each person are shown below in a table:

	Registrations (R)	Licences (L)	Learner's Permits (P)	Roadworthiness Certificates (W)
Alf	11	15	12	10
Barbara	15	18	14	11
Clarice	16	17	15	16
Donald	15	16	13	12

The allocation of one job per person is required to ensure that the minimum time is taken on tasks. The minimum allocation is completed using the Hungarian Algorithm with a row reduction followed by a column reduction to result in the matrix V below:

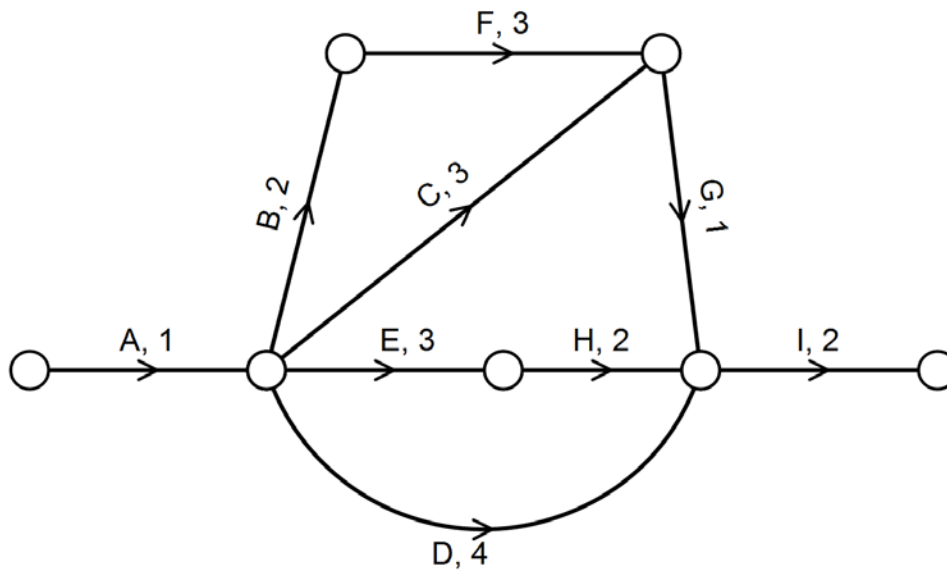
$$V = \begin{matrix} & R & L & P & W \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 3 & 2 & 0 \\ 3 & 5 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 2 & 1 & 0 \end{bmatrix} \end{matrix}$$

- (a) Explain why the tasks cannot be allocated from matrix V. **1 mark**
- (b) Complete the next steps of the Hungarian Algorithm to give a matrix that could result in an allocation. **1 mark**
- (c) Hence or otherwise state who should complete each task. **1 mark**

SECTION B – Module 2 - continued

Question Three (6 marks)

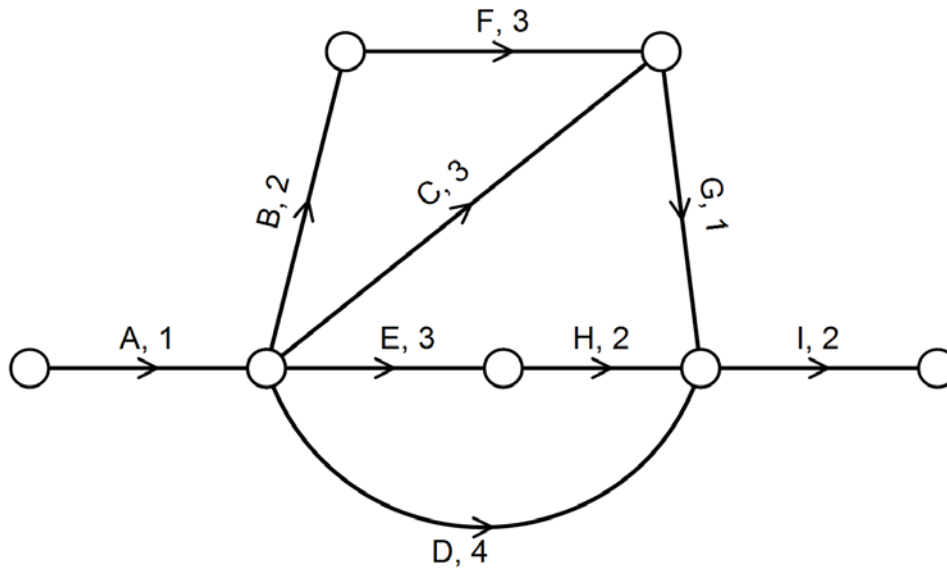
At the Vic Roads office there is a new staff member who must go through initial training procedures. An activity network showing all of the training tasks and the duration of each activity in hours is shown below:



