

**The Mathematical Association of Victoria  
FURTHER MATHEMATICS**

**SOLUTIONS: Trial Exam 2013**

**Written Examination 2**

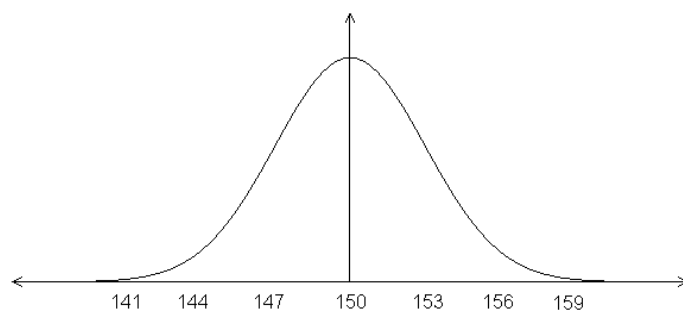
**SECTION A: Core--Data analysis**

**Question 1**

- a. Seasonal index =  $4 - 1.31 - 0.72 - 1.09 = 0.88$  A1
- b. Sales in Winter are 28% below the seasonal average A1
- c. Deseasonalised sales =  $\frac{\text{actual sales}}{\text{seasonal index}}$   
 $= \frac{654}{1.09} = 600$  A1
- d. Time period for Spring 2012 is  $t = 12$   
 Substituting gives Deseasonalised sales =  $540.1 + 4.25 \times 12 = 591.1$  M1  
 Residual = actual deseasonalised value – predicted deseasonalised value  
 $600 - 591.1 = 8.9$  A1
- e. Time period for Winter 2014 is  $t = 19$   
 Deseasonalised sales =  $540.1 + 4.25 \times 19 = 620.85$  M1  
 Actual sales will equal  $620.85 \times 0.72 = 447$  bikes (to nearest whole number) A1

**Question 2**

- a. numerical  
 categorical A1
- b. shape – Mountain Climber symmetric whereas Easy Rider is positively skewed  
 centre – Mountain Climber has a higher median than Easy Rider  
 spread – Both range and IQR are greater for Mountain Climber  
 outliers - Mountain Climber has no outliers, Easy Rider has one at the upper end  
4 x ½ marks (round down)    A2

**Question 3****a.**

A1

**b.** 153 is one standard deviation above the mean

16% are slower therefore 84% of riders finished ahead of Hunter

A1

**c.**  $-1.2 = \frac{x - 150}{3}$

 $x = 146.4$  therefore 146.4 minutes

A1

**d.** 2.5% are faster therefore  $z = -2$ 

M1

Solve  $-2 = \frac{116.4 - 120}{\sigma}$

$$\sigma = 1.8$$

A1

**SECTION B Module 1: Number Patterns****Question 1**

a.  $\frac{t_3}{t_2} = \frac{2430}{2700} = 0.9$  and  $\frac{t_2}{t_1} = \frac{2700}{3000} = 0.9$  (must show both calculations) A1

since  $\frac{t_3}{t_2} = \frac{t_2}{t_1} = 0.9$  therefore the sequence is geometric

b.  $t_4 = ar^3 = 3000 \times 0.9^3 = 2187$  A1

Alternatively the sequence can be generated on the calculator by using the general rule  $t_n = 3000 \times 0.9^{n-1}$

| n | anE    | ΣanE    |
|---|--------|---------|
| 1 | 3000   | 3000    |
| 2 | 2700   | 5700    |
| 3 | 2430   | 8130    |
| 4 | 2187   | 10317   |
| 5 | 1968.3 | 12285.3 |

