2017
VCE
Further Mathematics
Trial Examination 2

Kilbaha Multimedia Publishing **PO Box 2227** **Kew Vic 3101** **Australia**	**Tel: (03) 9018 5376** **Fax: (03) 9817 4334** **kilbaha@gmail.com** **http://kilbaha.com.au**

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Letter

STUDENT NUMBER									

VICTORIAN CERTIFICATE OF EDUCATION 2017

FURTHER MATHEMATICS

Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Section A - Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	9	9	36
Section B - Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to Be cleared. For approved computer based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 30 pages.
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Section A – Core

Instructions for Section A

Answer all questions in the spaces provided. Write using blue or black pen.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data Analysis

Question 1 (12marks)

Sydney		Melbourne
	1	8
	2	0 1
	3	1 2 2 4 5
	4	0 1 1 2 3

Key 1/8 = 18^0C

The maximum temperatures over 13 days for Melbourne are given in the above stem and leaf graph. The maximum temperatures in degrees Celsius for Sydney over these same days were

25, 26, 19, 11, 35, 40, 31, 28, 33, 25, 27, 40, 31.

a. Complete the back-to-back stem and leaf diagram above to include the maximum temperatures for Sydney.

1 mark

b.

i. On what percentage of these days was the Melbourne maximum temperature at least 32^0C? Give your answer to three significant figures.

1 mark

_____

ii. What was the range of the maximum temperatures for Melbourne over these 13 days?

1 mark

_____

Question 1 (continued)

b.

iii. What was the mean of the maximum temperatures for Melbourne over these 13 days? Give your answer to four significant figures.

1 mark

_____

c. Describe the shape of the stem and leaf data for

i. Sydney _____

1 mark

ii. Melbourne _____

1 mark

d.

i. What was the median for Sydney's maximum temperatures over this time?

1 mark

_____

ii. What was the interquartile range for Sydney's maximum temperatures over this time?

1 mark

_____

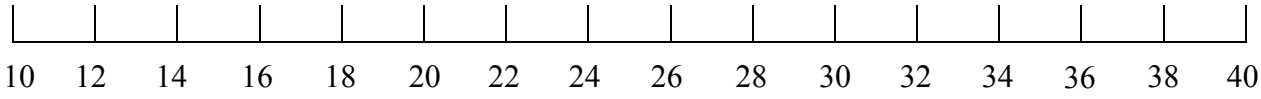
iii. Are there any outliers for Sydney's maximum temperatures over this time? Show how you arrived at your answer.

2 marks

_____

iv. On the grid below, draw the box plot to show Sydney's maximum temperatures over this time.

2 marks



Sydney's Maximum Temperatures

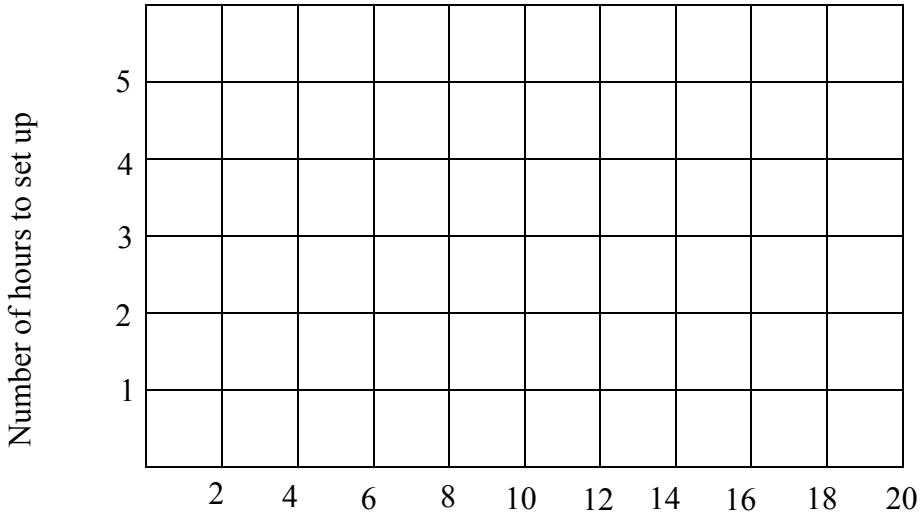
Question 2 (6 marks)

The time taken in hours by six groups of workers to set up a concert venue and the number of weeks that the group has worked together are given in the table below.

