

The Mathematical Association of Victoria
Trial Exam 2014
FURTHER MATHEMATICS
Written Examination 2

STUDENT NAME _____

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of Book

Core		
<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
5	5	15
Module		
<i>Number of modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 49 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

INSTRUCTIONS

This examination consists of core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions with the modules selected. You need not give numerical answer as decimals unless instructed to do so. Alternative forms may involve, for example, π , surds or fractions.

Diagrams are not to scale unless specified otherwise.

	Page
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Module	
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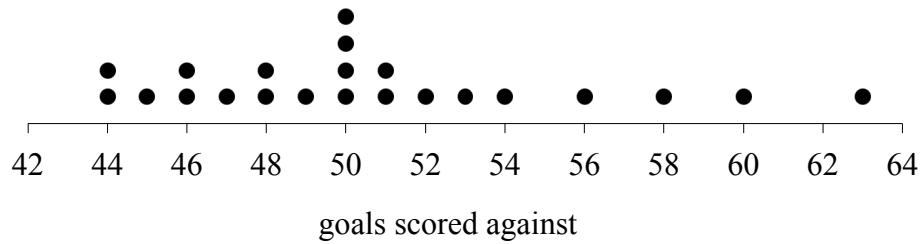
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TURN OVER

Core

Question 1(4 marks)

In her end-of-season review, the coach of the Senior B netball team of the Wombat Creek Sports and Social Club produced the following dot plot. The dot plot displays the goals scored against the Senior B netball team in every one of their 22 Home and Away games last season.



- (a) Using the information in the dot plot, determine the following :
- (i) median.

- (ii) Interquartile Range

1 + 1 = 2 marks

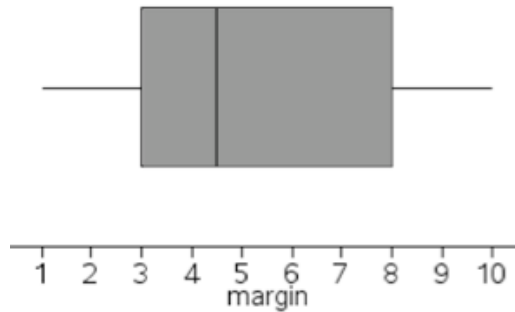
- (b) Write down an appropriate calculation and use it to explain why the goals against score of 63 is an outlier for this group of scores.

2 marks

Core - continued

Question 2 (2 marks)

Unfortunately, during the season, the Senior B team lost ten games. When their coach constructed a boxplot of the losing margins (the number of goals that the team lost by in those ten games), she obtained the graph as shown below.



The five number summary was :

Minimum = 1 $Q_1 = 3$ Median = 4.5 $Q_3 = 8$ Maximum = 10

Given that all margins were integer values (whole numbers), complete the table below to give one set of losing margin values that would give the boxplot shown above.

(Note : there is more than one correct set of values, you only have to find one of them)

No.	1	2	3	4	5	6	7	8	9	10
Value	1	2							9	10

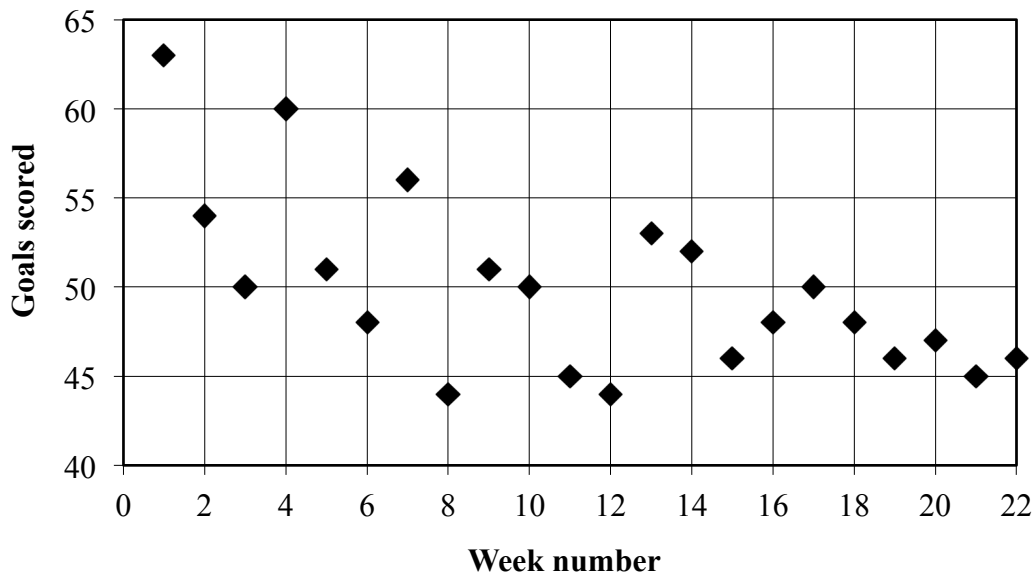
2 marks

Core – continued

TURN OVER

Question 3 (5 marks)

The Senior B netball team started their season strongly scoring over 60 goals in their first game, but as injuries mounted (their star goal shooter missed a large number of games), their ability to score winning totals diminished greatly. The scatterplot of goals scored in each game is shown below.



The least squares regression equation for this data is

$$\text{Goals scored} = 55 - 0.48 \times \text{week number}$$

- (a) Draw the line of this regression on the graph above.

1 mark

- (b) The coefficient of determination for this data set is 38.8%.

- (i) Interpret the coefficient of determination in terms of the variables goals scored and week number.

- (ii) Determine the value of Pearson's product moment correlation coefficient. Write your answer correct to three decimal places.

1 + 1 = 2 marks

Core – continued

- (c) Use the least squares regression equation to predict the number of goals scored in the twelfth week, correct to the nearest whole number.

1 mark

- (d) Given that the team actually scored 44 goals in the twelfth week, calculate the residual value.

