# 2016
# VCE
# Further Mathematics
# Trial Examination 1
# Suggested Answers

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**Section A Core**

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| **Question 1  B**<br>5 outlets sold more than 50 pairs of jeans.<br><br>$\% = \dfrac{5}{20} \times 100 = 25\%$ | **Question 2  D**<br>You could have two bars, one for 'yes' and one for 'no'. This is categorical data. Then, you would have the frequencies for females stacked on top of the frequencies for males.<br>This is a segmented bar graph.<br>Histograms, scatter plots, box plots and stem and leaf plots are used when the independent variable is numerical. |
| **Question 3  B**<br>95% are within 2 standard deviations of the mean. This means that 95% lie between $170 - 10$ and $170 + 10$, between 160 and 180. Greater than 160 would be the 95% up to 180 and the 2.5% greater than $180 = 97.5\%$ | **Question 4  A**<br>For history, the IQR $= 70 - 50 = 20$.<br>$1.5 \times 20 = 30$<br>$50 - 30 = 20$. Any score less than 20 would be an outlier, so A is not true. |
| **Question 5  E**<br>$Z = \dfrac{90 - 70}{10} = 2$<br>$\Pr(X) > 2 = 2.5\%$<br>$\dfrac{2.5}{100} \times 5000 = 125$ | **Question 6  C**<br>Two points on the line are (60,7) and (85,12)<br>$Gradient = \dfrac{12 - 7}{85 - 60} = 0.2$<br><br>Rating $= 0.2 \times$ final exam result $+ b$<br>$7 = 0.2 \times 60 + b$<br>$b = $ -5<br>Rating $= 0.2 \times$ final exam result -5 |
| **Question 7  A**<br>The slope of the regression line is negative, so Pearson's correlation coefficient will be negative.<br>$r = -\sqrt{0.278} = -0.53$<br>We cannot say that the variability is caused by anything. The relationship is negative and moderate. | **Question 8  C**<br>Could use $\log(y), \log(x), \dfrac{1}{y}$ or $\dfrac{1}{x}$ to linearise the given graph. |

**Section A Core**

**Section A Core**

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| **Question 9  E**<br>Not A or C because they do not have 14 points shown.<br>When x = 2, the actual value – the predicted value has two positive points and one negative point, so not B or D. | **Question 10  E**<br>Use the 5 values for February, March, April, May, June.  These are 6, 5, 2, 7, 4.<br>Writing them in ascending order we get 2, 4, 5, 6, 7.  The median is the middle term, which is 5.  The smoothed value for sales for April was $5000 |
| **Question 11  B**<br>The seasonal indices add to 4.<br>4 – (0.89 + 1.06 + 1.13) = 0.92 | **Question 12  C**<br>$$\text{deseasonalised figure} = \frac{\text{actual figure}}{\text{seasonal index}}$$<br>$$= \frac{70,500}{1.13} = \$62,389$$ |
| **Question 13  E**<br>Seasonal Index for summer<br>= Summer value ÷ season average<br>Season average<br> = (2073 + 2414 + 2339 + 1967) ÷ 4<br>= 2198.25<br>SI for summer = 2414 ÷ 2198.25 = 1.098<br>This is close to 1.10, which means the occupancy was about 0.1 or 10% above the average in this season. | **Question 14  A**<br>The values would be<br>1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,2,3,3,3,3,3,4,4,4,4, 4,5,5,9.<br>Median is 2, lower quartile is 1,<br>upper quartile is 4.  Hence, not B or C.<br>IQR = 4 – 1 = 3<br>1.5 × 3 = 4.5.<br>4.5 + 4 = 8.5, so 9 is an outlier. |
| **Question 15  A**<br>Maths is the $y$ axis and Biology is the $x$ axis.<br>Using the calculator we get the equation<br>Maths = 0.94  × Biology + 4.6.  this means that when Biology = 10, Maths = 9.4 + 4.6 = 14.<br>The gradient is positive. | **Question 16  D**<br>Put the y values and the $x^2$ values of 1, 4, 9, 16 and 25 into calculator.  The linear regression line is $y = -0.68 \times x^2 + 21.5$<br>This gives $y = -0.68 \times 3.2^2 + 21.5 = 14.5$ |