Number of weeks group has worked together	Number of hours to set up venue
10	3
20	2
12	4
15	2
8	5
16	4

a. Draw the scatter plot for the data on the grid below.

1 mark



b. i. Determine the equation of the least squares line that can be used to predict the time taken to set up a concert venue from the number of weeks that the group has worked together. Give your answers to one decimal place.

1 mark

Number of hours to set up = ☐ × Number of weeks group has worked together + ☐

Question 2 (continued)

b. ii. Draw this least squares line on the above scatter plot.

1 mark

iii. Interpret the gradient of this line in terms of hours taken to set up the concert venue and the number of weeks the group has worked together.

1 mark

_____

_____

c. Interpret the coefficient of determination in terms of these variables. Give your answer to three significant figures.

1 mark

_____

_____

d. Give one reason why it is not sensible to use your line of best fit to predict the length of time it will take a group to set up a concert venue if the group has been together for 3 weeks.

1 mark

_____

_____

Question 3 (2 marks)

At an athletics carnival for 11 and 12 year old students, the mean and standard deviation for each group's performance in the long jump was calculated. The results are given in the table below.

Age	Mean	Standard Deviation
11	4.8 m	0.6 m
12	5.2 m	0.8 m

2000 children in each of the age groups competed in this event. The results were normally distributed.

a. What was the standard score for an 11 year old who jumped 3.6 m?

1 mark

_____

b. Nathan is 12 years of age. He was placed 1997th in his age group in the long jump. How far did Nathan jump?

1 mark

_____

Question 4 (4 marks)

Quarterly Profits	Q_1	Q_2	Q_3	Q_4
2014	40000	52000		70000
2015	42000	42000	58000	64000
2016	46000	54000	56000	74000

The above table shows the quarterly profits in dollars for a toy store over a three-year period.

The table below shows the seasonal indices for the same quarters.

Seasonal Indices	Q_1	Q_2	Q_3	Q_4
2014	0.74	0.96	1	1.3
2015	0.82			1.24
2016	0.80	0.94	0.98	1.28

a. Show that the quarterly profit for the third quarter of 2014 was \$54000 to the nearest thousand.

1 mark

_____

b. What is the seasonal index for the third quarter of 2015?

1 mark

_____

c. What is the four-mean smoothed profit with centring for the third quarter in 2015?

2 marks

_____

Recursion and financial modelling

Question 5 (6 marks)

Aunt Joan opens an account for her niece, Zoe, on the day she is born. On this day she puts $500 into the account, which pays 10% simple interest per annum. Aunt Joan then adds $150 to this account each year on Zoe's birthday.

a. Write down a recurrence relation to model this situation.

_____

1 mark

b. How much money will Zoe have in her account the day after her tenth birthday?

_____

1 mark

c. Write a rule to show the amount of money, $A, that Zoe will have in her account after n years.

_____

1 mark

d. How much money will Zoe have in her account on the day after her 50^{th} birthday?

_____

1 mark

If Aunt Joan had invested the money at 5% annual compound interest when Zoe was born, and continued her annual birthday gift

e.

i. Write down a recurrence relation to model this situation.

_____

1 mark

ii. How much extra money would Zoe have in her account on the day after her tenth birthday? Give your answer to the nearest dollar.

_____

1 mark

Question 6 (3 marks)

Aunt Joan borrows $200000 from a bank at a rate of 6% per annum compounding monthly. She repays this money in equal monthly repayments over 12 years.

a. What is the monthly interest rate she is charged by the bank?

_____

1 mark

b. How much does she repay each month? Give your answer to the nearest cent.

_____

1 mark

c. What is the total mount of interest that she pays? Give your answer to the nearest dollar.

_____

1 mark

Question 7 (3 marks)

Ten years before her retirement, Aunt Joan started making monthly deposits of $500 into her superannuation account, which already contained $150000. The interest paid on this money was 5% per annum compounding monthly.

a. How much did she have in this account when she retired?