- (a) Which activity or activities must be completed immediately before activity G? **1 mark**
- (b) Given that all training activities must be completed, what is the minimum time to complete the initial training? **1 mark**
- (c) The trainer for activity C is running late. How many hours after the start of the training can he arrive without delaying the training? **1 mark**
- (d) Activities B and F can be reduced in time to only one hour each by using an internet training package. What would be the new minimum time to complete the training if this package is used? **1 mark**

SECTION - B – Module 2 – Question 3 - continued
TURN OVER

- e) It has been decided that Activity G will make more sense to trainees if it is done after Activity E. Alter the activity network below to reflect this change. **1 mark**

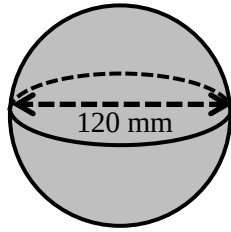


- f) Given that the internet package is used for Activities B and F, and Activity E must be completed before Activity G, which activities would be able to be delayed by one hour only without affecting the overall time for the training? **1 mark**

End of Module 2 - SECTION B - continued

Module 3 – Geometry**Question One (4 marks)**

A shot put ball, as used in the shot put field event, is spherical in shape, has a smooth surface and has a diameter of 120 mm. as shown in the diagram below.



- (a) What is the volume of the shot put ball as shown ?
Round your answer to the nearest cubic centimetre. **1 mark**
- (b) The shot put ball used in international mens competition must weigh 7260 g.
If density = $\frac{\text{weight}}{\text{volume}}$, what is the density of this shot put ball, in grams per cubic centimetre correct to two decimal places? **1 mark**
- (c) The shot put ball used in international womens competition must weigh 4 000 g.
If the density of a particular shot put ball is 8.00 gram per cubic centimetre, what will be the **diameter** of this shot put ball, in centimetres correct to one decimal place ? **2 marks**

SECTION B – Module 3 - continued
TURN OVER

Question Two (4 marks)

In this question, assume that the radius of Earth is 6400 km.

A shot put event is being held in Nairobi, Kenya at location (2° S, 37° E).

- (a) Irina is flying from Moscow (55° N, 37° E) directly to the Kenyan capital Nairobi (2° S, 37° E) to compete in a major shot put event.

If her plane flies the great circle route along the 37° E meridian of longitude, how far will it travel, correct to the nearest kilometre ?

1 mark

- (b) The qualifying round for this event, to select the twelve best throwers, will be held on a Friday evening in Nairobi, starting at 8.30 pm.
Irina's aunt will be watching the event live on television while visiting her friends in Melbourne. Assume that the time difference between Melbourne (38° S, 145° E) and Nairobi (2° S, 37° E) is seven hours.

On what day and at what time will the qualifying round begin in Melbourne ?

1 mark

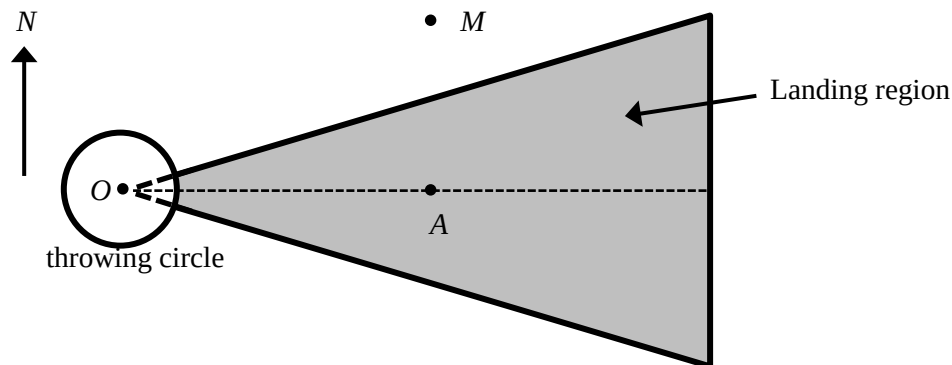
- (c) Jacqui, the Australian champion, is flying direct from Sydney (Australia) to Cape Town (South Africa) to compete in a warm-up event before flying on to Nairobi. Both Sydney (Australia) and Cape Town (South Africa) are on the 34° S parallel of latitude, but Sydney is at 151° E whereas Cape Town is at 18° E. **Show** that the distance along the 34° parallel of latitude between Sydney and Cape Town is 12 320 km, correct to four significant figures.