2187

c. 19539 tickets were sold A1

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{10} = \frac{3000(1-0.9^{10})}{1-0.9}$$

$$= 19539.65$$

| n  | anE    | ΣanE    |
|----|--------|---------|
| 1  | 3000   | 3000    |
| 2  | 2700   | 5700    |
| 3  | 2430   | 8130    |
| 4  | 2187   | 10317   |
| 5  | 1968.3 | 12285.3 |
| 6  | 1771.5 | 14056.8 |
| 7  | 1594.3 | 15651.1 |
| 8  | 1434.9 | 17086.0 |
| 9  | 1291.4 | 18377.4 |
| 10 | 1162.3 | 19540.0 |
| 11 | 1046.0 | 20586.0 |
| 12 | 941.4  | 21527.4 |
| 13 | 847.2  | 22374.6 |
| 14 | 762.5  | 23137.1 |
| 15 | 686.3  | 23823.4 |

19539.646797

- d. During the 26<sup>th</sup> minute

A1

The crowd is first over 28 000 after the 26<sup>th</sup> minute so it will reach capacity during the 26<sup>th</sup> minute

| n  | $a_nE$ | $\Sigma a_nE$  |
|----|--------|----------------|
| 15 | 686.30 | 23823.         |
| 16 | 617.67 | 24441.         |
| 17 | 555.91 | 24997.         |
| 18 | 500.32 | 25497.         |
| 19 | 450.28 | 25947.         |
| 20 | 405.26 | 26353.         |
| 21 | 364.73 | 26717.         |
| 22 | 328.26 | 27046.         |
| 23 | 295.43 | 27341.         |
| 24 | 265.89 | 27607.         |
| 25 | 239.30 | 27846.         |
| 26 | 215.37 | 28061.67543323 |
| 27 | 193.83 | 28256.         |
| 28 | 174.45 | 28430.         |
| 29 | 157.00 | 28587.         |

### Question 2

- a.  $200 + 400 = 600$  people

A1

- b.  $200 + 10 \times 400 = 4200$

A1

- c.  $b = 400$   $c = 200$

A2

- d.  $400n + 200 = 28000$

$$n = 69.5 \text{ minutes}$$

A1

### Question 3

- a. 737 spaces

A1

$$750 + 2\% \times 750 - 28 = 750 + 0.02 \times 750 - 28 = 737$$

- b.  $P_{n+1} = 1.02P_n - 28$ ,  $P_0 = 750$

$$b = 1.02$$

A1

$$c = -28$$

A1

$$a = 750$$

A1

c. 39<sup>th</sup> minute 5.39 pm

A1

The recursive equation is entered in the calculator

$$P_{n+1} = 1.02P_n - 28, \quad P_0 = 750$$

to reveal that after 38 minutes only 20 car spaces are left so the car park

will be full during the 39<sup>th</sup> minute.

| n  | a <sub>n</sub> |
|----|----------------|
| 26 | 312.28         |
| 27 | 290.52         |
| 28 | 268.33         |
| 29 | 245.70         |
| 30 | 222.61         |
| 31 | 199.07         |
| 32 | 175.05         |
| 33 | 150.55         |
| 34 | 125.56         |
| 35 | 100.07         |
| 36 | 74.073         |
| 37 | 47.555         |
| 38 | 20.50578493893 |
| 39 | -7.084         |
| 40 | -35.23         |

Sequence

Formula:  $u(n) = 1.02 \cdot u(n-1) - 28$

Initial Terms: 737

n0: 1

nMax: 50

nStep: 1

Ceiling Value:

OK Cancel

| A                       | B | C |
|-------------------------|---|---|
| =seqgen(1.02*u(n-1)-28, |   |   |
| 125.560579186           |   |   |
| 100.071790769           |   |   |
| 74.0732265848           |   |   |
| 47.5546911165           |   |   |
| 20.5057849388           |   |   |
| -7.08409936239          |   |   |