1 mark

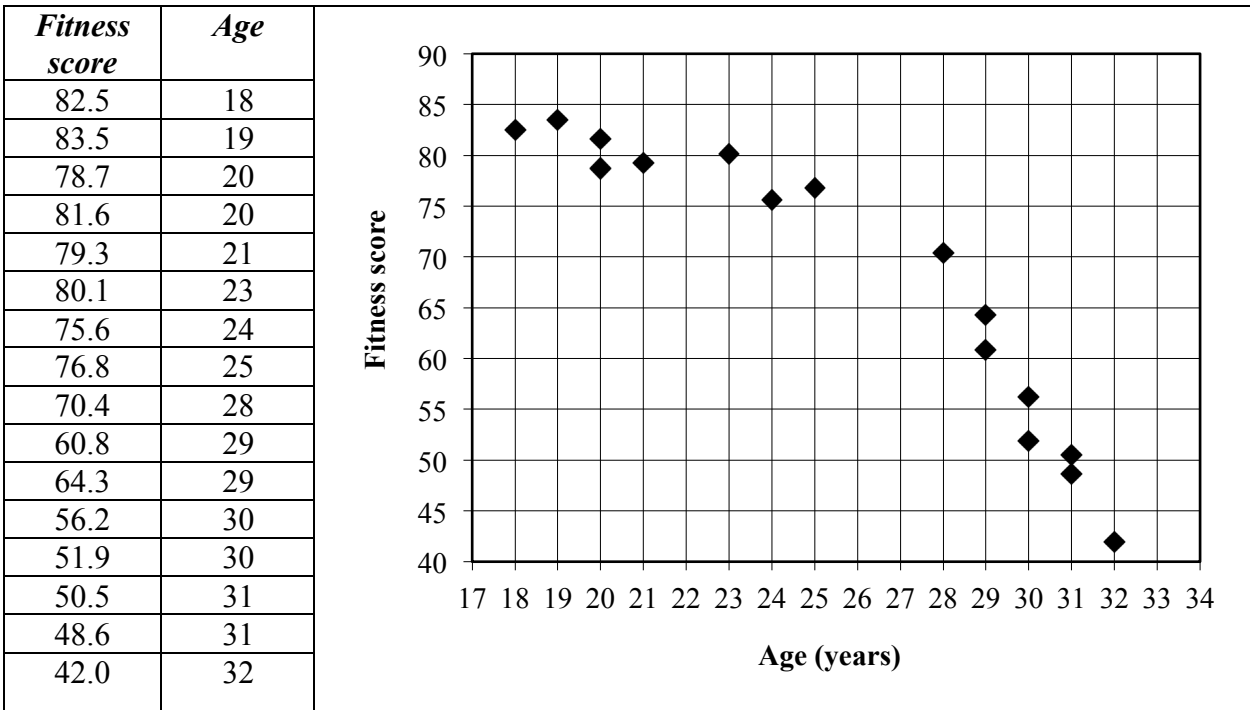
Core – continued

TURN OVER

Question 4 (2 marks)

The senior netball coach ran a fitness clinic, in which players undertook a series of activities. From their efforts a “fitness score” was derived, being a number between 0 (very unfit) and 100.0 (super-fit).

The *fitness score* and *age* of sixteen players are given in the table below. A scatterplot showing fitness score versus age of these results is also shown below.



The relationship between *fitness score* and *age* shown is nonlinear.

A **squared transformation** can be applied to the variable *fitness score* to linearise the data in the scatterplot.

- (a) Apply the squared transformation to the variable *fitness score* and determine the equation of the least squares regression line that allows $(\text{fitness score})^2$ to be predicted from *age*. Write the coefficients for this equation, correct to one decimal place, in the spaces provided below.

$$(\text{fitness score})^2 = \dots\dots\dots = \dots\dots\dots \times \text{age}$$

1 mark

Core - continued

- (b) Use this equation to predict the fitness score of a netballer who is 27 years old.
Write your answer, correct to one decimal place.

1 mark

Question 5 (2 marks)

Over the season, all 240 members of the Wombat Creek Sports and Social Club netball teams undertook the fitness analysis. The *fitness score* values were approximately normally distributed with a mean score value of 60.6 and a standard deviation value of 14.7.

Calculate the number of netballers who had a fitness score above 90.

2 marks

END OF CORE

Module 1: Number Patterns**Question 1** (7 marks)

The Wombat Creek Sports and Social Club runs a number of exercise programs. Jennifer has decided to participate in the walking program.

The distance (in kilometres) that Jennifer covers each day forms an arithmetic sequence.

The distances for the first three days of this program are shown in Table 1.

Table 1

Day	1	2	3
Distance (km)	1.3	1.65	2.0

- (a) What distance, in kilometres, will Jennifer walk on Day 5 ?

1 mark

- (b) On which day will Jennifer first walk more than 4 kilometres ?

1 mark

- (c) Determine the total distance that Jennifer will walk from Day 4 to Day 10 inclusive.
Write your answer in kilometres, correct to one decimal place.

2 marks

Module 1 - continued

- (d) The organisers of the program claim that if a walker completes thirty-one days of the walking program, they will have covered a total distance of more than 200 kilometres.

Show full working of a calculation that confirms this statement.

1 mark

- (e) An expression for the n th term of this sequence could be written as $t_n = An + B$.

Write in the values of A and B in the equation below.

$$t_n = \dots \times n + \dots$$

2 marks

Module 1 – continued

TURN OVER

Question 2 (4 marks)

Katie is participating in the Wombat Creek Sports and Social Club jogging program. The distance (in kilometres) that Katie jogs each day is always 7% greater than the previous day. The distances jogged in the first and third days of this program are shown in Table 1, correct to two decimal places.

Table 1

Day	1	2	3
Distance (km)	2.00		2.29

- (a) Complete the table above.

1 mark

- (b) Calculate the total distance that Katie will jog in 31 days.
Write your answer correct to the nearest kilometre.

1 mark

- (c) Katie is a good friend of Jennifer, and they always share the details of their exercise programs.
After 31 days, who will have travelled the furthest in total, and by how much, correct to the nearest kilometre?

2 marks

Question 3 (2 marks)

Brenda wants to do well in the 2014 Wombat Creek Big Hike.
She has decided to combine elements of both the walking and the jogging programs offered by the Wombat Creek Sports and Social Club in her preparation.

On Day 1 she will walk 500 m and jog 1200 m.
Her walking distance will increase by 150 m each day.
Her jogging distance will increase by 4% of the distance that she ran on the previous day.

On which day will the distance that Brenda walks first be greater than the distance she jogs ?

2 marks

Module 1 – continued
TURN OVER

Question 4 (2 marks)

Josie is another member of the Wombat Creek community intent on a good performance in the 2014 Wombat Creek Big Hike.

In her training program, she will run 3 km on the first day.

Each day thereafter she will travel 95% of the distance she ran on the previous day plus an additional 450 m.

- (a) Write a difference equation, using D_{n+1} and D_n , that specifies the distance (in kilometres) Josie will run on the $(n + 1)$ th day.

1 mark

Josie follows this training program for many months.

- (b) Explain why she will never run more than 9 km on any day.