2 marks

SECTION B – Module 3 - continued

Question Three (4 marks)

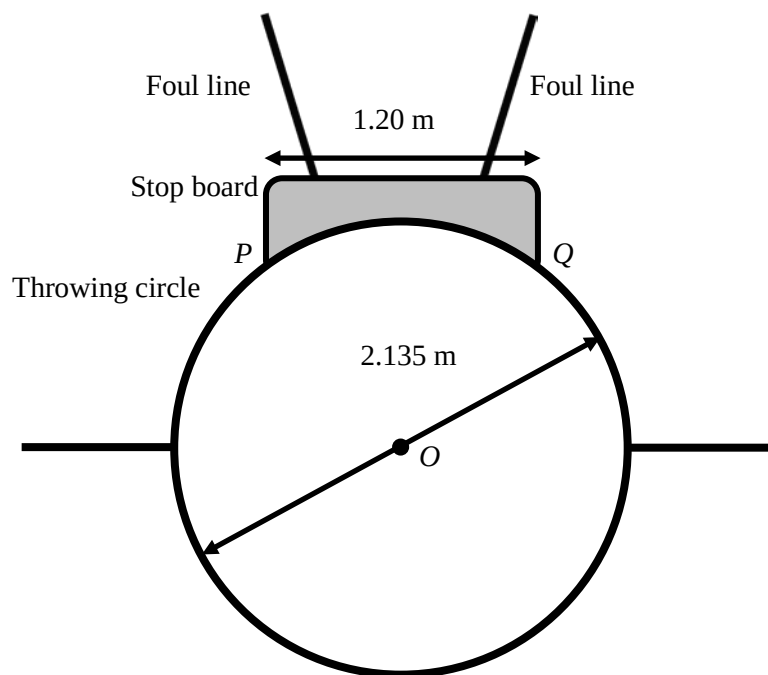
Michael, the chief supervisor of officials, is conducting a training session on a practice field as shown in the diagram below. The field markings for the shot put event essentially consist of an isosceles triangle with a circle centred on the apex of the two equal-length sides. The shaded area on the diagram is the legal landing region for a shot put throw made from the circle. The centre line of the landing region on the practice field runs due west-to-east.



Michael is currently standing at a point (marked M on the diagram above) to the north of the landing region and 30.0 m from point O (the centre of the circle). When Abebe, the chief Kenyan official, stands on the centre line of the landing region at point A which is a distance of 23.0 m from point O , Abebe is due south of Michael.

- (a) If another official stood at point O , at what bearing, correct to the nearest whole degree, would Michael be from this official? **2 marks**

SECTION B – Module 3 - Question 3 - continued
TURN OVER



The throwing circle in the main arena is shown in greater detail in the diagram above. The centre of the circle is marked O , and its diameter is 2.135 m .

At the front is a stop board (shaded grey) which the throwers are not allowed to step on or across. It is 1.20 m long. The sides of the stop board meet the circumference of the circle at points P and Q .

- (b) What is the magnitude of the angle POQ ?
Give your answer correct to two decimal places.

