

2015 VCE Further Mathematics Trial Examination 2 Suggested Answers



Kilbaha Multimedia Publishing
PO Box 2227
Kew Vic 3101
Australia

Tel: (03) 9018 5376
Fax: (03) 9817 4334
kilbaha@gmail.com
<http://kilbaha.com.au>

IMPORTANT COPYRIGHT NOTICE

- This material is copyright. Subject to statutory exception and to the provisions of the relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Kilbaha Multimedia Publishing.
- The contents of this work are copyrighted. Unauthorised copying of any part of this work is illegal and detrimental to the interests of the author.
- For authorised copying within Australia please check that your institution has a licence from <http://copyright.com.au>. This permits the copying of small parts of the material, in limited quantities, within the conditions set out in the licence.

Reproduction and communication for educational purposes The Australian Copyright Act 1968 (the Act) allows a maximum of one chapter or 10% of the pages of this work, to be reproduced and/or communicated by any educational institution for its educational purposes provided that educational institution (or the body that administers it) has given a remuneration notice to Copyright Agency Limited (CAL) under the Act.

For details of the CAL licence for educational institutions contact
CAL, Level 15, 233 Castlereagh Street, Sydney, NSW, 2000
Tel: (02) 9394 7600
Fax: (02) 9394 7601
Email: info@copyright.com.au

- All of these pages must be counted in Copyright Agency Limited (CAL) surveys
- This file must not be uploaded to the Internet.

These suggested answers have no official status.

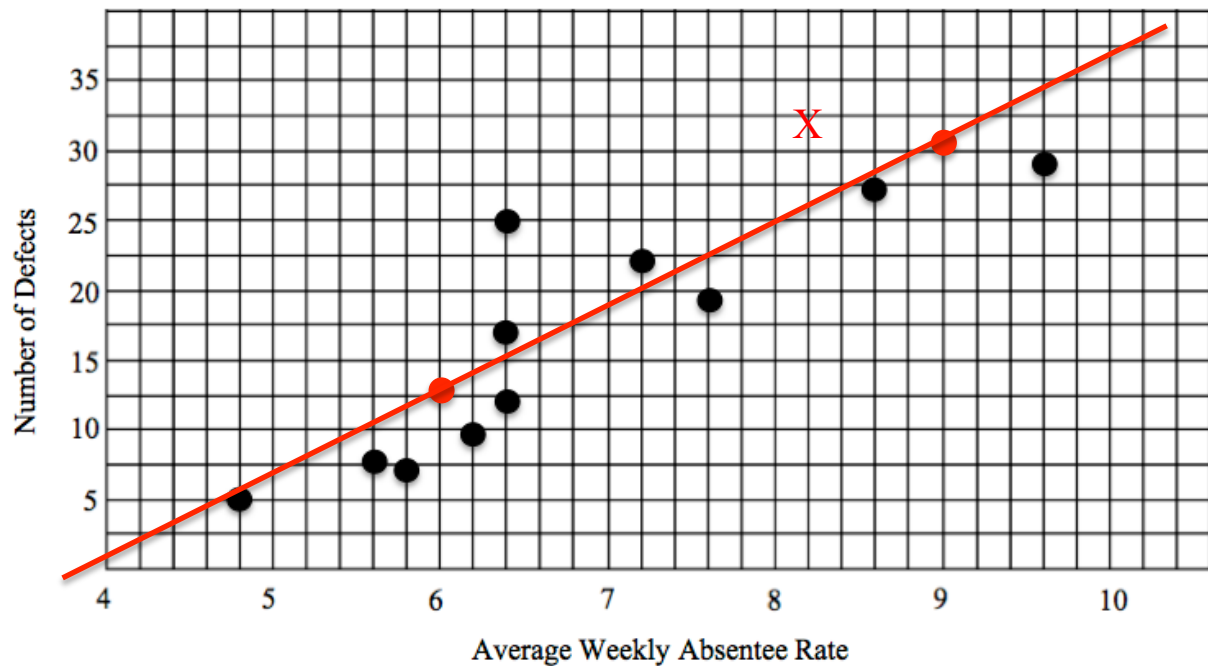
While every care has been taken, no guarantee is given that these answers are free from error. Please contact us if you believe you have found an error.

CAUTION NEEDED!

All Web Links when created linked to appropriate Web Sites. Teachers and parents must always check links before using them with students to ensure that students are protected from unsuitable Web Content. Kilbaha Multimedia Publishing is not responsible for links that have been changed in this document or links that have been redirected.

Core: Data analysis**Question 1**

a. X shows this point



(1 mark)

b.

Use calculator to get

$$\text{Number of defects} = 6 \times \text{Average weekly absentee rate} - 23$$

(1 mark)

c.