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**Section A Core**

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| **Question 17  B**<br>$I = 5120 - 4000 = 1120$<br><br>$R = \dfrac{100I}{PT} = \dfrac{100 \times 1120}{4000 \times 5} = 5.6\%$ | **Question 18  A**<br>$w_{n+1} = 3w_n - 4$<br>$w_1 = 3 \times 2 - 4 = 2$<br>$w_2 = 3 \times 2 - 4 = 2$<br>Each term will be 2. |
| **Question 19  C**<br>$V_0 = 50000$<br>$V_n = 0.9 \times V_{n-1}$<br><br>Use calculator and keep multiplying previous answer by 0.9 and count how many times you do this till you reach \$23,914.85.  This requires 7 multiplications. | **Question 20  B**<br>Effective interest rate =<br><br>$\left[ (1 + \dfrac{1.625}{100})^4 - 1 \right] \times 100 = 6.66\%$ |
| **Question 21  D**<br>$V_0 = 50000$ as seen on the graph so the choices are between D and E.<br>Graph decreases by 40,000 in 10 years, which is 4000 per year.  Each term is 4000 less than the term before it. | **Question 22  A**<br>A perpetuity account keeps the amount invested forever, so E. |
| **Question 23  C**<br>Amount lost = \$5000 for 100,000 km<br>= 5,000 ÷ 100,000 = 5 ÷ 100 = \$0.05 per km | **Question 24  B**<br>$N = 60$<br>$I = 6.2$<br>$PV = 200,000$<br>$PMT = -1600$<br>$FV =$<br>$PpY = 12$<br>$CpY = 12$<br>*End*<br>This gives $FV = \$160259.767$<br>$N =$<br>$I = 5.8$<br>$PV = 160259.767$<br>$PMT = -2000$<br>$FV = 0$<br>$PpY = 12$<br>$CpY = 12$<br>This gives $N = 101.597$<br>$101.597 ÷ 12 = 8.5$ years.  The money would be paid back 9 years from 2016, which is 2025 |

**Module 1 Matrices**

| **Question 1  B**<br>This is an upper triangular matrix because all the values below the main diagonal are zero | **Question 2  E**<br>For multiplication to be defined, the number of columns of the first matrix must equal the number of rows of the second matrix.  $P$ has 3 columns but $R$ has only 2 rows. |
|---|---|
| **Question 3  D**<br><br>$X = \begin{bmatrix} 18 & -3 \\ 9 & -6 \end{bmatrix}$<br><br>The determinant $= 18 \times -6 - 9 \times -3 = -81$<br>or you can use calculator.<br><br>$X^{-1} = -\dfrac{1}{81}\begin{bmatrix} -6 & 3 \\ -9 & 18 \end{bmatrix} = -\dfrac{3}{81}\begin{bmatrix} -2 & 1 \\ -3 & 6 \end{bmatrix}$<br><br>$= -\dfrac{1}{27}\begin{bmatrix} -2 & 1 \\ -3 & 6 \end{bmatrix}$ | **Question 4  D**<br>B and C cannot be multiplied since the number of columns of the first matrix does not equal the number of rows of the second matrix.<br>Since we multiply rows by columns, we want tomatoes, carrots, zucchinis, going across in the first matrix and tomatoes, carrots and zucchinis going down in the second matrix. |
| **Question 5  C**<br>Inconsistent means that the equations have no solution, i.e. they are parallel lines.<br>Start by checking the determinant for each set of equations.  Use the calculator to do this.  The first set, the third set and the fourth set of equations have the determinant = 0.<br>The first set can be written $x + y =$ a constant, where the constant is different for each equation.  This means that the lines are parallel and have no solution.  The same is true for the third system of equations where $x - 2y =$ a constant.<br>The fourth system of equations is not inconsistent because they are the same straight line and have an infinite number of solutions. | **Question 6  C**<br>$R = 6P \times Q$ must be a $3 \times m \times p \times q$<br>$= 3 \times q$ matrix.<br>R is a square matrix, so $q = 3$.<br>$Q + S$ is defined, so $p \times 3$ is same order as $2 \times t$ so $t = 3$ and $p = 2$<br>$PQ = 3 \times m \times 2 \times 3 = 3 \times 3$ matrix<br>so $m = p = 2$<br>$R^{-1}$ is of the same order as R, so it is $3 \times 3$ |