1 mark

END OF MODULE 1

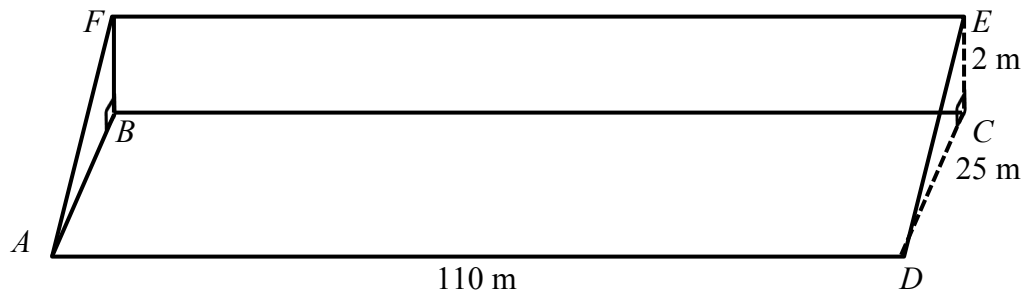
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TURN OVER

Module 2: Geometry and Trigonometry**Question 1** (7 marks)

The Wombat Creek Sports and Social Club will build a new set of netball courts on a site which will be carved into the side of a hill.

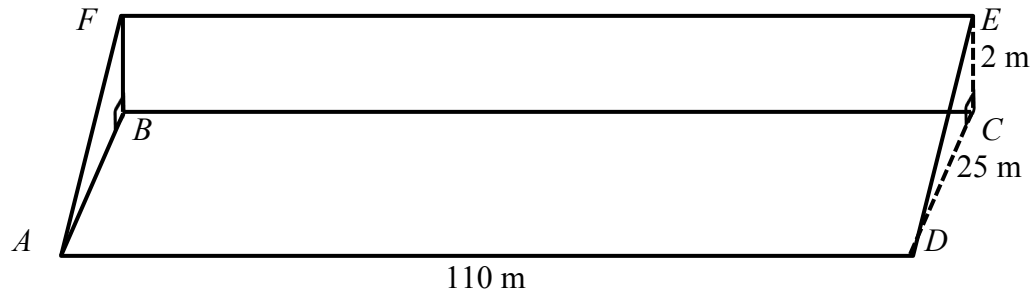
The site has the shape of a right-triangular prism $ABCDEF$, with a length of 110 m, a width of 25 m and a height on the uphill side of 2 m. This is shown in the diagram below.



- (a) Calculate the volume of earth to be removed, writing your answer in m^3 . 1 mark

Module 2 - continued

Once the hole has been dug, a safety fence will be erected around the site – along the two sloping sides AF and DE, and along the edges AD and FE.



- (b) Calculate the total length of safety fencing that will be required, writing your answer, in metres, correct to one decimal place.

2 marks

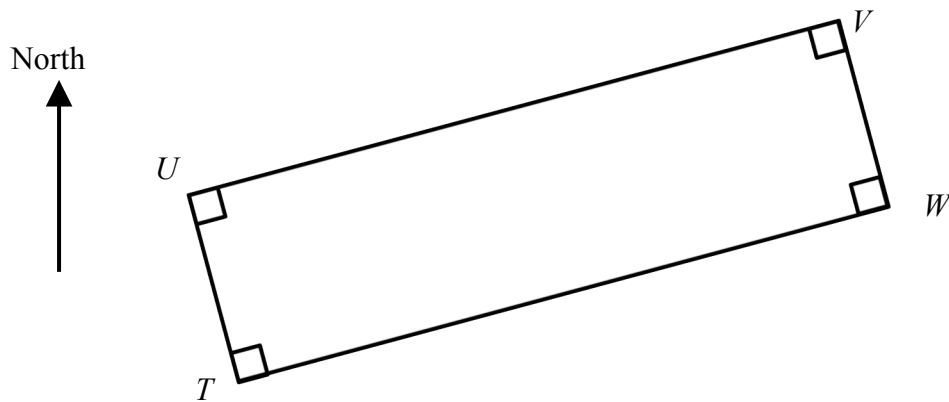
The site will accommodate three netball courts end-to-end, with a 5 m gap between adjacent courts. The entire horizontal surface (ABCD) will be surfaced with a special black-coloured rubber material that is 0.1 m thick.

- (c) Find the volume in cubic metres, of special rubber material that is needed to surface the site.

2 marks

Module 2 – continued
TURN OVER

During the construction process, four key survey points forming a rectangle were established.



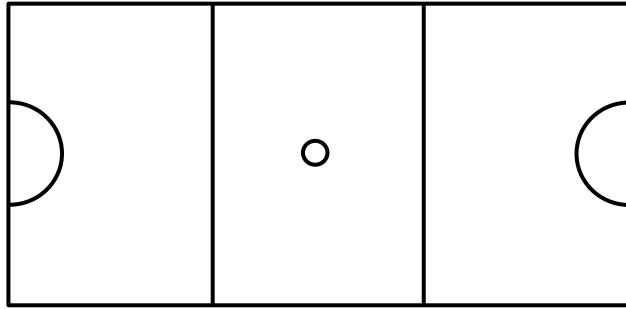
- (d) The bearing of point U from point T is 327° .

Determine the bearing of :

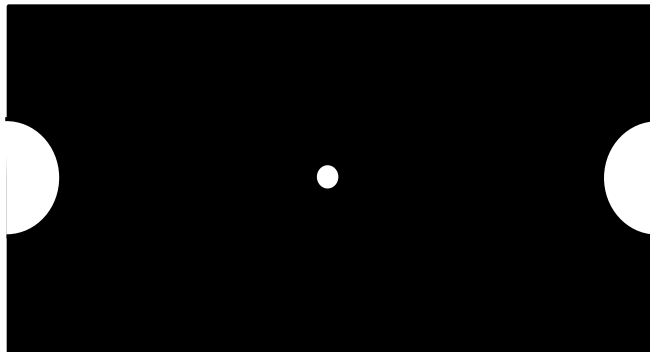
- (i) point V from point U

- (ii) point W from point V

1 + 1 = 2 marks

Question 2 (2 marks)

The 0.9 m diameter centre circle and the two 4.9 m radius semi-circular “shooting circles” or “goal circles” on **each** court will be painted white to aid players and umpires. This is shown in the diagram below.



- (a) Calculate the total area to be painted white on **all three courts**.
Give your answer in square metres correct to two decimal places.

1 mark

The special white paint required only comes in four-litre cans, each litre of paint will cover 12 m^2 of the rubber material, and to ensure durability, two coats of paint will required.

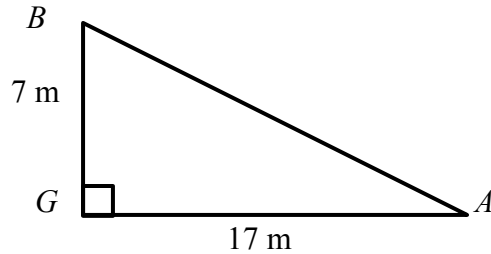
- (b) Calculate the number of four-litre paint cans needed to paint all three courts as required.