To draw the line, find 2 points.

e.g. When $x = 9$, $y = 54 - 23 = 31$. Point is (9,31)

When $x = 6$, $y = 36 - 23 = 13$. Point is (6,13)

Mark these 2 points on the graph and draw the line through them.

(1 mark)

d.

The gradient predicts that the number of weekly defects increases by 6 for each additional average weekly absentee rate.

(1 mark)

e.

Residual value = Actual value – Predicted value.

Actual value = 32

Predicted value = $6 \times 8.2 - 23 = 26.2$

Residual = $32 - 26.2 = 5.8$

(1 mark)

Core: Data analysis**Question 2****a.**

$$\text{Unacceptable} = \frac{7}{20} \times 100 = 35\%$$

(1 mark)

b.

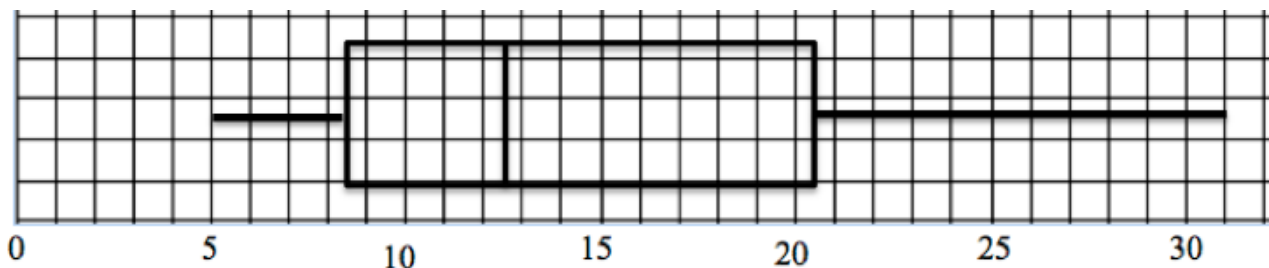
Use calculator to get

Mean = 16.3

(1 mark)

Standard deviation = 7.7

(1 mark)

c.

Waiting times (Days)

Minimum = 5

LQ = 8.5

Median = 12.5

UQ = 20.5

Maximum = 31

1 mark for correct values

1 mark for correct positioning.

(2 marks)

Core: Data analysis**Question 3**

a. Because the normal distribution graph is symmetrical about the mean, 50% will have waiting times of less than 15 days. (1 mark)	b. $z = \frac{19 - 15}{2} = 2$ 19 is 2 standard deviations above the mean of the standard normal curve. 95% lie within ± 2 standard deviations of the mean. So 5% lie outside. Half of these, i.e. 2.5% lie above 19. 2.5% have to wait more than 19 days. (1 mark)
c. $z_1 = \frac{11 - 15}{2} = -2$ $z_2 = \frac{21 - 15}{2} = 3$ Less than -2 is 2.5% Greater than 3 is 0.15% Outside -2 and 3 is $2.5 + 0.15 = 2.65$ Between -2 and 3 is $100 - 2.65 = 97.35\%$ (1 mark)	d. 2.5% has $Z_1 = 2$ on standard normal curve and 12 on the normal curve. 16% has $Z_2 = -1$ on standard normal curve and 3 on the normal curve. $Z_1 = 2 = \frac{12 - \bar{x}}{s} \Rightarrow 2s = 12 - \bar{x}$ $Z_2 = -1 = \frac{3 - \bar{x}}{s} \Rightarrow -s = 3 - \bar{x}$ Use calculator to solve these 2 simultaneous equations This gives Mean = 6, Standard deviation = 3 (2 marks)

Module 1: Number patterns and applications**Question 1**

a. From the graph it can be seen that when the number of years is 1, the projected number of car sales is 1500. (1 mark)	b. From year 1 to year 3, the number of car sales increases by 1000. This is an arithmetic sequence because the graph is a straight line. Hence, the increase in sales each year is a constant of 500. (1 mark)
c. $a = 1500$ $d = 500$ $n = 10$ $t_n = a + (n - 1) \times d$ $t_{10} = 1500 + (10 - 1) \times 500 = 6000$ (1 mark)	d. $S_n = \frac{n}{2}[2a + (n - 1) \times d]$ $S_{10} = \frac{10}{2}[2 \times 1500 + (10 - 1) \times 500] = 37500$ (1 mark)
e. $1500 + (n - 1) \times 500 > 15600$ $15 + 5n - 5 > 156$ $5n > 146$ $n > 29.2$ In the 30th year of production. (1 mark)	