_____

1 mark

b. When she retired she invested her money at 6% per annum, compounding annually and withdrew $30000 per annum for living expenses. For how many years would she be able to do this?

_____

1 mark

c. At the end of 5 years of her retirement, Aunt Joan inherited some money. She decided to put some of this inheritance into a perpetuity account, paying 8% per annum, in order to provide the Multiple Sclerosis foundation with $3000 per year. How much money would she have to invest in the perpetuity to accomplish this?

_____

1 mark

Section B – Modules

Instructions for Section B

Select two modules and answer **all** questions within the selected modules.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Contents Page

Module 1 - Matrices

Question 1 (4 marks)

The streaming service, *Movieflix*, keeps records of the number of males and females who watch different genres of movies. The genres offered by this company are Thrillers, Westerns, Science Fiction and Comedy. Matrix X below shows the number of movies watched by each group in one town on one particular evening.

$$X = \begin{array}{c} \\ F \\ M \end{array} \begin{array}{c} \begin{array}{cccc} T & W & SF & C \end{array} \\ \begin{bmatrix} 200 & 50 & 95 & 180 \\ 126 & 174 & 140 & 108 \end{bmatrix} \end{array}$$

a. What is the order of matrix X?

_____

1 mark

Matrix Y below shows the average length in hours of each genre of movie.

$$Y = \begin{bmatrix} 1.8 \\ 2.0 \\ 2.5 \\ 1.6 \end{bmatrix}$$

b. Evaluate matrix Z where $Z = XY$.

_____

1 mark

c. Explain why YX is not defined.

_____

_____

1 mark

Question 1 (continued)

d. What information do the elements of matrix Z provide?

_____

1 mark

Question 2 (4 marks)

The matrix below shows the number of movies of each genre that are available on *Movieflix* in 2017.

$$\begin{matrix} T \\ W \\ SF \\ C \end{matrix}\begin{bmatrix} 500 \\ 480 \\ 220 \\ 540 \end{bmatrix}$$

Each year *Movieflix* increases the number of movies it has in each genre by 10% of the number in stock at the end of the previous year. They also discard 40 Thrillers, 80 Westerns and 30 Science fiction movies. The matrix P used to model this is given below.

$$P = a\begin{bmatrix} 500 \\ 480 \\ 220 \\ 540 \end{bmatrix} + \begin{bmatrix} b \\ c \\ d \\ e \end{bmatrix}$$

a. What are the values of a, b, c, d and e?

_____

_____

2 marks

Question 2 (continued)

b. How many Westerns will be available in 2020? Give your answer to the nearest whole number.

_____

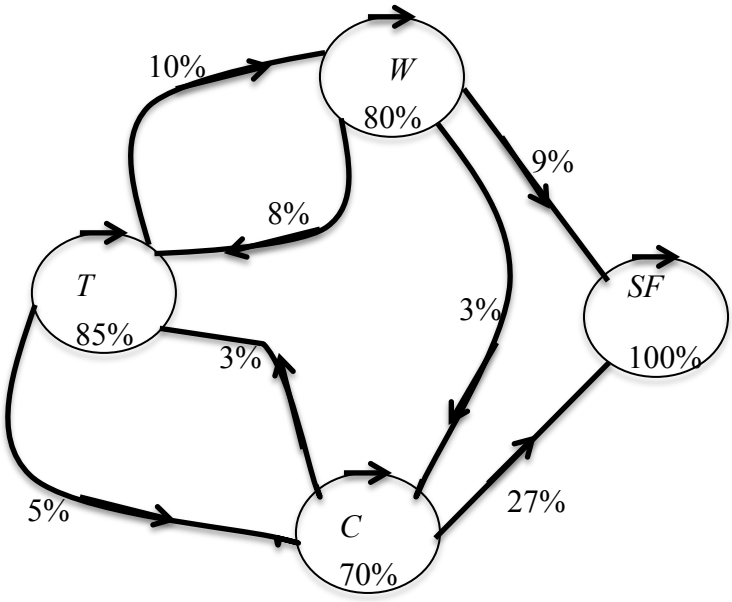
1 mark

c. How long will it take the company to have less than 200 Westerns left?