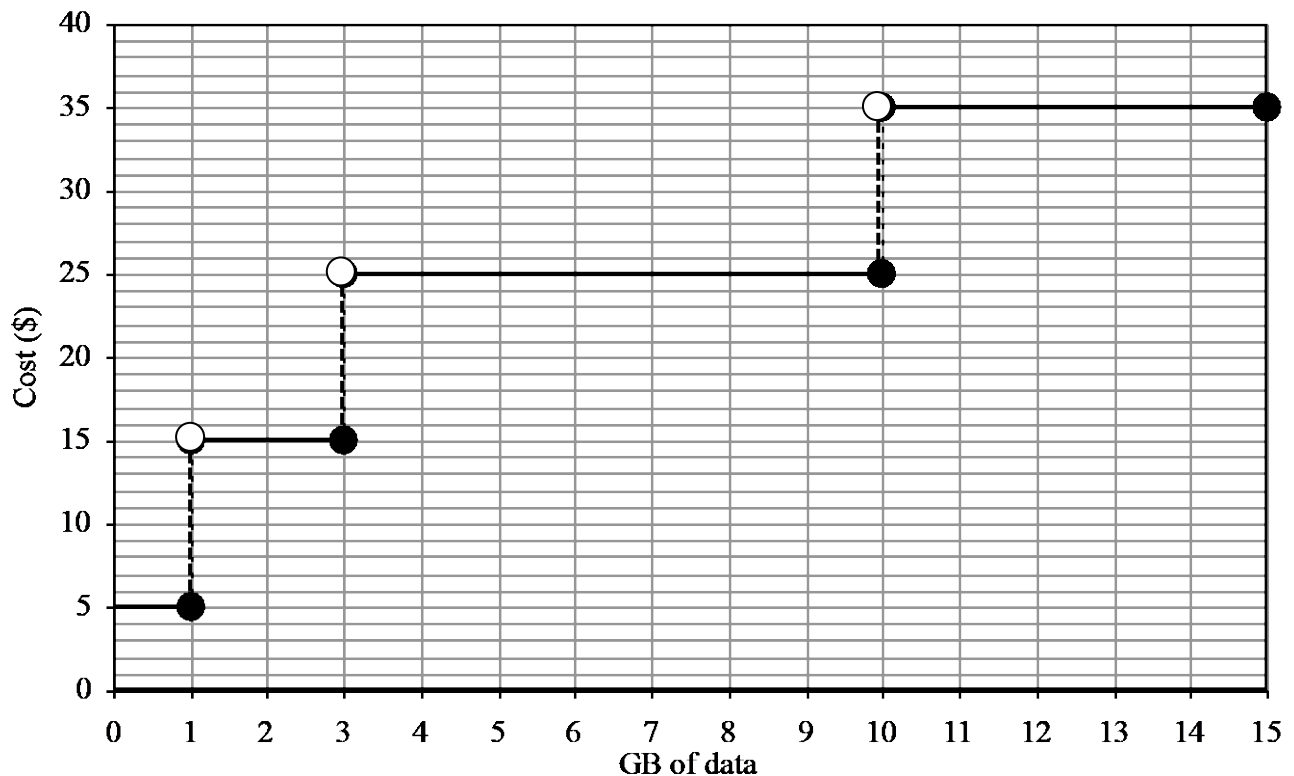
2 marks

End of Module 3 - SECTION B – continued

Module 4 – Graphs and Relations

Question One (5 marks)

Bill is investigating his mobile phone and internet plans with Modaphone. He first explores his data plan on his mobile and he is shown the graph below to explain his costs per month based on his per GB usage:



- (a) How much will Bill pay if he uses 10 GB of data each month?

1 mark

Bill determines that the graph can be modelled using the following set of equations where C is the cost in dollars and d is the number of GB of data:

$$C = \begin{cases} 5, & 0 < d \leq 1 \\ 15, & 1 < d \leq 3 \\ e, & f < d \leq g \\ 35, & 10 < d \leq 15 \end{cases}$$

- (b) State the values of e , f and g .

1 mark

- (c) Modaphone have an alternative plan where they charge \$2.50 per GB. Using C for cost in dollars and d for the number of GB of data, write an equation that would model the cost of this alternate plan.