1 mark

Module 2 – continued
TURN OVER

Question 3 (4 marks)

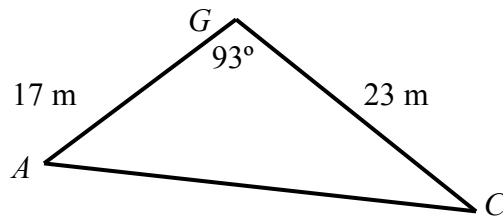
Briony (B) is sitting in the newly completed seating 7 m above the ground level (G) of the new netball courts. One of her friends, Ally (A), is playing on one of the courts, at a horizontal distance of 17m away from point G. This is shown in the diagram below.



- (a) Find the distance BA , correct to the nearest metre.

1 mark

Another of Briony's friends, Chloe (C), is playing on another court at a distance of 23m from G, and the top view of their relative positions is shown below.



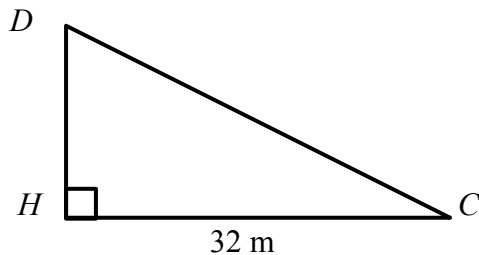
- (b) Show that the distance between Ally and Chloe is 29.3 m, correct to one decimal place.

1 mark

Chloe fell over where she was standing, and when Briony shrieked in fear of Chloe hurting herself, Chloe looked up at Briony, who is still sitting above point G.

- (c) At what elevation was her line of sight to Briony, correct to the nearest whole degree ?
1 mark

Delia, (D), in another part of the grandstand above the court level H which is 32m from C , also shrieked when Chloe fell over.

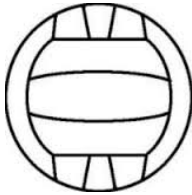


- (d) If Chloe looked up at Delia at an angle of 15° , how high was Delia above court level, in metres correct to one decimal place?
1 mark

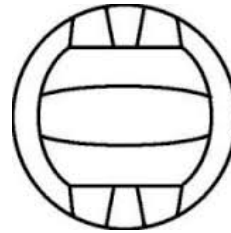
Module 2 – continued
TURN OVER

Question 4 (2 marks)

The netball used in senior competition has a volume of 6044 cm^3 . The junior netballers at Wombat Creek (the Under 12s and below) train with a ball whose volume is 4540 cm^3 .



Junior netball



Senior netball

By what value can the surface area of the junior netball be multiplied to give the surface area of the senior netball? Give your answer correct to two decimal places.

2 marks

END OF MODULE 2

Module 3: Graphs and Relations**Question 1** (3 marks)

During the football season, the Wombat Creek Sports and Social Club has a canteen that is run by volunteers. The canteen sells hotdogs for \$3.50 and cans of soft drink for \$2.20.

- (a) Write an equation for the revenue (R) obtained from selling x hotdogs and y cans of soft drink.

1 mark

- (b) One Saturday the canteen took in a revenue of \$579.50. If they sold 125 cans of soft drink, how many hotdogs were sold?

1 mark

The cost to make each hotdog is \$1.20 and the cost of each can of soft drink is \$0.80.

An equation for the canteen's cost (C) for x hotdogs and y cans of soft drink would be:

$$C = 1.2x + 0.8y$$

- (c) The canteen's profits (P) is given by the difference between the revenue and the costs. Write an equation for the profit made from selling x hotdogs and y cans of soft drink.

1 mark

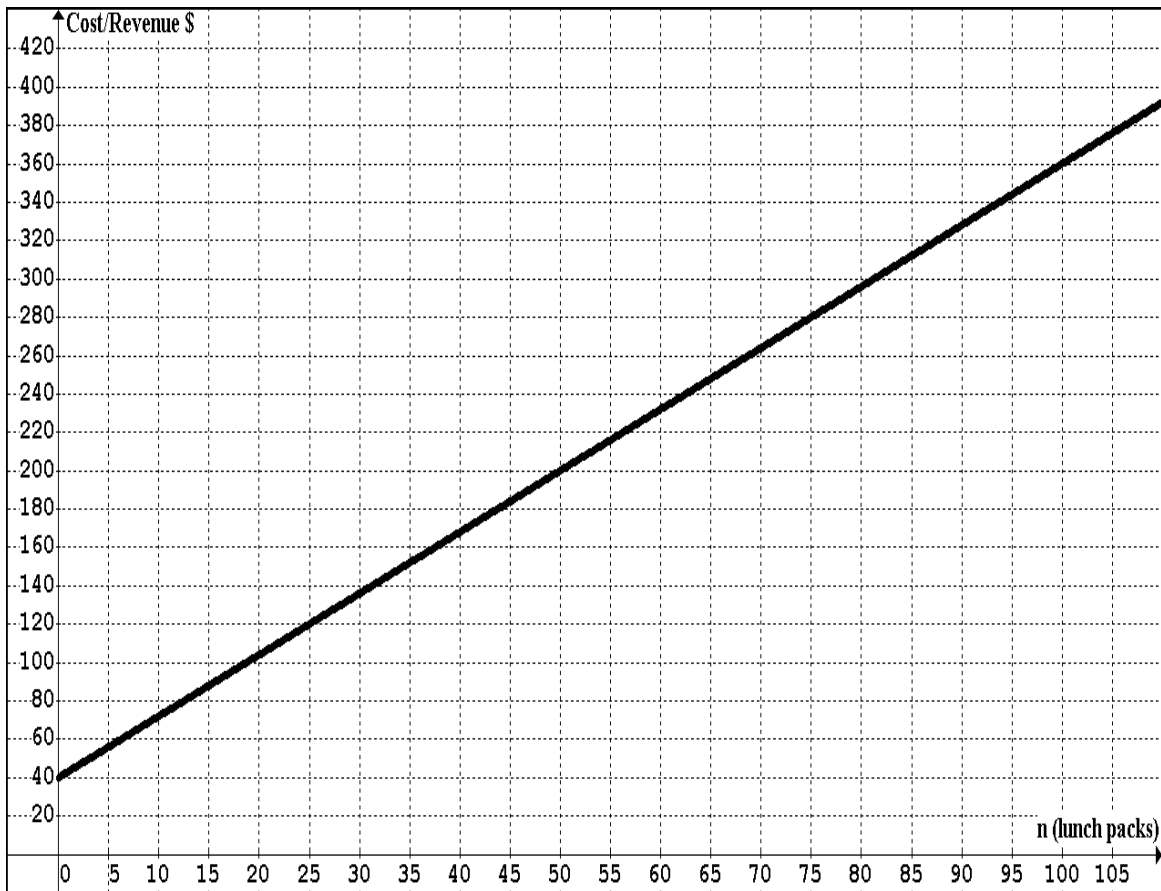
Module 3 - continued

Question 2 (5 marks)

It has been suggested that the canteen should start selling healthier food.