_____

1 mark

Question 3 (4 marks)



Movieflix records the percentage of people who change from one movie genre to another. The results are shown in the diagram above.

a. Complete the transition matrix below for the above data.

$$\begin{array}{cc} & \begin{array}{cccc} From & & & \\ T & W & SF & C \end{array} \\ \begin{array}{c} To \\ \\ \\ \\ \end{array} & \begin{array}{c} T \\ W \\ SF \\ C \end{array} \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

1 mark

b.

$$S_0 = \begin{array}{c} \\ T \\ W \\ SF \\ C \end{array} \begin{bmatrix} \\ 1000 \\ 2000 \\ 500 \\ 3000 \end{bmatrix}$$

What information does the matrix S_0 provide?

_____

_____

1 mark

Question 3 (continued)

c. Predict the number of people who will watch a Western for their fourth movie.

_____

1 mark

d. In the long term how many people would be expected to watch a comedy?

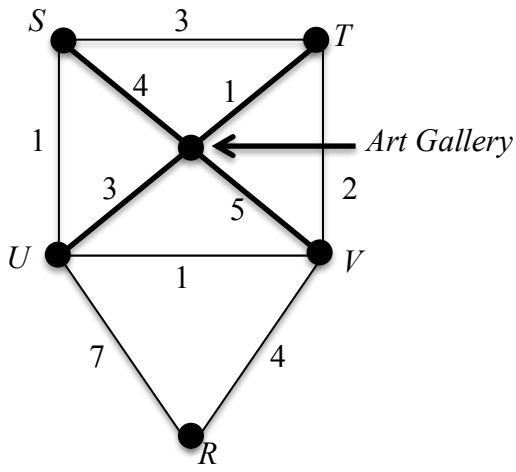
_____

1 mark

End of Module 1: Matrices

Module 2: Networks and decision mathematics

Question 1 (4 marks)



The graph above shows Riza's house, R, and the Art Gallery. It also shows the distances in kilometres between various points in the region.

a. How many paths are there from Riza's house to the Art Gallery?

_____

1 mark

b. What is the length of the shortest path from Riza's house to the Art Gallery?

_____

1 mark

c. How can you tell that an Euler path and not an Euler circuit exists for the given graph?

_____

_____

1 mark

Question 1 (continued)

d. To trace an Euler path on the above graph, where would you need to begin and end?

_____

1 mark

Question 2 (4 marks)

The Art Gallery employs five people, Riza, Sue, Tom, Umberto and Val to clean the establishment. The tasks that they are expected to perform include Vacuuming, dusting, cleaning the wet areas, polishing and moving paintings. The job allocation analyst at the gallery has found the times, in hours, that these five people take to complete each task and has listed them in table 1 below.

Table 1

	Vacuum	**Dust**	**Wet Areas**	**Polish**	**Move Paintings**
Riza	9	6	11	7	12
Sue	11	15	9	12	13
Tom	14	13	10	14	6
Umberto	11	13	12	10	8
Val	7	10	7	14	8

The job allocation analyst used the Hungarian algorithm to find the minimum time it would take to complete all these tasks. Table 2 shows the results of the first step of subtracting the smallest element in a row from each element in that row.

Table 2

	Vacuum	**Dust**	**Wet Areas**	**Polish**	**Move Paintings**
Riza	3	0	5	1	6
Sue	2	6	0	3	4
Tom	8	7	4	8	0
Umberto	3	5	4	2	0
Val	0	3	0	7	1

Question 2 (continued)

Table 3 shows the results of the second step of subtracting the smallest element in a column from all the elements in that column.

Table 3

	Vacuum	**Dust**	**Wet Areas**	**Polish**	**Move Paintings**
Riza	3	0	5		6
Sue	2	6	0		4
Tom	8	7	4		0
Umberto	3	5	4		0
Val	0	3	0		1

a. Complete **Table 3** by filling in the missing elements from the Polish column.

_____

1 mark

b. Explain why it is not possible at this stage of the Hungarian algorithm to allocate the tasks.

_____

_____

1 mark

Question 2 (continued)

Table 4 gives the final results after adding the minimum uncovered number to the intersection values and subtracting this number from the uncovered numbers.