**Module 1 Matrices**

| **Question 7  B** | **Question 8  D** |
|---|---|
| $$\begin{array}{c} \quad R \quad R' \\ \begin{array}{c} R \\ R' \end{array} \begin{bmatrix} 0.3 & 0.4 \\ 0.7 & 0.6 \end{bmatrix}^3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{array}{c} R \\ R' \end{array} = \begin{bmatrix} 0.363 \\ 0.637 \end{bmatrix} \end{array}$$ Probability it will rain on Thursday is 0.363 | P gets 1 win, 0 draws and 3 losses so the first row of the matrix is 1, 0. 3. Q gets 1 win, 1 draw and 2 losses so the second row of the matrix is 1, 1, 2. R gets 2 wins, 0 draws and 2 losses so the third row of the matrix is 2, 0, 2. S gets 2 wins, 1 draw and 1 loss so the fourth row of the matrix is 2, 1, 1.  This must be multiplied by a column matrix that is 3 by 1. |

## Module 2 Networks and decision mathematics

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| **Question 1   D**<br>There are 9 edges, 6 vertices and 5 faces.  There are 2 odd vertices and 4 even vertices. | **Question 2   A**<br>A tree has no loops or cycles. |
| **Question 3   B**<br>For an Eulerian circuit, all the vertices must be even.  Only T and S are odd, so another road connecting these two points would make them even. | **Question 4   E**<br>P, Q, R and V all feed into W either directly or indirectly, so must all be finished before W begins.  S, T and U do not have to be completed before W begins. |

**Question 5   E**
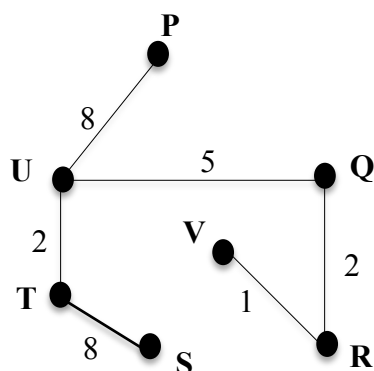


Length of cable
= 1 + 2  +  5 + 2 + 8 + 8 = 26 km

**Question 6  B**
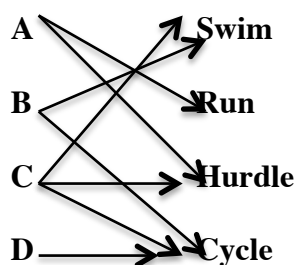


Dianne must cycle.  This means that Ben must swim.  This leaves Carla to hurdle and then Adam must run.