One suggestion is that they could sell salad rolls and orange juice as a lunch pack. The packs can be purchased for \$3.20 each, but the supplying company charges a delivery fee each time that they deliver the packs.

A graph is shown below for the cost of supplying and delivering n lunch packs:



- (a) The equation for the cost of supplying and delivering n lunch packs is $C = 3.2n + b$.

State the value of b in this equation.

1 mark

Module 3 - continued

- (b) What does the value of b in this equation represent? 1 mark

The suggested sale price for the lunch packs is \$4.00. This means that the equation for the revenue (R) would be:

$$R = 4n$$

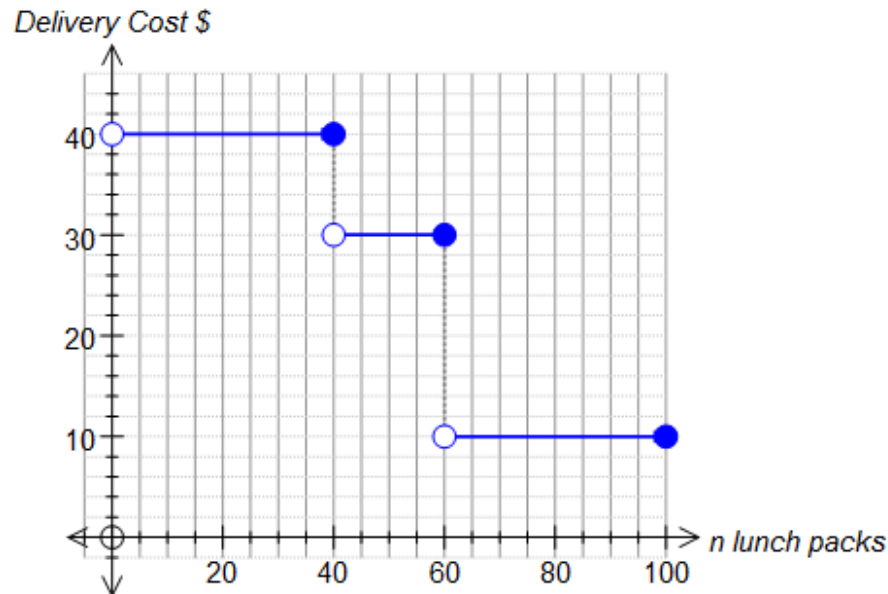
- (c) Add the line $R = 4n, 0 < n \leq 100$ to the graph above. 1 mark

- (d) What is the minimum number of lunch packs that the canteen would need to sell in order to make a profit? 2 marks

Module 3 – continued
TURN OVER

Question 3 (3 marks)

The Wombat Creek Sports and Social Club have decided to negotiate with another supplier for their lunch packs. This supplier agrees to reduce the delivery fee if a greater number of packs is delivered. A graph showing the relationship between the delivery fee and the number of packs delivered is shown below for up to 100 lunch packs, the maximum number allowed per delivery:



- (a) On one particular day the club has 150 lunch packs delivered in more than one delivery. What is the minimum cost for delivery on this day? 1 mark

A relationship connecting the maximum number of lunch packs for the three delivery costs is to be investigated.

The rule is found to be of the form $cost = a \times \text{number of lunch packs} + b$

- (b) Determine the values of a and b . 2 marks

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Module 3 – continued

TURN OVER

Question 4 (4 marks)

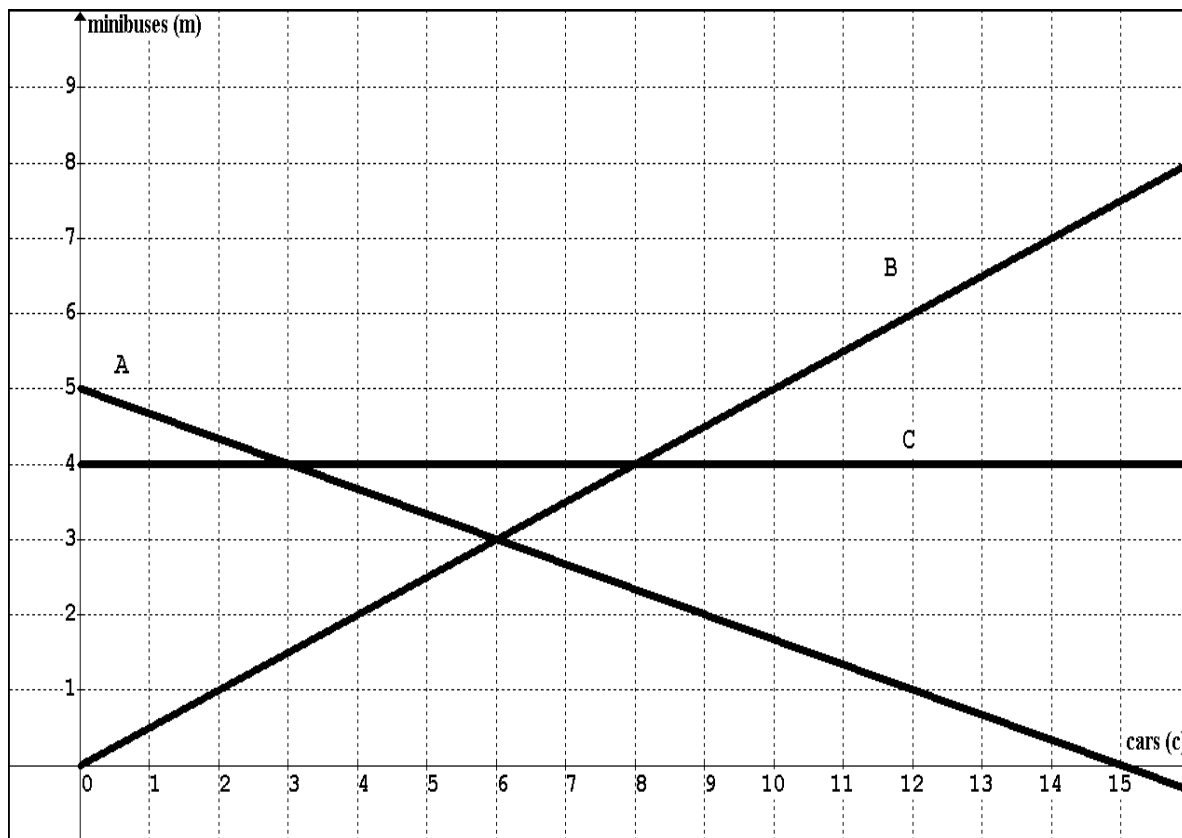
The Wombat Creek Sports and Social Club are planning for the annual football clash at the home ground of their great rivals, Platypus Flat. They plan to take a combination of cars (c) and minibuses (m) to transport the participants.