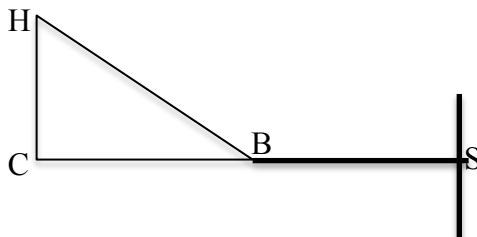
Module 1: Number patterns and applications**Question 2**

a. $2100 \div 1500 = 1.4$ $2940 \div 2100 = 1.4$ Since these both give the answer 1.4, there is a common ratio, and so the sequence is geometric. (1 mark)	b. $t_n = ar^{n-1}$ $t_6 = 1500 \times (1.4)^5 = 8067$ (1 mark)
c. Number of cars expected to be sold = 37500 Number of cars produced = S_{10} $S_n = \frac{a(r^n - 1)}{r - 1}$ $S_{10} = \frac{1500(1.4^{10} - 1)}{1.4 - 1} = 104720$ Number of cars not sold = $104720 - 37500$ = 67220 (1 mark)	d. $95\% \text{ of } 8067.36 = 7664$ (1 mark)
e. $4000 + (n - 1) \times 500 > 8067 \times 0.95^{n-1}$ Use solve on calculator to solve equation $4000 + (n - 1) \times 500 = 8067 \times 0.95^{n-1}$ This gives $n = 5.7$ n is an integer greater than 5.7 $n = 6$ The sixth year of production is the first year, so the 11th year of production is when $n = 6$. 11th year of production is the answer. (1 mark)	

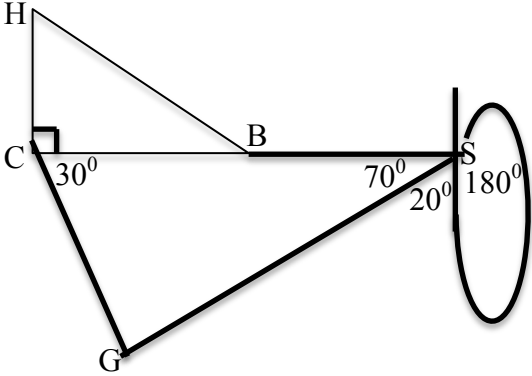
Module 1: Number patterns and applications**Question 3**

a. $p = 1.1$ and $q = 2000$, $k = 45000$ $I_2 = 1.1 \times 45000 + 2000 = \\51500 (2 marks)	b. Salary forms a geometric sequence and bonus form an arithmetic sequence. Geometric sequence has $a = 48000$, $r = 1.08$ Arithmetic sequence has $a = 2000$, $d = 180$ $48000 \times 1.08^{n-1} + 2000 + (n-1) \times 180 > 200000$ On calculator use solve to solve the equation $48000 \times 1.08^{n-1} + 2000 + (n-1) \times 180 = 200000$ This gives $n = 19.2$ n is an integer greater than 19.2 $n = 20$ Her income will first exceed \$200000 in her 20th year of working for the company. (2 marks)
c. $I_2 = 1.05 \times 30000 + k$ $I_3 = 1.05(1.05 \times 30000 + k) + k = 41751$ Use solve on calculator to get $k = 4232$ (1 mark)	

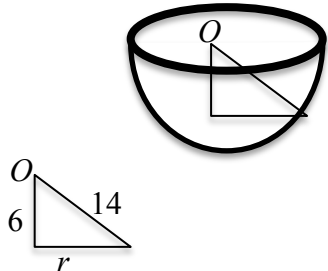
Module 2: Geometry and trigonometry**Question 1**

a.  $HB^2 = HC^2 + CB^2 = 80^2 + 30^2$ $HB = \sqrt{80^2 + 30^2} = 85.4 \text{ m}$ (1 mark)	b. $\tan \angle HBC = \frac{80}{30}$ $\angle HBC = \tan^{-1}\left(\frac{80}{30}\right) = 69.44$ $\angle HBS = 180 - 69.44 = 111^\circ$ (1 mark)
c. Triangle HBS is isosceles, so base angles are equal. All angles in this triangle add to 180°. $110.56 + 2 \times \angle HSB = 180$ $\angle HSB = 35^\circ$ (1 mark)	d. Measuring the angle in a clockwise direction from north at S we get $270 + 35 = 305^\circ$ (1 mark)

Module 2: Geometry and trigonometry**Question 2**

a.  $\angle CSG = 70^\circ$ (1 mark)	b. $\angle CGS = 180 - (70 + 30) = 80^\circ$ (1 mark)
c. $CS = 30 + 85.4 = 115.4$ $\frac{115.4}{\sin 80^\circ} = \frac{GS}{\sin 30^\circ}$ $GS = 59 \text{ m}$ (1 mark)	d. $\text{Area} = \frac{1}{2} \times 80 \times 30 + \frac{1}{2} \times 115.4 \times 58.6 \times \sin 70^\circ$ $\text{Area} = 4377 \text{ m}^2$ (1 mark)