**Question 7  B**
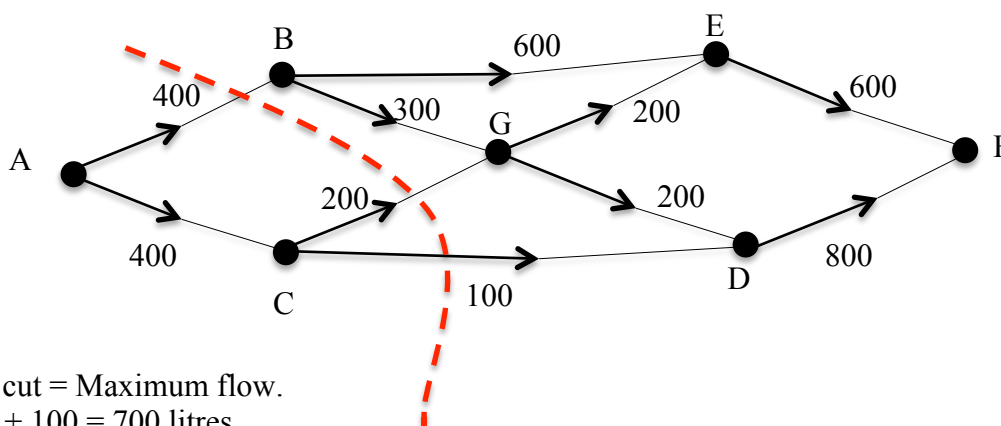


Minimum cut = Maximum flow.
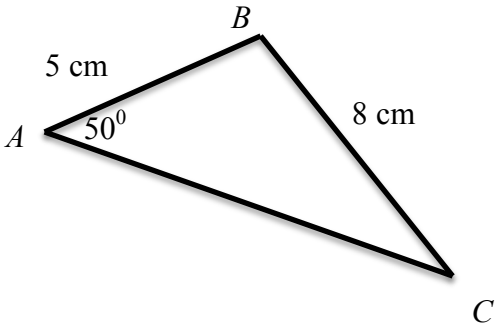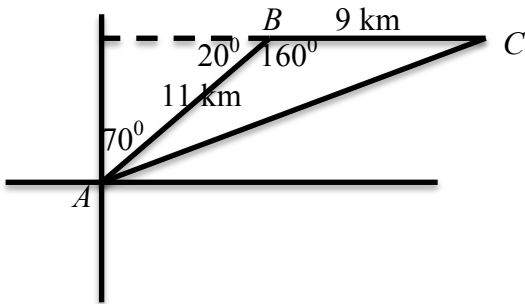400 + 200 + 100 = 700 litres.

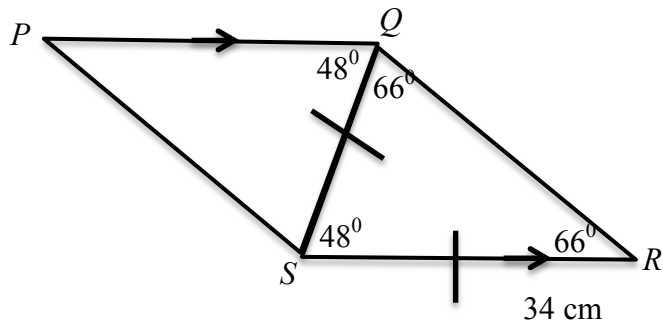**Question 8  C**
The critical path is ABDGHJ which takes 14 days
Activity I takes 1 day to complete, so the latest starting time for activity I is 14 – 1 = 13.
The earliest starting time for activity I is when ABDG is completed.  This takes 9 days.
Float time = 13 – 9 = 4