A35 =100.07179076942

- d. 1482 cars

A1

The recursive equation is entered in the calculator

$$C_{n+1} = 0.97C_n + 18, \quad C_0 = 2800$$

After 30 minutes there are 1482 cars

| n  | a <sub>n</sub> |
|----|----------------|
| 22 | 1725.6         |
| 23 | 1691.9         |
| 24 | 1659.1         |
| 25 | 1627.3         |
| 26 | 1596.5         |
| 27 | 1566.6         |
| 28 | 1537.6         |
| 29 | 1509.5         |
| 30 | 1482.2         |
| 31 | 1455.7         |
| 32 | 1430.1         |
| 33 | 1405.2         |
| 34 | 1381.0         |
| 35 | 1357.6         |
| 36 | 1334.9         |

1482.215550795

$$C(1) = 0.97 \times 2800 + 18 = 2734.$$

Find C(30)

Sequence

Formula: u(n)= 0.97\*u(n-1)+18

Initial Terms: 2734

n0: 1

nMax: 40

nStep: 1

Ceiling Value:

OK Cancel

| n  | u(n)          |
|----|---------------|
| 26 | 1596.52402101 |
| 27 | 1566.62830038 |
| 28 | 1537.62945137 |
| 29 | 1509.50056783 |
| 30 | 1482.21555079 |
| 31 | 1455.74008427 |

1482.2155507949

- e. When
- $C_n = 600$
- then
- $C_{n+1} = 0.97 \times 600 + 18 = 600$

A1

The number of cars in the car park is reduced by 3%

When there are 600 cars in the car park 3% leave, that is,

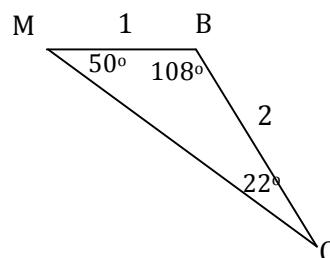
$$3\% \times 600 = 18 \text{ cars leave per minute} = 18 \text{ cars arriving per minute to fill a car space}$$

**SECTION B Module 2: Geometry and Trigonometry****Question 1**

- a. The interior angle of the pentagon  $\angle MBC = 180 - \frac{360}{5} = 180 - 72 = 108^\circ$

This means that  $\angle BCM = 180 - 108 - 50 = 22^\circ$

M1



- b.

Using the cosine rule

$$\text{distance PQ} = \sqrt{1^2 + 2^2 - 2 \times 1 \times 1 \times \cos 108^\circ}$$

$$= 2.5 \text{ m}$$

A1

- c.i.

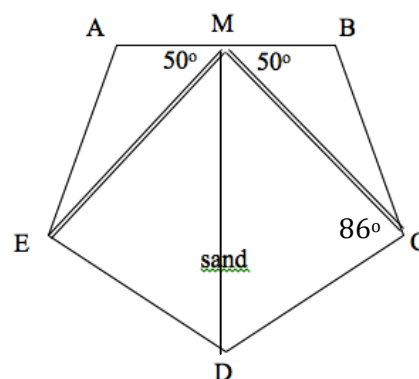
The quadrilateral can be divided into two triangles.

Triangle MCD has the same area as triangle MDE

$$\text{Angle MCD} = 108^\circ - 22^\circ = 86^\circ$$

$$\text{Length CD} = 2 \text{ m}$$

$$\text{Length MC} = 2.5 \text{ m (from part b)}$$



$$\begin{aligned} \text{The Area of triangle MCD} &= \frac{1}{2} \times MC \times CD \times \sin C \\ &= \frac{1}{2} \times 2.5 \times 2 \times \sin 86^\circ \end{aligned}$$

M1

$$\text{Area of MCDE} = 2 \times \frac{1}{2} \times 2.5 \times 2 \times \sin 86^\circ = 4.9878... = 5 \text{ m}^2$$