Module 2: Geometry and trigonometry**Question 3**

a. OA is the radius of the largest bowl. $OA = 14$ cm. (1 mark)	b. $V = \frac{1}{2} \times \frac{4}{3} \times \pi \times 14^3$ $V = 5747 \text{ cm}^3$ (1 mark)
c. Ratio of lengths = $\sqrt[3]{64} : \sqrt[3]{1} = 4 : 1$ Diameter of small bowl = $\frac{1}{4} \times 28 = 7$ Radius = 3.5 cm. (1 mark)	d.  r is the radius of the water. Since the radius of the dish is 14 and the depth of the water is 8, then the vertical part of the triangle is $14 - 8 = 6$. Using Pythagoras, $r^2 + 6^2 = 14^2$ $r = 12.65$ cm (1 mark)

Module 2: Geometry and trigonometry**Question 4****a.**

Let the height of the small pyramid that is cut off the top be h .

The small pyramid cut off and the large original pyramid are similar figures, so their sides are in the same ratio.

Height of large pyramid = $h + 40$

Base of small pyramid = 30.

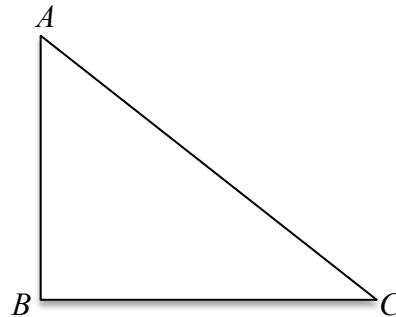
Base of large pyramid = 50

$$\frac{h}{h + 40} = \frac{30}{50}$$

$$h = 60$$

Height of original pyramid = $60 + 40 = 100$ cm.

(1 mark)

b.

AB is the height of the pyramid = 100

BC is half the distance across the side of the base = 25

AC is the height of a face of the pyramid.

$$AC^2 = 100^2 + 25^2$$

$$AC = 103.08$$

$$\text{Area of triangular face} = \frac{1}{2} \times 50 \times 103.08$$

$$\text{Area of triangular face} = 2577 \text{ cm}^2$$

(1 mark)

c.

Ratio of lengths = 3 : 5

Ratio of areas = 9 : 25

$$\text{Area of face of small pyramid that was cut off} = \frac{9}{25} \times 2576.94 = 927.7$$

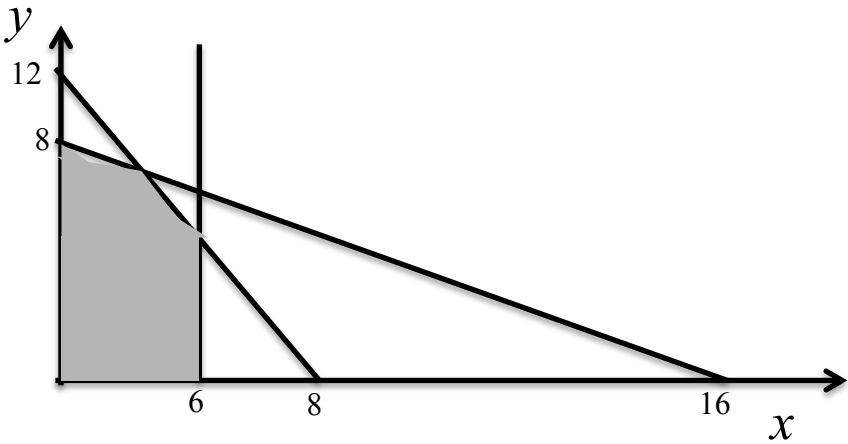
$$\text{Area of side of glass case} = 2576.94 - 927.7 = 1649.24$$

$$\text{Area of 4 sides of glass case} = 4 \times 1649.24 = 6596.96$$

$$\begin{aligned} \text{Area of 4 sides of glass case and top and bottom} &= 1649.24 + 50^2 + 30^2 = 9996.96 \text{ cm}^2 \\ &= 1 \text{ m}^2 \end{aligned}$$

(1 mark)

Module 3: Graphs and relations**Question 1**

a. $10x + 20y \leq 160$ $\Rightarrow x + 2y \leq 16$ (1 mark)	b. $6x \leq 36$ $\Rightarrow x \leq 6$ (1 mark)
c. and d.  Missing inequality is $x \leq 6$ (1 mark)	
e. $P = 200x + 120y$ (1 mark)	f. Corner points satisfying the constraints. $(0,0)$, $(0,8)$, $(6,0)$ and the point of intersection of the lines, $3x + 2y = 24$ and $x = 6$. This gives the point $x = 6$, $y = 3$ Also, the point of intersection of the lines $3x + 2y = 24$ and $x + 2y = 16$ Using a calculator to solve these simultaneous equations gives $x = 4$ and $y = 6$. Profits are \$0, \$960, \$1200, \$1560 and \$1520 Maximum profit is \$1560 (2 marks)