There are a number of factors that may affect the number of each of the vehicles used including that $m \geq 0$ and $c \geq 0$.

These can be summarised in a number of constraint inequations:

Line 1	There are only 4 minibuses available	$m \leq 4$
Line 2	Each car can take 4 passengers and each minibus can take 12 passengers. There are at least 60 people who are going on the trip	$4c + 12m \geq 60$
Line 3	For every two cars at least one minibus should be taken	$m \geq \frac{1}{2}c$

These three lines are shown on the graph below and labelled A, B and C.



- (a) Write down the letter associated with each of these lines in the table below. 1 mark

Line 1	
Line 2	
Line 3	

- (b) Another constraint inequation applies to this situation. There are only 12 people who have offered to drive cars. Write down the associated inequation below **and** add the line that borders the associated region to the graph above. 1 mark

- (c) Shade the feasible region for this problem **and** state the value of points in this region, given that the number of minibuses and the number of cars must each be whole numbers. 1 mark

- (d) The cost of transport must be minimised.
Given that hiring a minibus (including fuel) for the day is \$270, what is the maximum amount of money that could be allowed for each car's costs, if the cost is minimised only when there are 6 cars and 3 minibuses? 1 mark

END OF MODULE 3

Module 4 : Business-related Mathematics**Question 1** (5 marks)

The Wombat Creek Sports and Social Club has been updating its main clubrooms. In 2012, they purchased a new large refrigerator for the Canteen. The price listed for this refrigerator (including Goods and Services Tax (GST) of 10%) was \$2695.

- (a) If this refrigerator was sold at the listed price, how much GST would be paid ?

1 mark

The Club bought the refrigerator under a hire-purchase agreement. They paid a \$500 deposit, and agreed to pay \$70 per month for four years.

- (b) Calculate the total amount paid by the Club for this refrigerator.

1 mark

- (c) Find the total interest paid by the Club over the four years.

1 mark

Module 4 - continued

- (d) Determine the annual flat interest rate that is applied to this hire-purchase agreement.
Write your answer as a percentage, correct to one decimal place.

1 mark

In 2013, the price of this refrigerator increased by 3.5%.

In 2014, the price of this refrigerator increased by a further 2.5%.

- (e) Calculate the cash price of this refrigerator in 2014 after these two price increases, correct to the nearest dollar.

1 mark

Module 4 – continued
TURN OVER

Question 2 (2 marks)

The Club has invested a sum of money in a perpetuity at an interest rate of 5.75% per annum.

The club receives a monthly payment of \$1150 which it uses to support the education of financially disadvantaged students in the community.

- (a) Find the amount of money invested in the perpetuity, correct to the nearest dollar.

1 mark

- (b) After five years of monthly payments, how much money remains invested in this perpetuity?

1 mark

Question 3 (3 marks)

To maintain all the grassed areas it manages (golf course, sports oval, tennis courts), the Club has purchased a new lawn mower for \$27 000.

- (a) Use the flat rate depreciation method with a depreciation rate of 14% to find the depreciated value of the lawn mower after four years.

1 mark

- (b) Use the reducing balance depreciation method with a depreciation rate of 18% per annum to calculate the depreciated value of the lawn mower after four years. Write your answer in dollars correct to the nearest cent.

1 mark

- (c) The Club will replace the mower at the end of the year in which its flat rate depreciation value drops below \$5000. How many years will this take ?

1 mark

Module 4 – continued
TURN OVER

Question 4 (5 marks)

The Club takes out a reducing balance loan of \$540 000 to finance a major building project. Interest on the loan will be calculated and paid monthly at the rate of 5.95% per annum.

- (a) The loan will be fully repaid in equal monthly instalments over 15 years.
- (i) In the table below, write in the values used in your calculator's Finance Menu/function to show that the monthly repayment, in dollars, correct to the nearest cent is \$4542.25.

1 mark

No. of instalment periods/payments	Interest rate p. a.	Present value	Future Value	Payments per year	Compounds per year

- (ii) Calculate the total interest that will be paid over the 15 year term of the loan. Write your answer correct to the nearest dollar.

1 mark

- (b) After 90 monthly repayments have been made, what will be the outstanding principal on the loan?

Write your answer correct to the nearest dollar.

1 mark

At this point in time (after 90 repayments have been made), the Club received a State Government grant which enabled the club to reduce the outstanding principal on the loan to \$125 000.

The Club continues to repay the loan at \$4542.25 per month, with interest remaining at 5.95% per annum.

- (c) With this change, how many months, **in total**, will it take the Club to fully repay the \$540 000 loan ?

2 marks

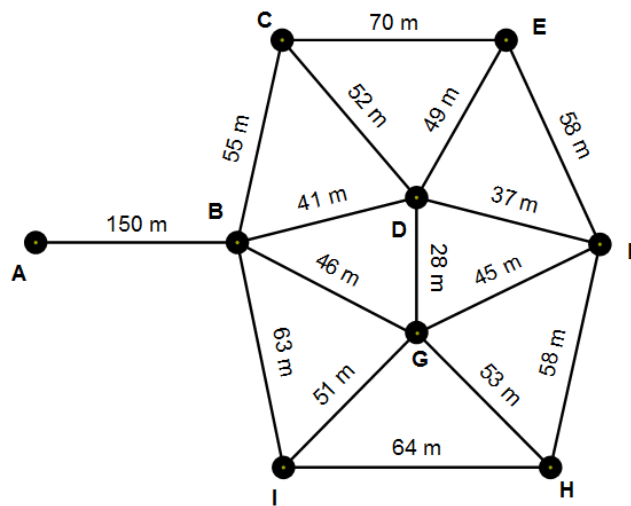
END OF MODULE 4

Module 5: Networks and Decision Mathematics

Question 1 (6 marks)

The Wombat Creek Sports and Social club has been given a grant to put in a watering system. The sprinklers need to be positioned so that the entire field is watered. A survey of the ground and suitable positions results in the proposed sprinkler system, including the source tap, as shown in the network below.

The vertices B to I represent sprinklers, while vertex A represents the source tap. The edges represent the connecting pipes between sprinklers and the tap:



- (a) The degree of vertex G in this network is _____ 1 mark

The groundsman wants to be able to walk along every pipe in the network exactly once in order to check for leaks.