A1

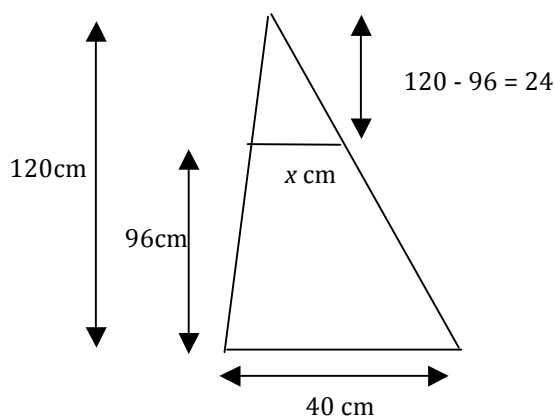
- c.ii. Volume =  $5 \times 0.45 = 2.25 \text{ m}^3$

H1

**Question 2****a.** Using similar triangles

$$\frac{\text{base}}{\text{height}} = \frac{x}{24} = \frac{40}{120}$$

$$x = \frac{40}{120} \times 24 = 8 \text{ cm}$$

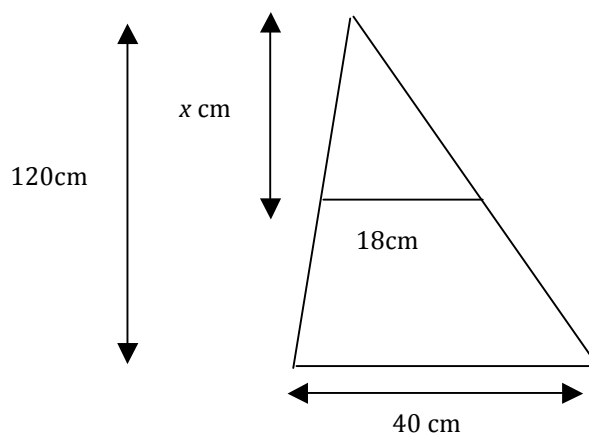


A1

**b.** Using similar triangles

$$\frac{\text{height}}{\text{base}} = \frac{x}{18} = \frac{120}{40}$$

$$x = \frac{120}{40} \times 18 = 54 \text{ cm}$$

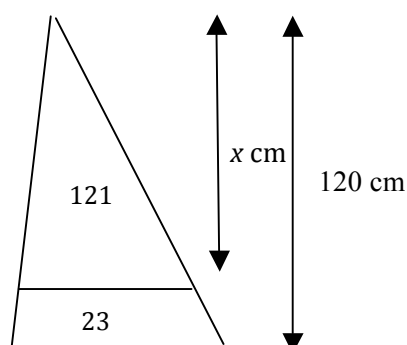
so the middle shelf is  $120 - 54 = 66$  cm above the ground

A1

**c.**

Area ratio of the similar triangles

$$= 121 : (23 + 121) = 121 : 144$$



$$\text{The height ratio} = \sqrt{121} : \sqrt{144} = 11 : 12$$

Attempt at finding length ratio

M1

$$\frac{x}{120} = \frac{11}{12} \quad \text{so } x = \frac{11}{12} \times 120 = 110$$

The height above the bottom shelf is 110 cm

The bottom shelf is  $120 - 110 = 10$  cm above the ground.

A1

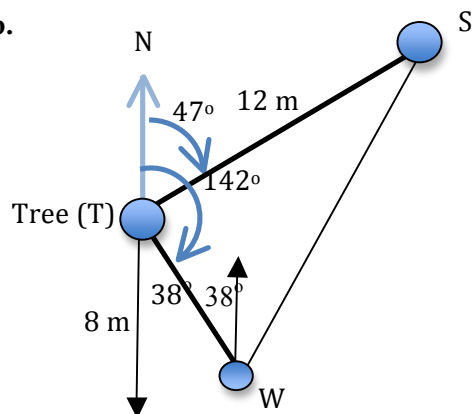
**Question 3**

- a. The angle  $STW = 142 - 47 = 95^\circ$

Using the cosine rule

$$\begin{aligned} \text{distance } SW &= \sqrt{12^2 + 8^2 - 2 \times 12 \times 8 \times \cos 95^\circ} \\ &= 15 \text{ metres} \end{aligned} \quad \text{M1}$$

- b.