Module 3: Graphs and relations**Question 1**

g. Make 6 snow jackets and 3 rain jackets. (1 mark)	h. The corner points would now be (0,8), (0,3), (4,6) and the point of intersection of the lines, $3x + 2y = 24$ and $y = 3$. This gives the point $x = 3, y = 6$ Profit = $260x + 100y$ By inspection, (4,6) will give a larger profit than the other 3 points. $P = \\$1640$ So, the profit will increase by $1640 - 1560 = \\$80$ (1 mark)
--	--

Question 2

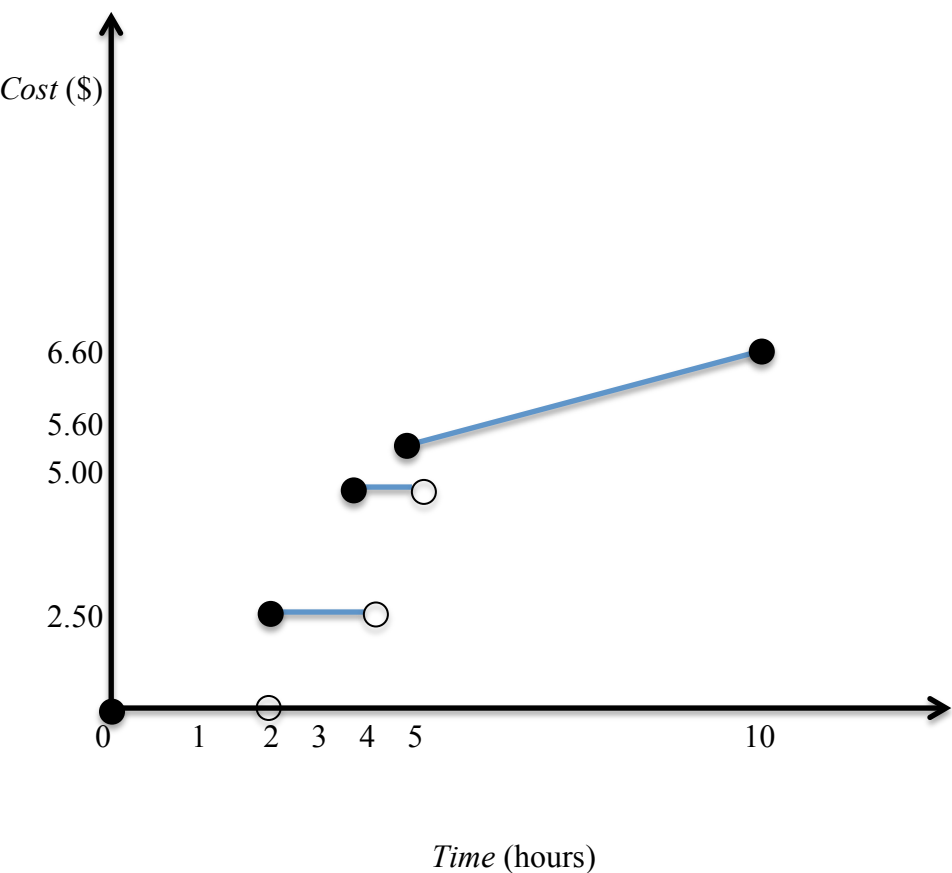
a. $180 + 50 = 230$. (1 mark)	b. Sale price = \$200 Profit = \$120 Cost price = $200 - 120 = \\$80$ (1 mark)
---	--

Question 3

a. Entry cost for week = $(10 + 4 \times 2) \times 7 = \\126 Cost of lifts = $20 \times 3 + 45 \times 2 = \\150 Total cost = $126 + 150 = \\$276$ (1 mark)	b. Cost for 8 hours = $5 + 0.1 \times 12 = \\$6.20$ (1 mark)
c. Cost of parking for 2 days = 10 Cost of parking for 5 days = $42 - 10 = 32$ This would average at more than \$6 for the other 5 days, so use $t \geq 5$. In this case cost for 5 days = $5 \times [5.00 + 0.1(2t - 4)] = 32$ Solve on calculator to get $t = 9$ hours. (1 mark)	

Module 3: Graphs and relations

Question 3 (continued)



(1 mark)