- (b) Give a mathematical name to the route that the groundsman would like to take. 1 mark

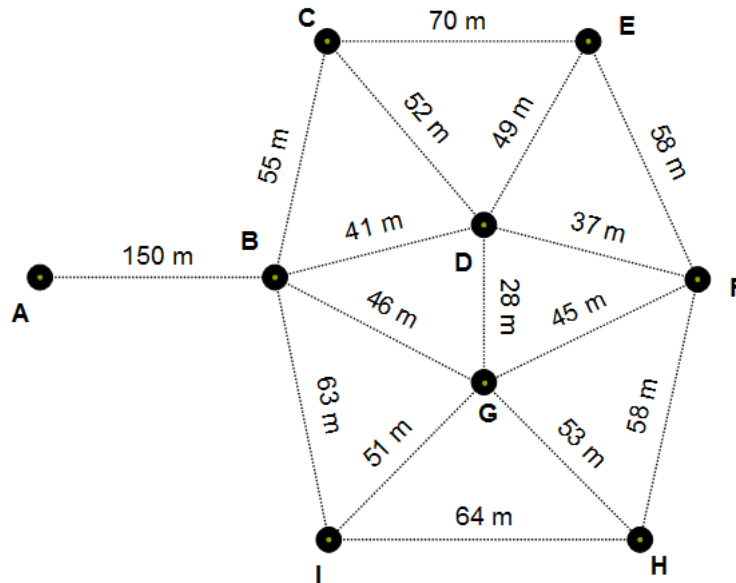
- (c) Explain why it is not possible for the groundsman to walk along every pipe exactly once. 1 mark

Module 5 - continued

- (d) It has been suggested that the groundsman could walk along every pipe exactly once starting at the tap (A) and finishing at sprinkler (B) if some of the pipes were removed. What is the **minimum** number of pipes that would need to be removed for this to happen? 1 mark
-
-

A more experienced irrigation company suggests that even fewer pipes would be required in order to simply connect these sprinklers.

- (e) Determine the minimal spanning tree that would connect this network of sprinklers and tap. Show your minimal spanning tree on the copy of the network below. 2 marks



Module 5 – continued

TURN OVER

Question 2 (4 marks)

One of the Wombat Creek football coaches has introduced a fitness program that involves a 6 km run, a 1km hurdle race, a 10 km cycle race and a 1 km race carrying a medicine ball.

The times that each of four players complete each of these races correct to the nearest minute is given in the table below:

	Player			
	Andrew (A)	Bruce (B)	Charles (C)	David (D)
Run (6 km) (R)	35	39	38	42
Hurdles (1 km) (H)	8	6	9	8
Cycle (10 km) (Y)	17	18	20	20
Medicine Ball (1 km) (M)	12	11	16	15

A competition is held in teams to complete each of these events one after the other. The team containing Andrew, Bruce, Charles and David needs to determine which part of the race should be completed by each player to minimize the time.

The Hungarian Algorithm is used to select players. The initial matrix to be used is shown below.

$$\begin{array}{c}
 R \\
 H \\
 Y \\
 M
 \end{array}
 \begin{array}{c}
 A \quad B \quad C \quad D \\
 \left[\begin{array}{cccc}
 35 & 39 & 38 & 42 \\
 8 & 6 & 9 & 8 \\
 17 & 18 & 20 & 20 \\
 12 & 11 & 16 & 15
 \end{array} \right]
 \end{array}$$

- (a) Complete the elements of the final reduced matrix that would be ready to determine the optimal allocation for this situation. 1 mark

$$\begin{array}{c}
 R \\
 H \\
 Y \\
 M
 \end{array}
 \begin{array}{c}
 A \quad B \quad C \quad D \\
 \left[\begin{array}{cccc}
 & & & \\
 & & & \\
 & & & \\
 & & &
 \end{array} \right]
 \end{array}
 \quad
 \begin{array}{c}
 R \\
 H \\
 Y \\
 M
 \end{array}
 \begin{array}{c}
 A \quad B \quad C \quad D \\
 \left[\begin{array}{cccc}
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 \end{array} \right]
 \end{array}$$

(b) There are two possible allocations. Complete the table below showing the optimal allocations that would have a **minimum** time:

2 marks

Event	Allocation 1 : Selected Player	Allocation 2 : Selected Player
Run (6 km) (R)		
Hurdles (1 km) (H)		
Cycle (10 km) (Y)		
Medicine Ball (1 km) (M)		

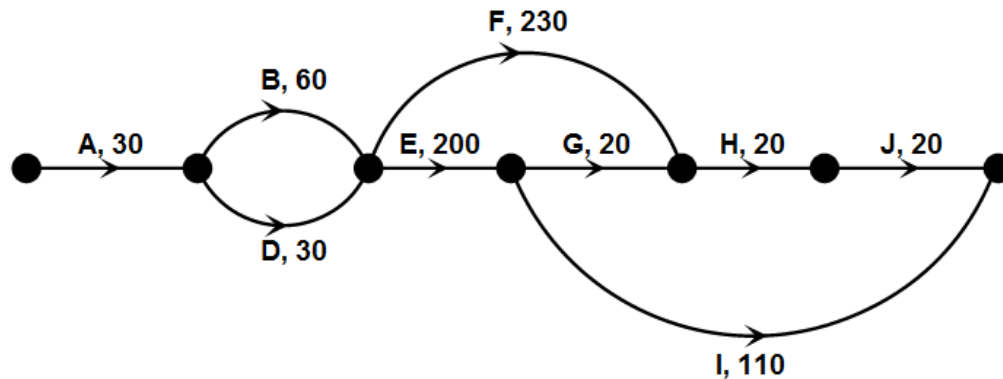
(c) Explain why there are two possible allocations for a minimum time.

1 mark

Module 5 – continued
TURN OVER

Question 3 (5 marks)

The Wombat Creek Sports and Social Club are planning the annual Ungulate Shire Netball Tournament featuring teams from all communities in the Shire. The planning committee have set up an activity network of tasks that must be completed on the day. The activity network and a table of tasks are shown below:

**Table 1**

Activity	Immediate Predecessor(s)	Duration (minutes)
A – Brief officials	–	30
B – Player registrations	A	60
C – Set up netball posts	–	40
D – Umpire registrations	A	30
E – Games	B, C, D	200
F – Recording & collating results	B, C, D	230
G – Umpires select Best & Fairest	E	20
H – Trophy preparation	F, G	20
I – Cleaning courts	E	110
J – Presentations	H	20

- (a) Activity C has not been included on the diagram. Add this activity to the diagram above.