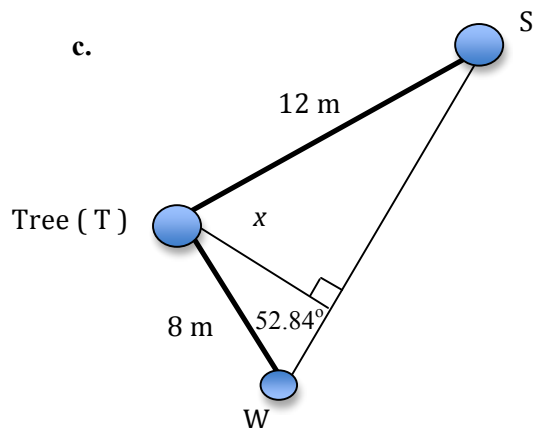
To find Angle TWS use sine rule

$$\sin W = \frac{12 \sin 95^\circ}{15} \quad \text{M1}$$

$$W = \sin^{-1}\left(\frac{12 \sin 95^\circ}{15}\right) = 52.84^\circ \quad \text{M1}$$

$$\begin{aligned} \text{Bearing} &= 52.84^\circ - 38^\circ = 14.84 \\ &= 015^\circ \text{ to the nearest degree} \end{aligned} \quad \text{A1}$$

- c.



The shortest distance from the tree is the line perpendicular to SW from T.

Using triangle TWB

$$\sin 52.84 = \frac{x}{8}$$

$$x = 8 \sin 52.84 = 6.4 \text{ m} \quad \text{A1}$$

The tree is 6 metres high and the shortest distance from the tree to the bench is 6.4 metres. This means that the tree will not hit the bench because its height is less than the shortest distance to the bench.

A1

**Module 3: Graphs and relations****Question 1**

a.  $30 \times \$1.15 + 18 \times 1.25 = \$57$  A1

b.  $\frac{45}{1.05} = 43$  or  $\frac{45}{1.15} = 39$   
 The distributor can purchase 43 kg of tomatoes at \$1.05 per kg.  
 or the distributor can purchase 39 kg of tomatoes at \$1.15 per kg. A1

**Question 2**

a. Find gradient of (260, 32.80) and (315, 75.70)

$$\frac{75.70 - 32.80}{315 - 260} = 0.78$$

78 cents per kilogram A1

b. Find the y-intercept

$$y = mx + c$$

$$32.80 = 0.78 \times 260 + c$$

$$c = -170$$

A loss of \$170 A1

c. Break even occurs when Profit = 0

$$\text{Profit} = 0.78x - 170$$

$$0 = 0.78x - 170$$

$$x = 217.9487$$

To break even 218 kg of apples must be sold. A1

**Question 3**

a.

$$38 + 2y \geq 140$$

$$2y \geq 102$$

$$y \geq 51$$

The minimum number of hectares of apples trimmed is 51. A1

b.  $0.3x + 0.1y \leq 30$  or  $3x + y \leq 300$  A1

c. 150 hectares of land is available to grow strawberries and apples . A1

d. Line A :  $3x + y = 300$

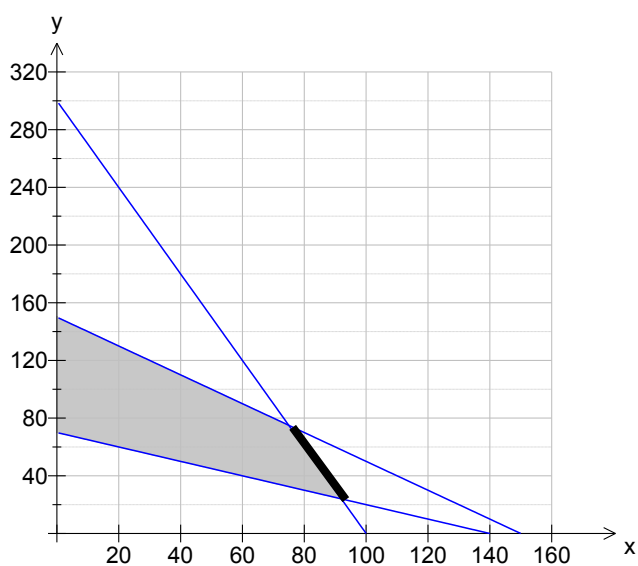
Line B :  $x + y = 150$

Line C :  $x + 2y = 140$

(One correct A1)

All correct A2

e.