Module 4: Business-related mathematics**Question 1**

a. $400,000 - 332,000 = \\$68,000$ She has to use \$68,000 of her own money. This leaves $80,000 - 68,000 = \\$12,000$ (1 mark)	b. $33\frac{1}{3}\% \times 18,000 = 100 \div 300 \times 18,000 = \\$6,000.$ (1 mark)
c. Amount paid = $6000 + 420 \times 36 = 21,120.$ Cash price = 18,000 Interest = $21,120 - 18,000 = \\$3,120.$ (1 mark)	d. $I = \frac{PRT}{100}$ $3120 = \frac{(18000 - 6000) \times R \times 3}{100}$ $R = 8.7\%$ (1 mark)
e. Effective interest rate $\simeq \frac{2n}{n+1} \times \text{flat rate} = \frac{2 \times 36}{36+1} \times 8.6667 = 17\%$	

Question 2

a. $6\% \text{ of } 18,000 = 0.06 \times 18,000 = \\1080 (1 mark)	b. Amount to be depreciated by $= 18,000 - 5040 = 12,960$ Number of years for this depreciation $= 12960 \div 1080 = 12 \text{ years}.$ (1 mark)
c. $A = P \left(1 - \frac{r}{100} \right)^n$ $5040 = 18000 \left(1 - \frac{r}{100} \right)^{10}$ Use solve on calculator to get $r = 12\%$ (1 mark)	

Module 4: Business-related mathematics**Question 3**

a. Use TVM solver $N = 18 \times 12$ $I = 5.2$ $PV = 332000$ $PMT =$ $FV = 0$ $P/Y = 12$ $C/Y = 12$ This gives $PMT = \\$2370$ (1 mark)	b. Use TVM solver $N = 60$ $I = 5.2$ $PV = 332000$ $PMT = -2370.0769$ $FV =$ $P/Y = 12$ $C/Y = 12$ This gives $FV = \\$268334$ Amount owing = $268334 - 150000 = \\$118,334$ (1 mark)
c. Use TVM solver $N =$ $I = 5.2$ $PV = 118334$ $PMT = -1916$ $FV = 0$ $P/Y = 12$ $C/Y = 12$ This gives $N = 72$ 72 months = 6 years. (1 mark)	d. Actual time of repayment will be a little over 72 months. Use TVM solver $N = 72$ $I = 5.2$ $PV = 118334$ $PMT = -1916$ $FV =$ $P/Y = 12$ $C/Y = 12$ This gives $FV = -63.93631138$ Use TVM solver $N = 1$ $I = 5.2$ $PV = 63.93631138$ $PMT =$ $FV = 0$ $P/Y = 12$ $C/Y = 12$ This gives $FV = \\$64.22$ (1 mark)

Module 4: Business-related mathematics**Question 4**

a. Use TVM solver $N = 12$ $I = 6.5$ $PV = 80000$ $PMT =$ $FV = -80000$ $P/Y = 12$ $C/Y = 12$ This gives $PMT = -433.33333$ Monthly repayments = \$433.33 (1 mark)	b. Amount paid to bank each month $= 24 \times 433.3333 = 10400$ Amount paid to bank = $10400 + 80000 = 90,400$ Profit = $120,000 - 90,400 = \\$29,600$ (1 mark)
c. $Q = 1530$ per month $= 1530 \times 12 = \\$18,360$ per year. $P = 200,000 + 29,600 = 229600$ $Q = \frac{P \times r}{100}$ $18360 = \frac{229600 \times r}{100}$ Use solve on calculator to get $r = 8\%$ (1 mark)	

Module 5: Networks and decision mathematics**Question 1****a.**

Teams defeated by Edithaven besides Carramatta were Aliceville and Beaconstown.

(1 mark)

b.

$$D^1 = \begin{matrix} & A & B & C & D & E \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

(1 mark)

c.

It gives the number of games won by each of the towns.

(1 mark)

d.

$$D^2 = \begin{matrix} & A & B & C & D & E \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{bmatrix} 0 & 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 2 & 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

(1 mark)

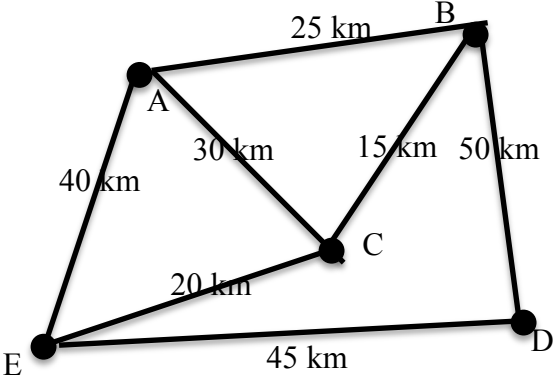
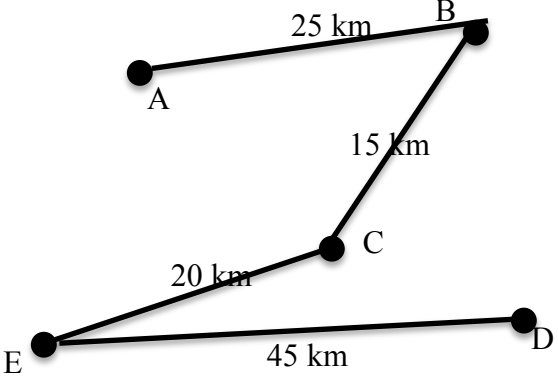
e.