**Module 3 Geometry and measurement**

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| **Question 1   D**<br><br>$V =$ Area of base $\times$ height<br>$V = \pi \times r^2 \times h$<br>$V = \pi \times 5^2 \times 15$<br>$V = 1178 \text{ cm}^3$ | **Question 2  E**<br><br><br><br>$\dfrac{8}{\sin 50^0} = \dfrac{5}{\sin C}$<br><br>$\sin C = \dfrac{5\sin 50^0}{8}$<br><br>$C = 28.6^0$<br>$\angle ABC = 180 - (50 + 28.6) = 101.4$<br>This is closest to $100^0$ |
| **Question 3   A**<br>$A =$ base $\times$ perpendicular height<br>$\sin 70^0 = \dfrac{h}{5}$<br>$h = 5\sin 70^0$<br>$A = 9 \times 5\sin 70^0 = 42$ | **Question 4   C**<br>$l = 12 \times \dfrac{\pi}{180} \times (360 - 300)$<br>$l = 4\pi$<br>$2\pi r = 4\pi$<br>$r = 2$<br>Diameter = 4 cm |
| **Question 5  D**<br><br><br><br>$AC^2 = 11^2 + 9^2 - 2 \times 9 \times 11\cos 160^0$<br>$AC = \sqrt{11^2 + 9^2 - 2 \times 9 \times 11\cos 160^0} = 19.7 km$ | **Question 6  E**<br><br>Ratio of volumes $= 1 : 156 \div 22.4 = 1 : 6.964$<br><br>Ratio of lengths $= 1 : \sqrt[3]{6.963} = 1 : 1.90965$<br><br>Ratio of areas $= 1 : 1.90965^2 = 1 : 3.65$<br><br>Surface area of A $= 3.65 \times 52 = 189.8 \text{ cm}^2$ |

**Question 7  B**



$QRS$ is an isosceles triangle and $PQS$ and $QSR$ are alternate angles, which are equal. This means that the other two angles in triangle $QRS$ are equal to $66^0$

$$\frac{34}{\sin 66^0} = \frac{QR}{\sin 48^0}$$

$QR = 27.7$

**or**

Use the cosine rule.

**Question 8  B**
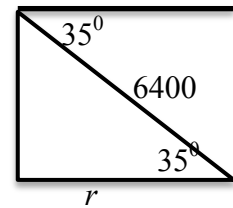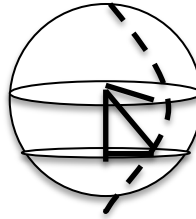


$$\cos 35^0 = \frac{r}{6400}$$

$r = 5242.57$

Radius of earth $= 6400$ km.
Angle between the 2 cities is $150 + 90 = 240$ or going the other way, $360 - 240 = 120^0$. This latter angle will give the shorter distance.

$$l = 5242.57 \times \frac{\pi}{180} \times 120 = 10980 \text{ km}$$

**Module 4 Graphs and relations**

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| **Question 1  C**<br>$50 for the 25 kg parcel and $30 for the 15 kg parcel = $80 | **Question 2  D**<br>Points on *AB* are $(0, 0)$ and $(1, -1)$.<br>Gradient of $AB = \dfrac{-1-0}{1-0} = -1$<br>Equation of *AB* is $y = -x$<br>*CD* is parallel to this with a *y* intercept of 5, s its equation is $y = -x + 5$<br>$x + y = 5$ |
| **Question 3  B**<br>speed = distance ÷ time = 30 ÷ 2 = 15 km/hr. | **Question 4   E**<br>$x = 4$ is the vertical line passing through $(4,6)$. The shaded region is less than or equal to this line, so not A or D.<br>The line $y = x - 2$ is the line passing through $(0, -2)$ and $(2, 0)$  The shaded region is greater than or equal to this line, so not C.<br>The remaining line is $y = 10 - x$.  The shaded region is less than or equal to this line, so not B. |
| **Question 5  E**<br><br>$C = 7500 + 20n$<br>$R = 35n$<br>$35n - 7500 - 20n = 6000$<br>$n = 900$ | **Question 6  A**<br>$y = 3x^{-1} = \dfrac{3}{x}$ |
| **Question 7  C**<br>Maximum value will occur at the last point that the line Z=0 would slide onto.  This is R. | **Question 8  B**<br>$x^2 + 2x = 3$<br>$= x^2 + x + x = 3$<br>$y = x^2 + x$<br>$y + x = 3$<br>$y = -x + 3$ |

**End of Suggested Solutions 2016 VCE Further Mathematics Trial Examination 1**