Feasible region is shaded

A1

f.

$$\begin{cases} x+y=150 \\ 3x+y=300 \end{cases} \left| \begin{array}{l} x, y \\ \{x=75, y=75\} \end{array} \right.$$

$$\begin{cases} x+2y=140 \\ 3x+y=300 \end{cases} \left| \begin{array}{l} x, y \\ \{x=92, y=24\} \end{array} \right.$$

| Extreme Points | Profit= $345x + 115y$                     |
|----------------|-------------------------------------------|
| (0 , 70)       | $115 \times 70 = \$8050$                  |
| (0 , 150)      | $115 \times 150 = \$17250$                |
| (75 , 75)      | $345 \times 75 + 115 \times 75 = \$34500$ |
| (92 , 24)      | $345 \times 92 + 115 \times 24 = \$34500$ |

Since the points line (75, 75) and (92, 24) both give a maximum profit, then

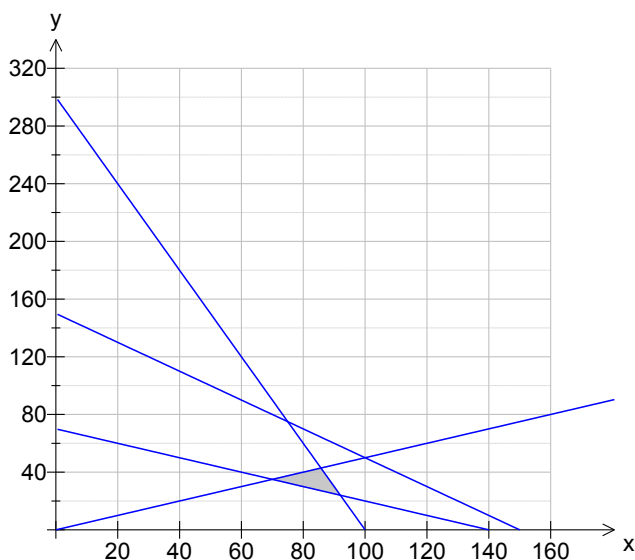
The maximum profit occurs for all points on the line joining the line (75, 75) and (92, 24) inclusive.

(Award A1 if end points given only)

A2

- g. i.  $x \geq 2y$  or  $y \leq \frac{x}{2}$   
 ii.

A1



$$\left\{ \begin{array}{l} x+2y=140 \\ y=x/2 \end{array} \right|_{x,y} \quad \{x=70, y=35\}$$

$$\left\{ \begin{array}{l} x+2y=140 \\ 3x+y=300 \end{array} \right|_{x,y} \quad \{x=92, y=24\}$$

$$\left\{ \begin{array}{l} 3x+y=300 \\ y=x/2 \end{array} \right|_{x,y} \quad \{x=85.71428571, y=42.8571\}$$

The maximum now occurs along the line joining the points  $(600/7, 300/7)$  and  $(92, 24)$  inclusive.