1 mark

- (b) State the minimum time in minutes that it would take to complete the entire program on the day.

1 mark

Module 5 – continued

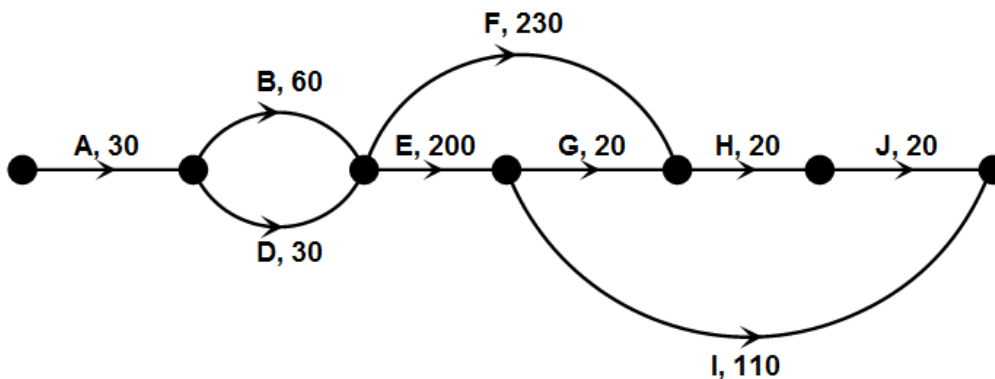
- (c) What is the latest time after the start that activity G could begin without delaying the entire program? 1 mark

A number of suggestions have been made to reduce the time using additional staff.

Table 2 below shows some possible reductions and the associated costs. All of the reductions must be done completely if chosen. Another copy of the activity network has been given if required.

Table 2

Activity	Original Duration	Duration after reduction	Cost of reduction
B	60	30	\$40
C	40	20	\$30
D	30	10	\$30
H	20	10	\$20
I	110	50	\$80



- (d) What is the minimum length of the entire program given that all of the reductions could be funded? 1 mark

- (e) What is the minimum cost associated with the greatest reduction of time? 1 mark

END OF MODULE 5

Module 6: Matrices**Question 1** (5 marks)

The Wombat Creek Sports and Social Club have members in a number of categories; Juniors(J), Intermediates(I), Adults(A) and Social (S) members.

The numbers of each type of member and their gender, male (M) and female (F), is given in the matrix, N, below:

$$N = \begin{matrix} & \begin{matrix} M & F \end{matrix} \\ \begin{matrix} J \\ I \\ A \\ S \end{matrix} & \begin{bmatrix} 30 & 21 \\ 40 & 15 \\ 20 & 25 \\ 45 & 60 \end{bmatrix} \end{matrix}$$

- (a) The annual club fees are \$40 for Juniors, \$60 for Intermediates, \$80 for Adults and \$30 for Social club members. Write this information in a 1×4 matrix, F. 1 mark

- (b) Explain why the matrix product, FN , is defined for multiplication. 1 mark

Module 6 - continued

- (c) Write down the matrix product, FN . Call this matrix M . 1 mark

- (d) Explain the meaning of the element, m_{12} . 1 mark

- (e) A matrix showing the total number of each of the Juniors, Intermediates, Adults and Social club members is required. It is obtained using the matrix product, NG .

Write down the matrix G that would produce the required matrix. 1 mark

Module 6 – continued
TURN OVER

Question 2 (3 marks)

The Wombat Creek Sports and Social Club sell caps (C), polo shirts (P) and hoodies (H) with their logo design.

One family buys 1 cap, 2 polo shirts and 2 hoodies, costing \$226.

Another family buys 2 caps and 1 polo shirt and 2 hoodies, costing \$206.

A third family buy 2 polo shirts and 1 hoodie and this costs \$139.

- (a) Complete the entries in the matrix equation below that could represent the situation described. 1 mark

$$\begin{bmatrix} & & \end{bmatrix} \times \begin{bmatrix} C \\ P \\ H \end{bmatrix} = \begin{bmatrix} 226 \\ 206 \\ 139 \end{bmatrix}$$

- (b) The values of C, P and H are calculated using a matrix multiplication, $D \times \begin{bmatrix} 226 \\ 206 \\ 139 \end{bmatrix}$. 1 mark
Complete matrix D below.

$$D = \begin{bmatrix} & & \end{bmatrix}$$

- (c) Show, using a calculation, that the cost of each polo shirt is \$32. 1 mark

CONTINUED OVER PAGE

Module 6 – continued
TURN OVER

Question 3 (3 marks)

The gym at the Wombat Creek Sports and Social Club has a number of fitness activities. Players are advised to vary their training each week to maximise their fitness outcomes.

The football players can choose between the treadmills (T), the weight circuit (W) or football skills training (F).

During the first week of the season, 100 players from various levels attended training. All of the players started on the treadmill. The state matrix for week 1, is:

$$W_1 = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} T \\ W \\ F \end{matrix}$$

The transition matrix showing the movement between training regimes each week is:

$$T = \begin{bmatrix} \mathbf{0} & 0.2 & 0.4 \\ 0.5 & \mathbf{0} & 0.6 \\ 0.5 & 0.8 & \mathbf{0} \end{bmatrix}$$

- (a) Three elements in the transition matrix have been highlighted. Explain the meaning of these three elements. 1 mark

- (b) Determine the number of players who will do football skills training in the third week of training. 1 mark

- (c) The pattern continues. Show that in the long run, 41 players will do football skills training each week if values are rounded to the nearest whole number. 1 mark

Module 6 – continued
TURN OVER

Question 4 (4 marks)

One year the Wombat Creek Sports and Social Club has football teams playing at each of three levels; firsts, second and thirds.

Some players are promoted or dropped a level during the season and others are unavailable due to injury or return from injury.

The number of players at each level each week is modelled by the equation:

$$P_{n+1} = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.2 & 0.6 & 0.4 \\ 0 & 0.2 & 0.6 \end{bmatrix} \times P_n + \begin{bmatrix} -2 \\ 5 \\ -3 \end{bmatrix}, P_2 = \begin{bmatrix} 38 \\ 53 \\ 29 \end{bmatrix}$$

- (a) How many players would be available at each level during the third week? 1 mark

- (b) How many players were available at each level during week 1? 2 marks

- (c) The club would like to retain the same number of players that they had at each level during week 2.

$\begin{bmatrix} -2 \\ 5 \\ -3 \end{bmatrix}$ represents the number of players lost or returned due to injury at that level each week.

Using the matrix $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ instead of $\begin{bmatrix} -2 \\ 5 \\ -3 \end{bmatrix}$ will ensure that the numbers do not change after week 2.

What are the values of x, y and z ?

1 mark

END OF QUESTION AND ANSWER BOOK