Table 4

	Vacuum	**Dust**	**Wet Areas**	**Polish**	**Move Paintings**
Riza	3	0	6	0	7
Sue	1	5	0	1	4
Tom	7	6	4	6	0
Umberto	2	4	4	0	0
Val	0	3	1	6	2

c. Which task should be allocated to Umberto?

_____

1 mark

d. What is the minimum time for all the tasks to be completed?

_____

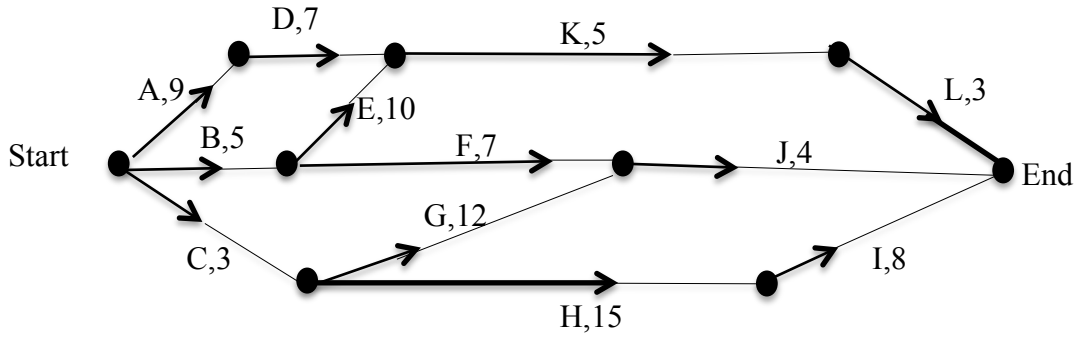
_____

_____

_____

1 mark

Question 3 (4 marks)



The Art Gallery is to have some extensions done. The directed graph above shows the 12 activities that have to be completed in order to build the extension and the time in weeks for each activity.

a. What is the minimum time for the extension to be completed?

_____

1 mark

The worker who has forecast the time that activity J will take, realizes that he has made a mistake and that activity J will actually take 12 weeks instead of 4 weeks.

b. How much longer will the extension take to complete?

_____

1 mark

Question 3 (continued)

The extension will cost $6,000 per week. Extra workers can be brought in to reduce the number of weeks an activity will take. The cost of an extra worker is $800 per week.

c.

i. If three extra workers are brought in for a week to reduce the time of activity A from 9 to 2 weeks, how would this affect the total cost of the project?

_____

1 mark

ii. If three extra workers are brought in for a week and one extra worker is brought in for two weeks to reduce the time of activity G from 12 to 4 weeks, how would this affect the total cost of the project?

_____

1 mark

End of Module 2: Networks and decision mathematics

Module 3: Geometry and measurement

Question 1 (2 marks)

Miles is organizing a race in the city of Riga (57^0N 24^0E) and Shayla is organizing a similar competition in the city of Haparanda (66^0N 24^0E). They both wish to start their competitions at the same time. Assume that the radius of the earth is 6400 km.

a. Why is there no time difference between the two cities?

_____

1 mark

b. Find the shortest distance along the great circle between the two cities. Give your answer to the nearest kilometre.

_____

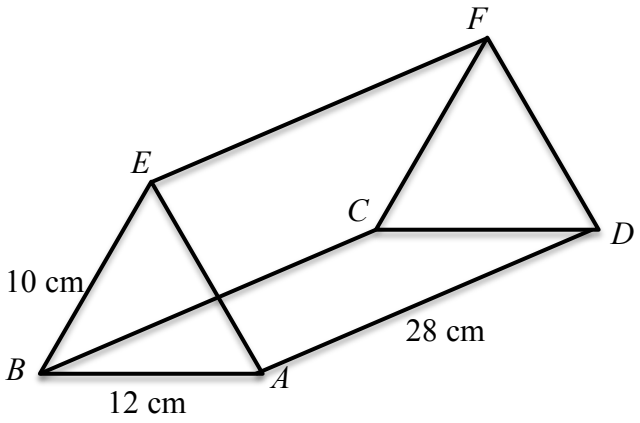
_____

_____

1 mark

Question 2 (3 marks)

Miles carries the trophies for the winners in Riga in the box below.