Aliceville is now $2 + 4 = 6$, Beaconstown is now $1 + 2 = 3$, Carramatta is now $1 + 1 = 2$,
 Dixonvale is now $5 + 3 = 8$, Edithaven is now $3 + 4 = 7$

First is Dixonvale
 Second is Edithaven
 Third is Aliceville
 Fourth is Beaconstown
 Fifth is Carramatta

(1 mark)

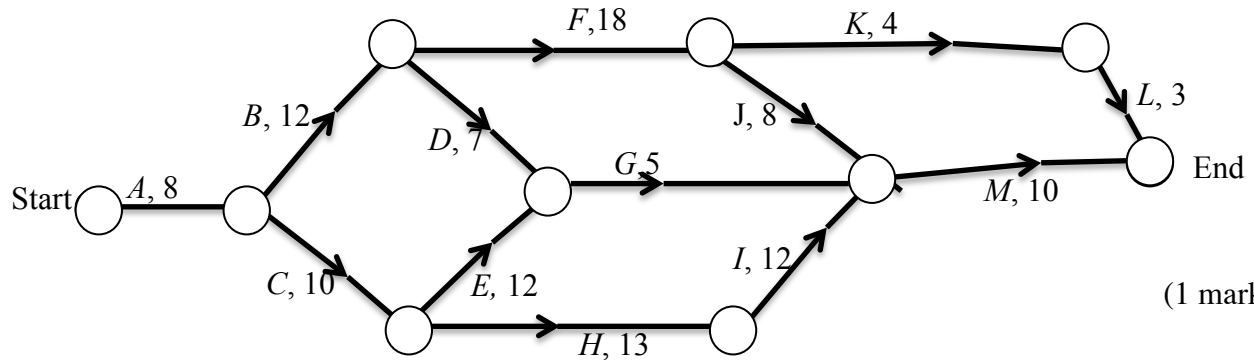
Module 5: Networks and decision mathematics**Question 2**

a.  (1 mark)	b.  (1 mark)
c. $25 + 15 + 20 + 45 = 105 \text{ km.}$ (1 mark)	d. There are more than 2 odd vertices, so it will not be possible for Mimi to travel on each road only once. (1 mark)

Module 5: Networks and decision mathematics**Question 3****a.**

D and E.

(1 mark)

b.

(1 mark)

c.

Critical path is A – B – F – J – M.
 Shortest time to complete project
 $= 8 + 12 + 18 + 8 + 10 = 56$ days.

(1 mark)

d.

Float time = Latest start time – Earliest start time
 Latest start time $= 56 - 10 - 5 = 41$
 Earliest start time $= 8 + 10 + 12 = 30$
 Float time $= 41 - 30 = 11$ days.

(1 mark)

e.

Extra workers reduce the number of days.
 Reducing the number of days on an activity that does not lie on the critical path will not reduce the total time. Hence, F, which lies on the critical path is the one to be reduced.

(1 mark)

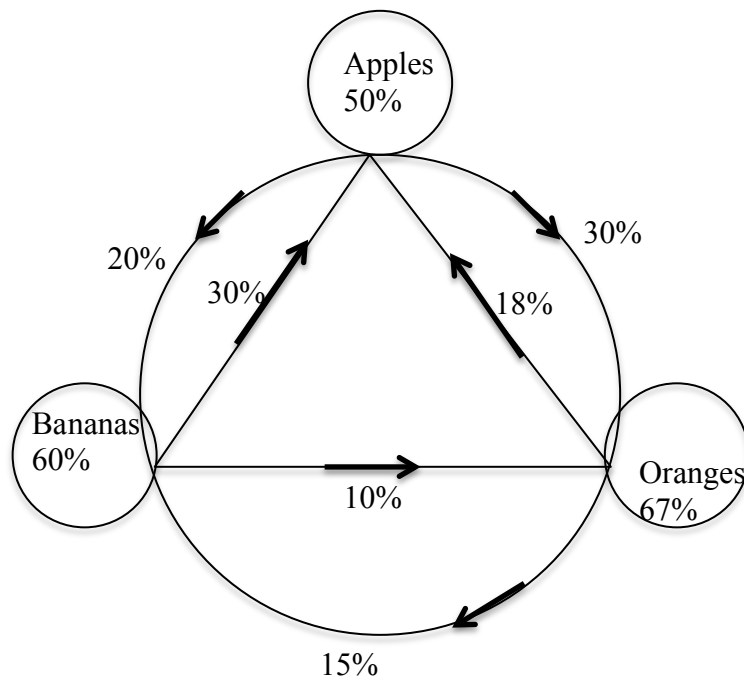
f.

