

FURTHER MATHEMATICS

Written Examination 2



(TSSM's 2015 trial exam updated for the current study design)

SOLUTIONS

Core – Data Analysis

Question 1

a. $Range = 42 - 7 = 35$ hours

1 mark

b. $Q_1 = 12.5, Q_3 = 23.5, IQR = 11$

$$Q_1 - 1.5 \times IQR = -4, Q_3 + 1.5 \times IQR = 40$$

Hence, 41 and 50 are outliers.

The data is positively skewed with outliers (41 and 50)

2 marks

c. $\frac{13}{23} \times 100 = 56.5 \%$

1 mark

d. $\bar{x} = 24.48$ hours

1 mark

Question 2

- a. Percentage of population living in towns.

1 mark

There is a strong negative linear relationship between the percentage of population living in towns and the percentage of population in farming jobs.

1 mark

- b. Slope – the percentage of population living in towns decreases by 0.4982 with every percent increase in population in farming jobs.

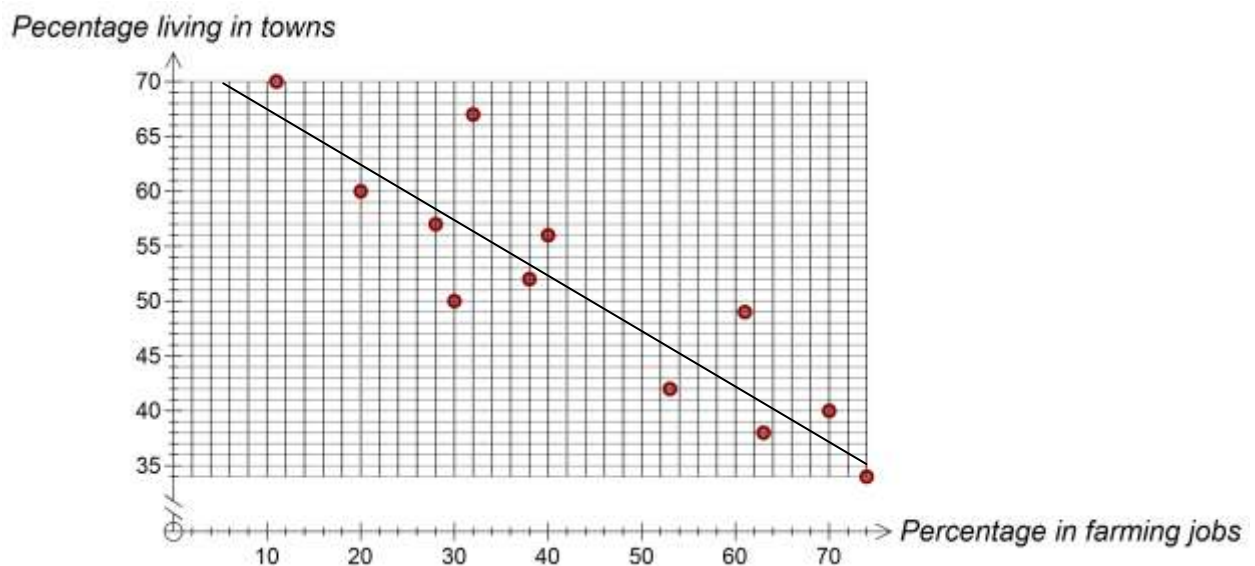
Vertical Intercept – 72.8384% of population was living in the towns when there was no farming job.

2 marks

- c. $r^2 = 0.8043$
80.43%

1 mark

- d.



1 mark

- e.

$$\text{Predicted value} = 72.8384 - 0.4982 \times 74 = 35.9716$$

$$\text{Actual value (from graph)} = 34$$

$$\text{Residual} = 34 - 35.9716 = -1.97$$

1 mark

Question 3

- a. $\log_{10}(\text{no of bacteria})$.

1 mark

- b. Use CAS to generate the lists.

$$\log_{10}(\text{Number of bacteria}) = 4.5941 - 0.0949 \times \text{Day number}.$$

2 marks

Total 15 marks

Core – Recursion and financial modelling

Question 4

- a. Increase = $45 \times 0.1 = \$4.50$

1 mark

- b. Price = $45 \times 1.1^3 = \$59.90$

1 mark

- c. $P_n = 1.1 P_{n-1}$, $P_0 = 45$

2 marks

- d. 2019. Generate the list on CAS.