$AB = 12$ cm, $BE = 10$ cm and $AD = 28$ cm.

a. What is the length of the diagonal AC? Give your answer to three significant figures.

_____

_____

1 mark

b. What is the surface area of the box?

_____

_____

_____

1 mark

c. What is the volume of the box?

_____

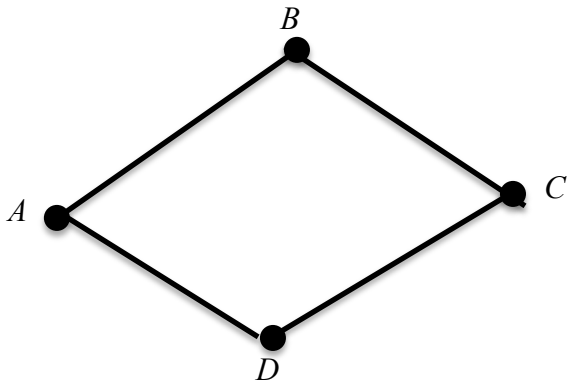
_____

_____

1 mark

Question 3 (4 marks)

The race in Riga begins at A. From here the contestants run for 6 km to B at a bearing of 060^0T. They then run 5 km on a bearing of 130^0 T to C. From here they run 9 km to D on a bearing of 190^0T. The final section is from D on a bearing of 320.6^0T to A.



a. How far east of *A* is *B*? Give your answer to one decimal place.

_____

_____

1 mark

b. What is the bearing of D from A?

_____

_____

1 mark

c. What is the length of AD? Give your answer to the nearest kilometre.

_____

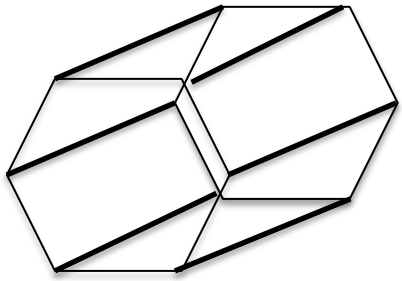
_____

_____

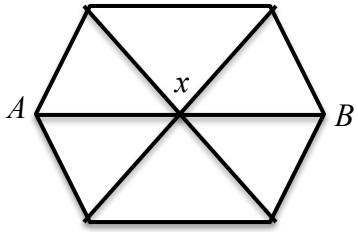
2 marks

Question 4 (3 marks)

The award for the first place getter is a regular hexagonal prism made of crystal. The depth of the crystal is the distance between the two hexagonal faces.



The distance from A to B for the hexagonal face of the crystal part for the first place getter is 16 cm and is shown below.



a. What is the size of the angle marked x?

_____

_____

1 mark

b. What is the area of the hexagonal face? Give your answer to one decimal place.

_____

_____

1 mark

Question 4 (continued)

c. The prize for second place is a similar shape to that for first place. The volume of crystal for the first place prize is 832 cm^3. The volume of crystal for the second prize is13 cm^3. What is the depth of the crystal for the second prize?