Therefore the least number of hectares of strawberries is  $\frac{600}{7} = 85.7$

A1

**Module 4 Business related mathematics****Question 1**

- a.  $\frac{2.4\%}{12} = 0.2\%$  A1
- b. 0.2% of January balance = 2.70.  
Solving  $\frac{0.2}{100} \times x = 2.70$  gives  $x = 1350$  M1  
Therefore he withdrew  $1525.50 - 1350 = \$175.50$  A1
- c. Balances \$1350, \$1850, \$1500, \$1100, \$2100 A1
- d. January: \$2.70 February:  $0.2\% \times 1350 = \$2.70$  March  $0.2\% \times 1100 = \$2.20$  M1  
Interest earned =  $2.70 + 2.70 + 2.20 = \$7.60$  A1

**Question 2**

- a. Finance Solver  
N = 12  
I(%) = 2.4  
PV = -2500  
Pmt = 0  
FV = ?  
PpY = 12  
CpY = 12  
Future value is \$2560.66 therefore interest earned is  $2560.66 - 2500 = \$60.66$  A1
- b.  $R = 1 + \frac{2.4}{1200} = 1.002$  A1  
 $n = 12$  A1
- c. Finance Solver  
N = 12  
I(%) = 2.4  
PV = -2500  
Pmt = -100  
FV = ?  
PpY = 12  
CpY = 12  
Future value is \$3773.95 A1

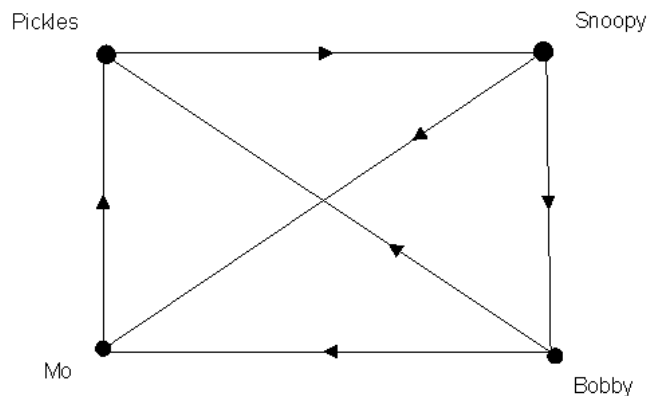
**Question 3**

a. Depreciation per annum =  $\frac{5000 - 400}{5} = 920$   
 Annual depreciation rate =  $\frac{920}{5000} \times 100 = 18.4\%$  A1

b.  $\frac{4600}{5000} \times 100 = 92\%$  A1

c.  $\frac{4600}{0.50} = 9200$  hours  
 $\frac{9200}{5} = 1840$  hours A1

d. Depreciated value = Purchase price  $\times (1 - \frac{\text{depreciation rate}}{100})^{\text{number of years}}$   
 $400 = 5000(1 - \frac{d}{100})^5$  M1  
 $1 - \frac{d}{100} = \sqrt[5]{0.08}$   
 $\frac{d}{100} = 0.39658$   
 depreciation rate = 39.7% A1

**Module 5: Networks and decision mathematics****Question 1****a.**

A1

**b.** Bobby has dominance over Pickles who in turn has dominance over Snoopy

A1

**c.**

| Dominance order | Total     |
|-----------------|-----------|
| Snoopy          | 5 (2 + 3) |
| Bobby           | 4 (2 + 2) |
| Pickles         | 3 (1 + 2) |
| Mo              | 2 (1 + 1) |

Any 2 correct

A1

All 4 correct

A1

**Question 2****a.** Since this would be an Euler circuit and this only exists if the degree of every vertex is even. The degrees of vertices B, C, D and E are all odd.