2 marks

Question 5

- a. Interest = $116395 - 75000 = \$41395$

1 mark

- b.

$$116395 = 75000 \left(1 + \frac{r}{400} \right)^{24} \text{ or use TVM Solver}$$

$$r = 7.4\%$$

1 mark

c. $116395\left(1 + \frac{7.8}{1200}\right) + 480 = \117632

1 mark

- d. Use TVM Solver or generate sequence on CAS.
\$131 776

2 marks

Question 6

- a. Use TVM Solver – $N = 36$, $I = 6.95$, $PV = 200000$, $Pmt = -1350$, $PpY = 12$, $CpY = 12$
\$192352.48

2 marks

- b. Use TVM solver
28 years and 3 months.

2 marks

Total 15 marks

Module 1: Matrices

Question 1

- a. 3×4 1 mark
- b. 3 1 mark
- c. There were 20 pasties sold on Tuesday. 1 mark
- d. The number of columns in matrix C is the same as the number of rows in matrix N. 1 mark
- e. $CN = \begin{bmatrix} 99.70 & 75.50 & 40.90 & 85.80 \end{bmatrix}$
This matrix represents the sales collected on each of the four days. 2 marks

Question 2

- a. $T = \begin{bmatrix} 0.90 & 0.15 & 0.25 \\ 0.075 & 0.80 & 0.25 \\ 0.025 & 0.05 & 0.5 \end{bmatrix}$ 2 marks
- b. 15% 1 mark
- c. Need to consider only those people who change from C, not those who stay shopping at C the following week
 $0.025 \times 240 + 0.05 \times 195 = 15.75$
16 customers change their preference to shop C 1 mark

d. $T^2 \times S_1 = \begin{bmatrix} 337.95 \\ 238.875 \\ 83.175 \end{bmatrix}$

This matrix shows the number of customers buying their bakery products from each of the three shops after 2 weeks.

2 marks

Question 3

a. The revenue collected in week 2.

1 mark

b.

$$\begin{bmatrix} 270 & 182 \\ 235 & 205 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{41}{2516} & \frac{-91}{6290} \\ \frac{-47}{2516} & \frac{27}{1258} \end{bmatrix}$$

2 marks

Total 15 marks

Module 2: Networks and decision mathematics

Question 1

a. $a = 35, b = 0$

2 marks

b. The last column does not have a zero entry.

1 mark

c. $1 \rightarrow B, 2 \rightarrow D, 3 \rightarrow C, 4 \rightarrow A \quad d = 75 + 65 + 90 + 45 = 275 \text{ km}$

$1 \rightarrow D, 2 \rightarrow C, 3 \rightarrow B, 4 \rightarrow A \quad d = 80 + 55 + 95 + 45 = 275 \text{ km}$

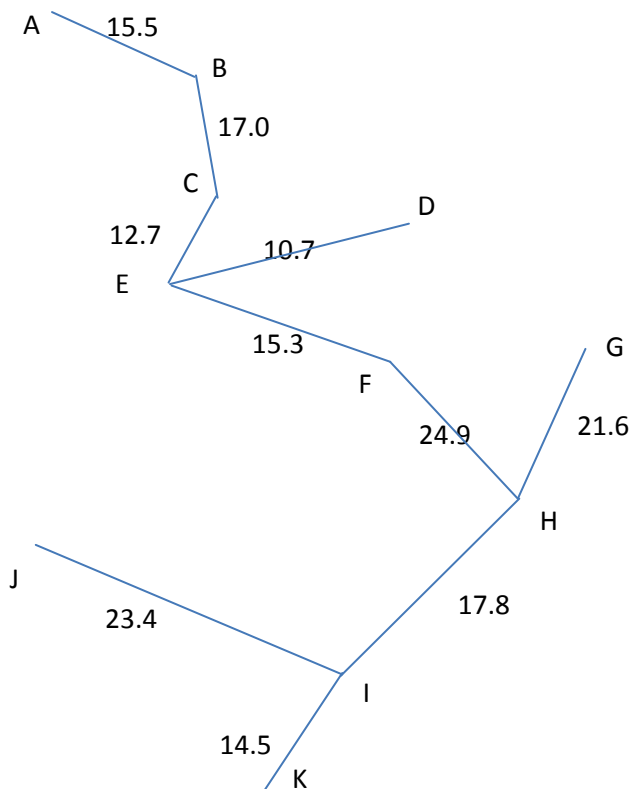
1 mark

d. $80 + 55 + 95 + 45 = 275 \text{ km}$

1 mark

Question 2

a.