_____

_____

_____

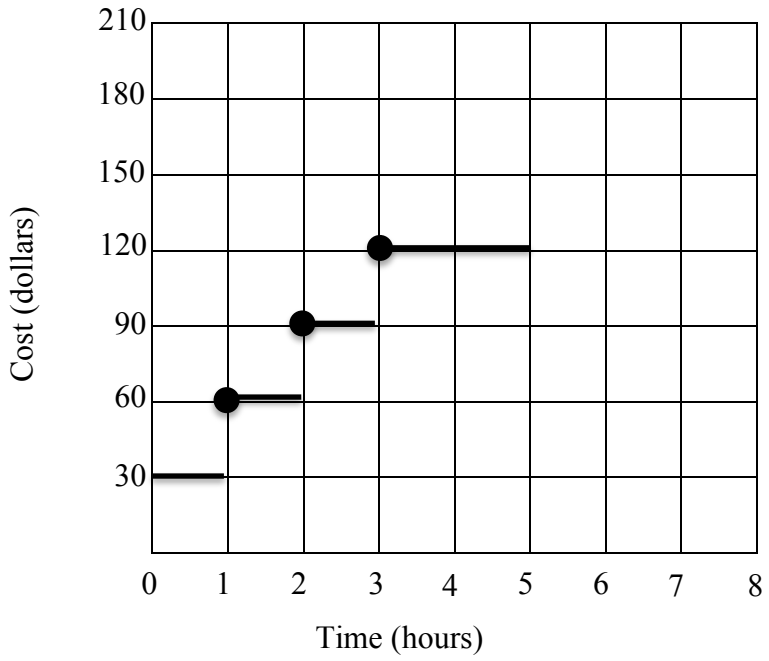
_____

1 mark

End of Module 3: Geometry and measurement

Module 4: Graphs and relations

Question 1 (3 marks)



Mr. Kingston wants to have his factory cleaned. The above graph shows part of the quote he received from the Eye Kleen Company. From five to eight hours inclusive the cost of the cleaning is $150.

a. Complete the above graph to show this information.

1 mark

b. How much would Mr. Kingston have to pay for three hours of cleaning?

_____

1 mark

Question 1 (continued)

c. Write the Cost function $\$C$ in terms of time, t.

$$C = \begin{cases} \\ \\ \\ \end{cases}$$

1 mark

Question 2 (6 marks)

Mr. Kingston manufactures gidgits. It costs him \$4.00 to produce one gidgit and the fixed cost for running the factory to do this is \$300.00

a. Write down the equation to show the total cost, $\$C$, of producing x gidgits.

_____

1 mark

b. How many gidgits can be produced for \$2300?

_____

1 mark

The sales revenue, $\$R$, for this product is given by the equation, $R = 10x$

c. How many gidgits must be sold for Mr. Kingston to break even?

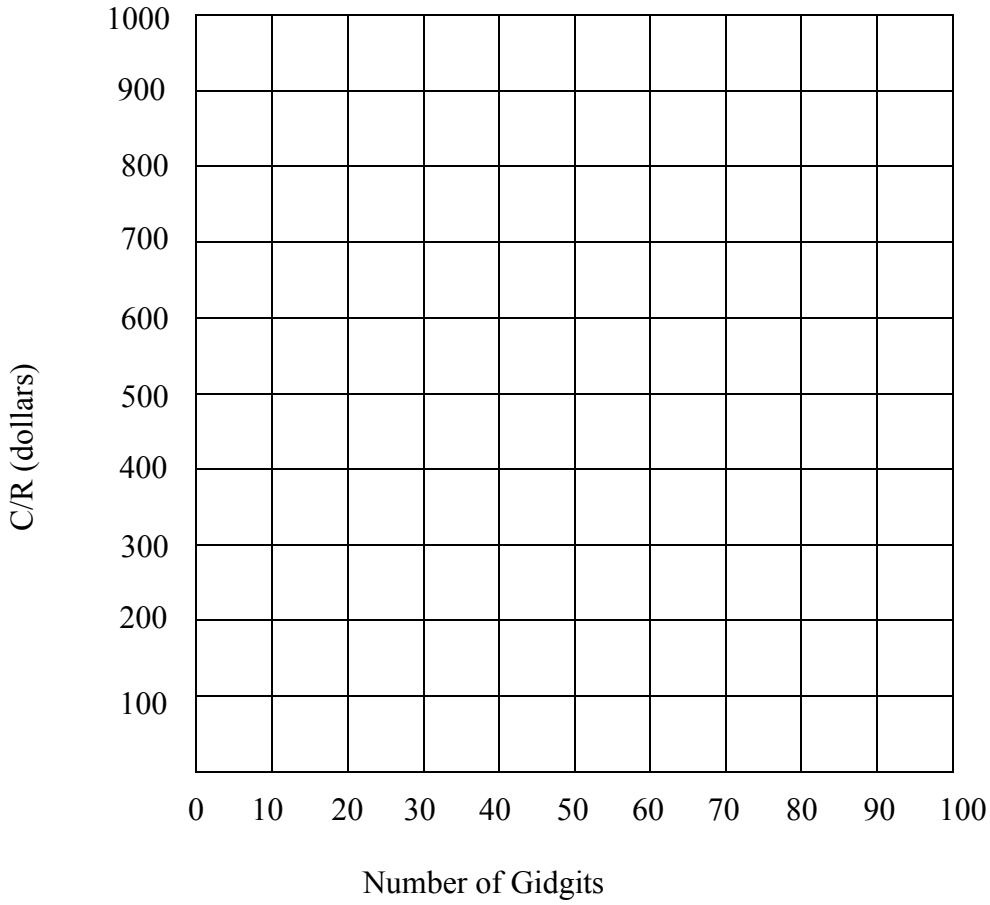
_____

1 mark

Question 2 (continued)