A1

**b. i.** Any 2 of vertices B, C, D and E.

A1

**ii.** Any appropriate path beginning and ending at two of B, C, D and E

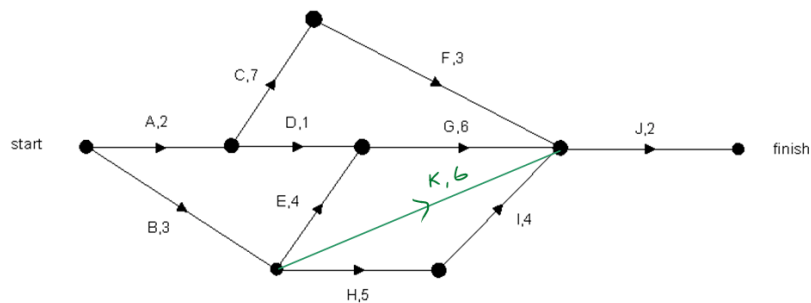
A1

**c.** 3550 metres (along path CBAFEDG)

A1

**Question 3**

- a. ACFJ and BHIJ (both have duration of 14 hours) A1
- b. 15 hours (along path BEGJ) A1
- c. A, C, D, F, H and I A1
- d. F has an earliest start time of 9 and a latest start time of 10  
Float time = latest start time – earliest start time =  $10 - 9 = 1$  M1
- e. 5 (A, C, H, I and F) A1
- f. Arrow as marked on diagram



- g. Initially K does not affect the critical path and has a float time of 4.  
If K takes 5 hours longer it will add 1 to the minimum completion time.  
Answer equals 16 hours (with new critical path BKJ) A1

**Module 6: Matrices****Question 1**

a.  $2 \times \begin{bmatrix} 20 \\ 15 \\ 10 \\ 12 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$  A1

b.  $2 \times \begin{bmatrix} 20 \\ 15 \\ 10 \\ 12 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 41 \\ 31 \\ 21 \\ 25 \end{bmatrix}$   $41+31+21+25= 118$  staff A1

**Question 2**

a.  $\begin{bmatrix} 1 & 1 \\ 250 & 150 \end{bmatrix} \begin{bmatrix} P \\ S \end{bmatrix} = \begin{bmatrix} 5500 \\ 975000 \end{bmatrix}$

Using matrices of correct order

M1

All correct

A1

- b. The determinant is non zero (can be 100 or – 100 depending on the row in which equations are entered) therefore there is a unique solution.

A1

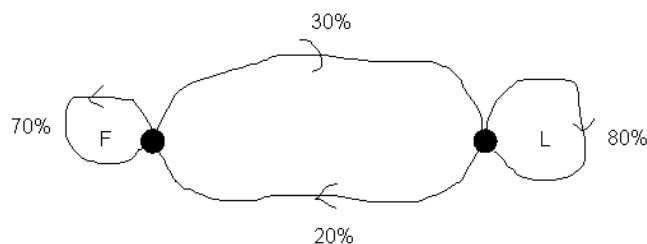
c.

$$\begin{bmatrix} P \\ S \end{bmatrix} = \begin{bmatrix} -1.5 & 0.01 \\ 2.5 & -0.01 \end{bmatrix} \begin{bmatrix} 5500 \\ 975000 \end{bmatrix}$$

A1

- d. Solving from c. gives  $P = 1500$  and  $S = 4000$   
1500 Platinum tickets are sold and 4000 Standard tickets are sold.

A1

**Question 3****a.**

A1

- b.** When he plays Like a Cyclone to open a concert, 80% of the time he will open the next concert with Like a Cyclone

A1

**c.**  $S_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

A1

**d.**  $S_4 = T^3 S_1 = \begin{bmatrix} 0.475 \\ 0.525 \end{bmatrix}$  therefore 52.5% chance

A1

- e.** Using two consecutive large values of  $n$ , the steady state can be found e.g.

$$T^{99} \times S_1 = T^{100} \times S_1 = \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}$$

M1

Chance of opening with Fingerpowder is 40%

- f.** No effect as the steady state is independent of the initial state.

Or demonstrate as in e.

A1

**g.** Let the transition matrix be  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

For this to occur  $\frac{b}{b+c} = 0.75$  and  $\frac{c}{b+c} = 0.25$

M1

Solve to give  $b = 0.75$  and  $c = 0.25$  therefore matrix is  $\begin{bmatrix} 0.75 & 0.75 \\ 0.25 & 0.25 \end{bmatrix}$

A1