2 marks

b. 173.4 km

1 mark

Question 3**a.**

Activity	Time (in hours)	Depends on
A	3	-
B	5	-
C	2	A
D	3	A
E	3	B, D
F	5	C, E
G	1	C
H	2	F, G

2 marks

b. A - D - E - F - H

2 marks

Question 4**a.** Euler trail has to start at B or C as they are the only vertices with odd degree.

2 marks

b. B-E-D-F-G-C-D-B-A-C or the reverse path from C to B.

1 mark

Total 15 marks

Module 3: Geometry and measurement

Question 1

a. $\frac{1}{2}(1 + 1.2) \times 0.8 = 0.88 \text{ m}^2$

1 mark

b. Side length $= \sqrt{0.8^2 + 0.1^2} = 0.806$

Perimeter $= 1 + 0.806 + 0.806 + 1.2 = 3.81 \text{ m}$

2 marks

Question 2

a. Distance $= \sqrt{1.2^2 + 0.806^2 - 2 \times 1.2 \times 0.806 \times \cos(83^\circ)} = 1.362 \text{ m}$

2 marks

b. $0.88 \times 0.015 \times n = 3.3$
 $n = 250$ table tops

1 mark

Question 3

a. $360^\circ - (180^\circ - 124^\circ) = 304^\circ$

1 mark

b. $\cos(56^\circ) = \frac{x}{25}$
 $x = 13.98 \text{ km} = 13980 \text{ m}$

1 mark

c. $BP = \sqrt{32^2 + 25^2 - 2 \times 32 \times 25 \times \cos(94^\circ)} = 42.0 \text{ km}$

1 mark

d. $\frac{\sin(PBA)}{25} = \frac{\sin(94^\circ)}{42}$
 Angle $PBA = 36.43^\circ$
 Bearing of P from $B = 180^\circ + 38^\circ + 36.43^\circ = 254^\circ T$

3 marks

Question 4

a. $\sqrt[3]{9} : \sqrt[3]{4}$

1 mark

b. SA of smaller bowl = $\frac{\sqrt[3]{4}}{\sqrt[3]{9}} \times 24$

$$4\pi r^2 = \frac{\sqrt[3]{4}}{\sqrt[3]{9}} \times 24$$

$$r = 1.2 \text{ cm}$$

2 marks

Total 15 marks

Module 4: Graphs and relations

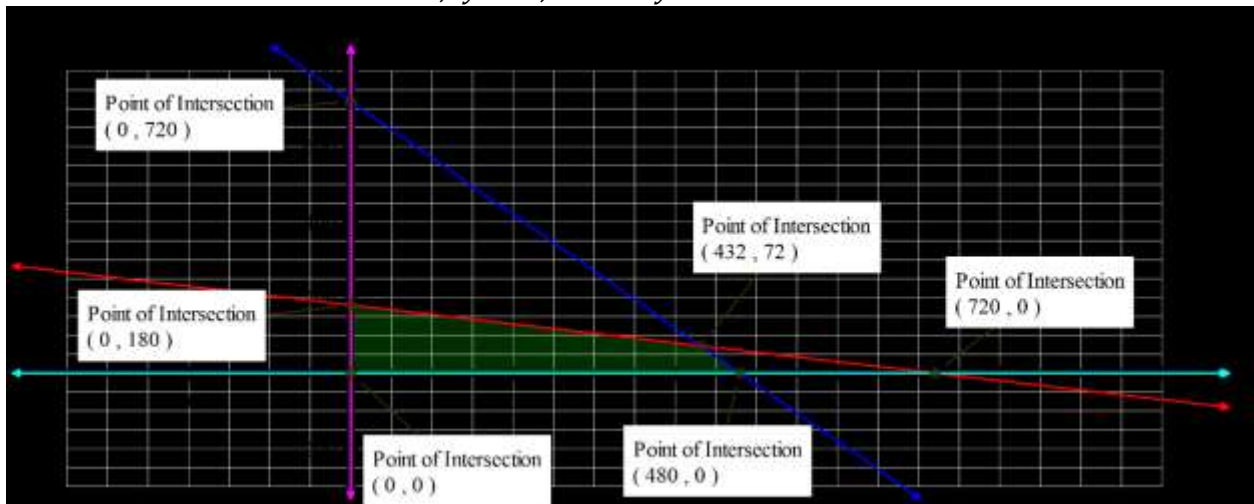
Question 1

a. $2x + 8y \leq 1440$ or $x + 4y \leq 720$

1 mark

b.