To get to the start of activity M.
 $A - C - H - I = 8 + 10 + 13 + 12 = 43$
 $A - C - E - G = 8 + 10 + 12 + 5 = 35$
 $A - B - D - G = 8 + 12 + 7 + 5 = 32$
 For A – B – F – J to still be part of the critical path it cannot be less than 43.
 Hence, $F = 15$ which means it is reduced by 3 days.
 Cost of 3 days reduction for F
 $= 800 \times 3$
 $= \$2,400$

(1 mark)

Module 6: Matrices**Question 1**

a. $X = \begin{matrix} & A & B & O & M \\ \begin{matrix} May \\ Nat \\ Owen \end{matrix} & \begin{bmatrix} 6 & 5 & 3 & 1 \\ 2 & 3 & 3 & 4 \\ 5 & 4 & 1 & 2 \end{bmatrix} \end{matrix}$ (1 mark)	b. $Y = \begin{matrix} & A & B \\ \begin{matrix} A \\ B \\ O \\ M \end{matrix} & \begin{bmatrix} 0.8 & 0.5 \\ 0.6 & 0.7 \\ 0.5 & 0.3 \\ 3.1 & 4.5 \end{bmatrix} \end{matrix}$ (1 mark)
c. $XY = \begin{bmatrix} 12.4 & 11.9 \\ 17.3 & 22 \\ 13.1 & 14.6 \end{bmatrix}$ (1 mark)	d. The cost of the fruit bought by Natalie in shop A. (1 mark)
e. $13.1 + 14.6 = \$27.70$ (1 mark)	

Module 6: Matrices**Question 2****a.**

(1 mark)

b.

		From		
		Apple	Banana	Orange
To	Apple	0.5	0.3	0.18
	Banana	0.2	0.6	0.15
	Orange	0.3	0.1	0.67

(1 mark)

c.

$$S_0 = \begin{matrix} \text{apples} \\ \text{bananas} \\ \text{oranges} \end{matrix} \begin{bmatrix} 300 \\ 100 \\ 500 \end{bmatrix}$$

(1 mark)

d.

$$\begin{bmatrix} 0.5 & 0.3 & 0.18 \\ 0.2 & 0.6 & 0.15 \\ 0.3 & 0.1 & 0.67 \end{bmatrix} \times \begin{bmatrix} 300 \\ 100 \\ 500 \end{bmatrix} = \begin{bmatrix} 270 \\ 195 \\ 435 \end{bmatrix}$$

195 people are expected to buy only bananas next week.

(1mark)

Module 6: Matrices**Question 2 (continued)**

e. $\begin{bmatrix} 0.5 & 0.3 & 0.18 \\ 0.2 & 0.6 & 0.15 \\ 0.3 & 0.1 & 0.67 \end{bmatrix}^3 \times \begin{bmatrix} 300 \\ 100 \\ 500 \end{bmatrix} = \begin{bmatrix} 277 \\ 255 \\ 368 \end{bmatrix}$ Expect 368 people to buy oranges only in 3 weeks time. (1mark)	f. $\begin{bmatrix} 0.5 & 0.3 & 0.18 \\ 0.2 & 0.6 & 0.15 \\ 0.3 & 0.1 & 0.67 \end{bmatrix}^{100} \times \begin{bmatrix} 300 \\ 100 \\ 500 \end{bmatrix} = \begin{bmatrix} 286 \\ 272 \\ 342 \end{bmatrix}$ Expect 286 people to buy apples only in the long term. (1mark)
--	--

Question 3

a. The decoding matrix will be the inverse of A. $D = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$ (1mark)	b. $AB = C$ (1mark)
c. $DAB = DC$ (1mark)	d. $DAB = DC$ $IB = DC$ $B = DC$ $B = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 20 & 10 & 67 \\ 35 & 19 & 114 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 20 \\ 5 & 7 & 7 \end{bmatrix}$ (1mark)

End of Suggested Solutions 2015 VCE Further Mathematics Trial Examination 2

Kilbaha Multimedia Publishing
PO Box 2227
Kew Vic 3101
Australia

Tel: (03) 9018 5376
Fax: (03) 9817 4334
kilbaha@gmail.com
<http://kilbaha.com.au>