1 mark

SECTION B – Module 4 – Question 1 - continued
TURN OVER

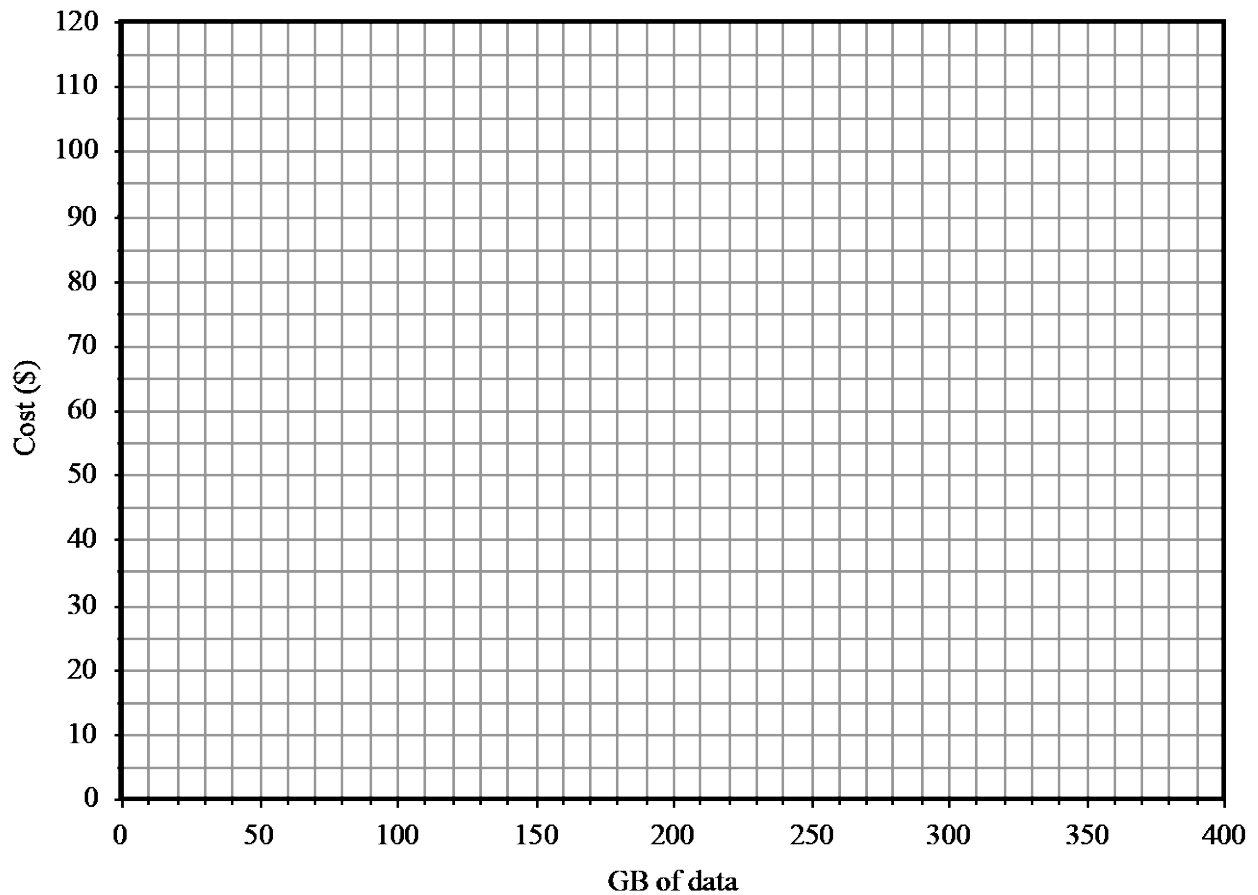
- (d) Add the graph of the relationship in part (c) to the graph above. **1 mark**
- (e) Explain exactly the circumstances when using the alternate plan at \$2.50 per GB of data would be less expensive for Bill. **1 mark**

SECTION B – Module 4 - continued

Question Two (5 marks)

Bill now explores his home internet plans. He is told that data is charged at 40 cents per GB up to and including 100 GB but that for every GB of data that he uses after the first 100 GB he will only be charged 20 cents. For this plan he cannot use more than 400 GB of internet per month.

- (a) Graph the relationship for Bill's internet plan on the provided axes below.

2 marks

The relationship between C , the cost in dollars for the internet and d , the number of GB of data can be modelled using the relationship below:

$$C = \begin{cases} 0.4d, & 0 < d \leq 100 \\ jd + k, & 100 < d \leq 400 \end{cases}$$

- (b) State the values of j and k for the equation given above.

1 mark

SECTION B – Module 4 – Question 2 - continued
TURN OVER

Bill is running an ancestry searching site through his internet and he charges a revenue, R , of 28 cents per GB of data he uses. The equation for his revenue is $R = 0.28d$.