The other constraints are $x \geq 0$, $y \geq 0$, $3x + 2y \leq 1440$



3 marks

c. Profit (0, 180) = 3060
 Profit (480, 0) = 4800
 Profit (432, 72) = 5544
 Max Profit = \$5544

1 mark

d. Wood screws = 432 boxes
 Metal screws = 72 boxes

1 mark

Question 2

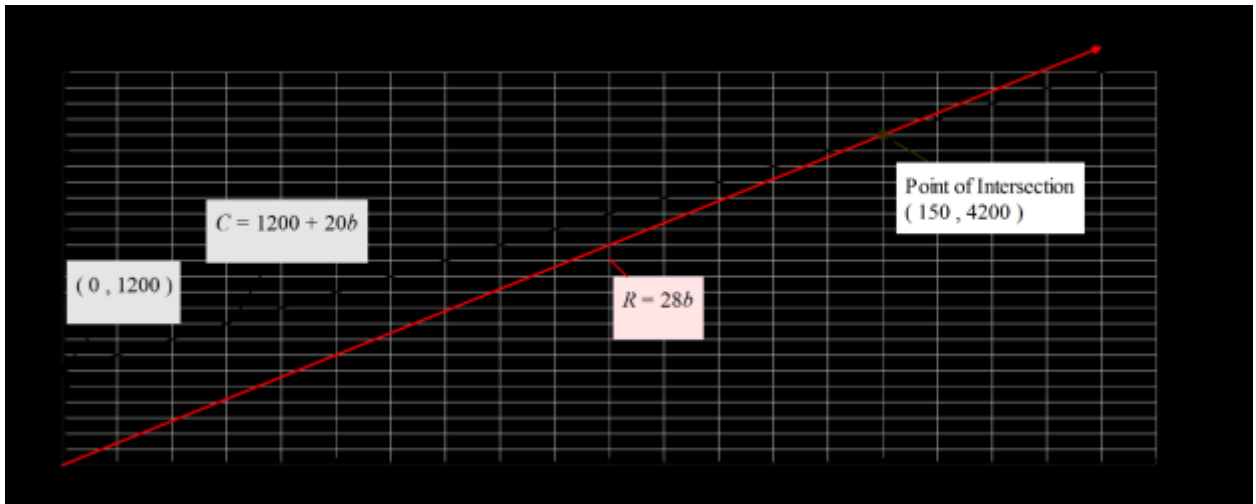
a. It will cost the manufacturer \$20 to produce the metal screw boxes.

1 mark

b. $28b - (1200 + 20b) = 8b - 1200$

1 mark

c.

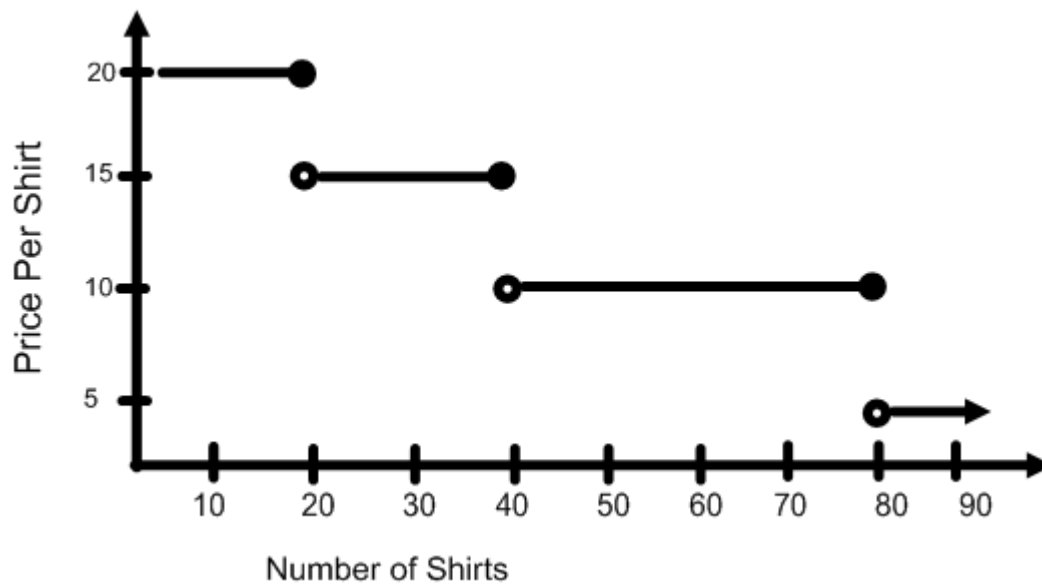


Break-even point is $(150, 4200)$

3 marks

Question 3

a.



2 marks

b. \$750

1 mark

c. $a = 20$ and $b = 80$

1 mark

Total 15 marks