d. On the axes below draw the graph of C and R.

2 marks



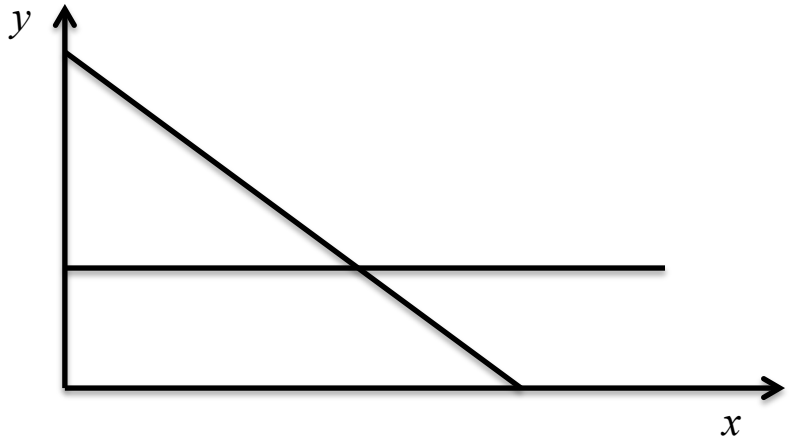
e. If the gidgits are packaged in batches of 40, what is the minimum number of packets that would have to be sold in order to make a profit?

_____

1 mark

Question 3 (3 marks)

Mr. Kingston also makes blicks and blocks in his factory where x = the number of blicks and y = the number of blocks.



The lines for two of the constraints, $y \geq 4$ and $x + 2y \leq 20$ for making these two products have been drawn above.

The third constraint is that the number of blocks made in the factory must not be more than twice the number of blicks that are made.

a. Write down this third constraint in terms of x and y.

_____

1 mark

b. Draw the required line and shade the region that satisfies the three constraints on the above graph.

1 mark

The profit equation is given by $P = 2x + 3y$

c. How many blicks and blocks should be produced to maximize the profit?

_____

1 mark

End of Module 4 : Graphs and relations

End of 2017 VCE Further Mathematics Trial Examination 2
Question and Answer Book

Kilbaha Multimedia Publishing **PO Box 2227** **Kew Vic 3101** **Australia**	**Tel: (03) 9018 5376** **Fax: (03) 9817 4334** **kilbaha@gmail.com** **http://kilbaha.com.au**

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score:	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line:	$y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value:	residual value = actual value – predicted value
seasonal index:	seasonal index$= \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core: Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1: Matrices

determinant of a 2×2 matrix:	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix:	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$
recurrence relation:	$S_0 =$ initial state, $\quad S_{n+1} = TS_n + B$

Module 2: Networks and decision mathematics

Euler's formula:	$v + f = e + 2$

Module 3: Geometry and measurement

area of a triangle:	$A = \frac{1}{2}bc\sin(\theta^0)$
Heron's formula:	$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$
sine rule:	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule:	$a^2 = b^2 + c^2 - 2bc\cos(A)$
circumference of a circle:	$2\pi r$
length of an arc:	$r \times \frac{\pi}{180} \times \theta^0$
area of a circle:	πr^2
area of sector	$\pi r^2 \times \frac{\theta^0}{360}$
volume of a sphere:	$\frac{4}{3}\pi r^3$
surface area of a sphere:	$4\pi r^2$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a prism:	area of base × height
volume of a pyramid:	$\frac{1}{3}$ × area of base × height

Module 4: Graphs and relations

gradient (slope) of a straight line:	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line:	$y = mx + c$

END OF FORMULA SHEET