- (c) Write a set of profit equations in terms of C and d that would model the profit from Bill's ancestry site. **1 mark**

- (d) How many GB of data would Bill need to use in order to make a profit from his ancestry site? **1 mark**

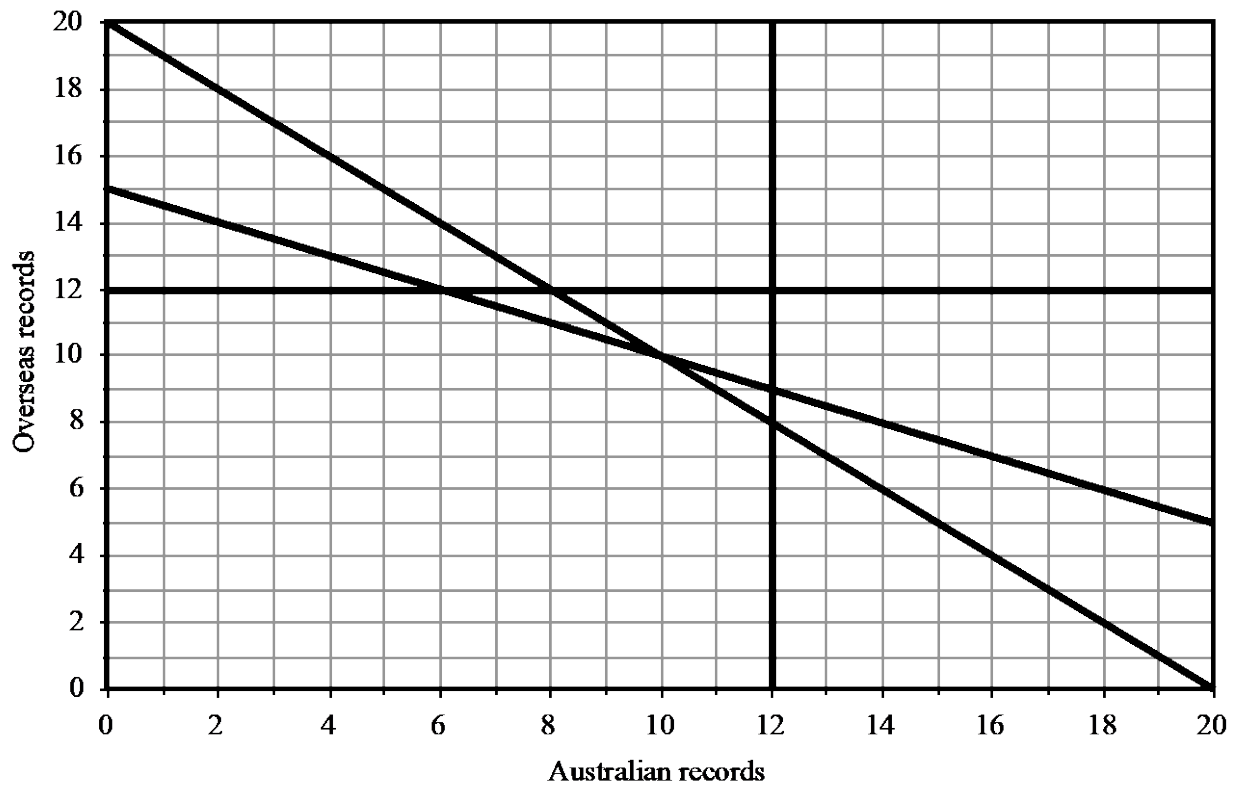
SECTION B – Module 4 - continued

Question Three (2 marks)

Bill's ancestry searches are limited by a number of factors:

- He is only allowed to search a maximum of 12 Australian records per day
- He is only allowed to search a maximum of 12 overseas records per day
- In total he cannot do more than 20 searches every day
- Each Australian search takes 20 minutes and each overseas search takes 40 minutes and he only has 10 hours (600 minutes) available for searching.

A graph showing the lines that form the boundaries of the feasible region is shown below:



Bill's maximum profit is made when he searches 10 Australian and 10 overseas records only.
What value(s) for the gradient of the profit function would ensure that this is true?

2 marks

END OF QUESTION AND ANSWER BOOK